# COMPRESSED BEAM-SELECTION IN MILLIMETER WAVE SYSTEMS WITH OUT-OF-BAND PARTIAL SUPPORT INFORMATION

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# ABSTRACT

Compressed beam-selection (CBS) exploits the limited scattering of the millimeter wave (mmWave) channel using compressed sensing and finds the best beam-pair with limited overhead. The CBS procedure can further benefit from the knowledge of some additional structure in the channel. As mmWave systems are envisioned to be deployed in conjunction with sub-6 GHz systems, we use the spatial information extracted at sub-6 GHz as out-of-band side information about the mmWave channel. In particular, we formulate beamselection as a *weighted* sparse signal recovery problem, and obtain the weights using sub-6 GHz angular information. Furthermore, we formalize the notion of spatial congruence between sub-6 GHz and mmWave, and numerically evaluate the degree of spatial congruence necessary for the success of the proposed approach. The simulation results illustrate that the proposed approach reduces the training overhead of the CBS approach by 3x.

*Index Terms*— Millimeter wave communication, analog beamforming, weighted compressed sensing, Root-MUSIC.

# 1. INTRODUCTION

MmWave systems use large antenna arrays and directional beamforming to provide sufficient link margin. Analog beamforming was proposed for mmWave to overcome the hardware limitations that preclude digital beamforming [1,2]. The CBS is one approach to reduce the overhead by exploiting the limited scattering in the mmWave channel [3]. The existing work on CBS (e.g., [4]), however, does not account for other structure in the channel besides sparsity.

MmWave systems will likely be deployed in conjunction with sub-6 GHz systems [5, 6]. As such, it is possible to exploit sub-6 GHz channel information to help establish the mmWave link. In [7], the information retrieved from legacy WiFi was used to reduce the beam steering overhead of 60 GHz WiFi. In [8], the correlation of the mmWave channel was approximated by transforming the sub-6 GHz correlation. The primary focus of [7] were line-of-sight links, whereas [8] considered only a single transmit antenna.

In this paper, we consider non line-of-sight channels with a few multipaths, and use sub-6 GHz angular information for beam-selection via weighted  $\ell_1$ -minimization [9, 10]. The proposed weighted-CBS approach exploits the limited scattering of the channel, as in [4], and leverages the out-ofband information, as in [7,8]. Simulation results show that weighted-CBS reduces the training overhead of exhaustive search and CBS by 128x and 3x, respectively. The success of weighted-CBS relies on the accuracy of the weighting information, which is correlated with the spatial congruence of sub-6 GHz and mmWave. We formalize the notion of spatial congruence and numerically quantify the spatial congruence necessary for the success of weighted-CBS. The weighted  $\ell_1$ -minimization was used in the context of sparse channel estimation for FDD massive-MIMO systems [11]. This work is different from [11] as we bypass explicit channel estimation and focus on beam-selection for analog beamforming.

**Notation:** X is a matrix, x is a vector,  $\mathcal{X}$  is a set, x and X are scalars,  $[\mathbf{x}]_{\mathcal{X}}$  are the elements of x indexed by the index set  $\mathcal{X}$ . Superscript T, \*, and c represent transpose, conjugate transpose, and complex conjugate, respectively.  $\mathbb{E}[\cdot]$  is the expectation, and  $\mathcal{CN}(\mathbf{x}, \mathbf{X})$  is a complex Normal with mean x and covariance X. 0 is the zero-vector, 1 is the one-vector, I is the identity matrix,  $\mathbf{X} \otimes \mathbf{Y}$  is the Kronecker product.  $\|\mathbf{x}\|_p$  is the *p*-norm,  $\|\mathbf{X}\|_F$  is the Frobenius norm,  $|\cdot|$  is the absolute value of the scalar/vector and cardinality of the set,  $\mathbb{1}\{\cdot\}$  is the indicator function. The sub-6 GHz variables are underlined to distinguish them from mmWave.

# 2. JOINT SUB-6 GHz AND MILLIMETER WAVE UPLINK

We consider the setup shown in Fig. 1, where uniform linear arrays (ULA)s of isotropic point sources are used at the userequipment (UE) and the base-station (BS). The ULAs are assumed for ease of exposition, whereas the algorithm developed in this work can be extended to other array geometries. The sub-6 GHz and mmWave arrays are co-located, aligned, and have comparable aperture. The mmWave system has a single RF chain, whereas the sub-6 GHz system has one RF chain per antenna. As such, mmWave system is limited to analog beamforming, whereas fully digital precoding is pos-

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sible in sub-6 GHz. We assume a narrowband signal model for both the sub-6 GHz and the mmWave systems. Further, the coherence time of the channel is long enough to permit (i) retrieving angular information at sub-6 GHz, and (ii) using it for mmWave CBS. This assumption is reasonable with directional beamforming at mmWave [12]. Both sub-6 GHz and mmWave systems operate simultaneously.



**Fig. 1**: The uplink setup with co-located sub-6 GHz and mmWave antenna arrays and a multipath channel.

#### 3. MILLIMETER WAVE SYSTEM

In the training phase, the UE and BS use a sequence of precoding and combining vectors. If the UE uses a training precoding vector  $\mathbf{f}_m$ , and the BS uses a training combining vector  $\mathbf{q}_n$ , then the received signal is

$$y_{n,m} = \mathbf{q}_n^* \mathbf{H} \mathbf{f}_m s_m + \mathbf{q}_n^* \mathbf{v}_{n,m}, \tag{1}$$

where **H** is the  $M_{\rm BS} \times M_{\rm UE}$  channel matrix,  $\mathbf{v}_{n,m}$  is the additive noise,  $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\mathbf{v}}^2 \mathbf{I})$ , and  $s_m$  is the training symbol on the beamforming vector  $\mathbf{f}_m$ . We use  $s_m = \sqrt{E_s}$ , where  $E_s$  is the transmit symbol energy. The UE transmits the training signal on  $N_{\rm UE}$  distinct precoding vectors. For each precoding vector, the BS uses  $N_{\rm BS}$  distinct combining vectors. The subscripts m and n index the distinct precoders and combiners, respectively. Collecting the received signals, we get an  $N_{\rm BS} \times N_{\rm UE}$  measurement matrix

$$\mathbf{Y} = \sqrt{E_s} \mathbf{Q}^* \mathbf{H} \mathbf{F} + \mathbf{V},\tag{2}$$

where  $\mathbf{Q} = [\mathbf{q}_1, \mathbf{q}_2, \cdots, \mathbf{q}_{N_{\mathrm{BS}}}]$  is the  $M_{\mathrm{BS}} \times N_{\mathrm{BS}}$  training combining matrix,  $\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_{N_{\mathrm{UE}}}]$  is the  $M_{\mathrm{UE}} \times N_{\mathrm{UE}}$  training precoding matrix, and  $\mathbf{V}$  is the  $N_{\mathrm{BS}} \times N_{\mathrm{UE}}$  noise matrix.

Due to the limited scattering of the mmWave channel [3], we adopt a geometric channel model for **H**. There are *P* multipaths in the channel, and each path is parameterized by  $\{\alpha_p, \phi_p, \theta_p\}$ , where  $\alpha_p$  is the complex gain (including pathloss), and the variables  $\{\phi_p, \theta_p\} \in (-\frac{\pi}{2}, \frac{\pi}{2}]$  are the physical AoD/AoA. The spatial angles are  $\omega \triangleq \sin(\phi)$  and  $\nu \triangleq \sin(\theta)$ . With these assumptions, the channel **H** is

$$\mathbf{H} = \sqrt{\frac{M_{\rm UE}M_{\rm BS}}{P}} \sum_{p=1}^{P} \alpha_p \mathbf{a}_{\rm BS}(\nu_p) \mathbf{a}_{\rm UE}^*(\omega_p), \qquad (3)$$

where  $\mathbf{a}_{UE}(\omega)$  and  $\mathbf{a}_{BS}(\nu)$  are the array response vectors of the UE and BS. The array response vector of the UE is

$$\mathbf{a}_{\mathrm{UE}}(\omega) = \frac{1}{\sqrt{M_{\mathrm{UE}}}} [1, e^{\mathrm{j}2\pi\Delta\omega}, \cdots, e^{\mathrm{j}(M_{\mathrm{UE}}-1)2\pi\Delta\omega}]^{\mathsf{T}}, \quad (4)$$

where  $\Delta$  is the inter-element spacing in wavelength. The array response vector of the BS is defined in a similar manner.

For analog beamforming, the phase of the signal transmitted from each antenna is controlled by a network of analog phase-shifters. If  $D_{\rm UE} = \log_2(M_{\rm UE})$  bit phase-shifters are used at the UE, and similarly  $D_{\rm BS} = \log_2(M_{\rm BS})$ , then the DFT codebooks can be realized. The *m*th codeword in the DFT codebook for the UE is  $\mathbf{a}_{\rm UE}(\bar{\omega}_m)$ , where  $\bar{\omega}_m = \frac{2m-1-M_{\rm UE}}{M_{\rm UE}}$ ,  $m = 1, 2, \cdots, M_{\rm UE}$ . The DFT codebook for the BS is similarly defined. Collectively the DFT codebook for the UE is  $\mathbf{A}_{\rm UE}$  and for the BS  $\mathbf{A}_{\rm BS}$ . If the DFT codebooks are used in the training phase, then the received measurement matrix (in the absence of noise) is

$$\mathbf{G} = \mathbf{A}_{\mathrm{BS}}^* \mathbf{H} \mathbf{A}_{\mathrm{UE}} \Rightarrow \mathbf{g} = \operatorname{vec}(\mathbf{G}) = (\mathbf{A}_{\mathrm{UE}}^{\mathsf{T}} \otimes \mathbf{A}_{\mathrm{BS}}^*) \operatorname{vec}(\mathbf{H}).$$
(5)

Due to the limited scattering of **H**, it is implicit that **G** is a sparse matrix, and is commonly referred to as the *beamspace* representation of the channel [13]. Under the assumption that  $\{\omega_1, \dots, \omega_p\} \in \{\bar{\omega}_1, \dots, \bar{\omega}_{M_{\text{UE}}}\}$ , and  $\{\nu_1, \dots, \nu_p\} \in \{\bar{\nu}_1, \dots, \bar{\nu}_{M_{\text{BS}}}\}$ , **g** is a *P*-sparse vector. We proceed by assuming that the multipath angles follow the aforementioned model. The index of the largest absolute entry in **g**, i.e.,  $i^* = \arg \max |[\mathbf{g}]_i|$ , determines the best beam-pair (or code-

words). Once the best beam-pair is known, the BS feedbacks the best transmit beam information to the UE. Reconstructing **G** (or g) by exhaustive search as in (5) incurs a training overhead of  $M_{\rm UE} \times M_{\rm BS}$  symbols. The training burden can be reduced by exploiting the sparsity of g. The resulting framework, called CBS, uses a few random measurements of the space to estimate  $i^*$ . Codebooks that randomly sample the space while respecting the analog beamforming constraints were reported in [14], where UE designs its  $M_{\rm UE} \times N_{\rm UE}$ training codebook such that  $[\mathbf{F}]_{n,m} = \frac{1}{\sqrt{M_{\rm UE}}} e^{j\zeta_{n,m}}$ , where  $\zeta_{n,m}$  is randomly and uniformly selected from the set of quantized angles  $\{0, \frac{2\pi}{2^{D} \text{UE}}, \cdots, \frac{2\pi (2^{D} \text{UE} - 1)}{2^{D} \text{UE}}\}$ . The BS similarly designs its  $M_{\rm BS} \times N_{\rm BS}$  training codebook **Q**. The received signal matrix **Y** in (2) is vectorized to get

$$\mathbf{y} = \sqrt{E_s} (\mathbf{F}^\mathsf{T} \otimes \mathbf{Q}^*) \operatorname{vec}(\mathbf{H}) + \operatorname{vec}(\mathbf{V}),$$
$$\stackrel{(b)}{=} \sqrt{E_s} (\mathbf{F}^\mathsf{T} \otimes \mathbf{Q}^*) (\mathbf{A}_{\mathrm{UE}}^c \otimes \mathbf{A}_{\mathrm{BS}}) \mathbf{g} + \operatorname{vec}(\mathbf{V}), \quad (6)$$

where in (b) we used (5) to note that  $\operatorname{vec}(\mathbf{H}) = (\mathbf{A}_{UE}^c \otimes \mathbf{A}_{BS})\mathbf{g}$ . For notational simplicity, we introduce the measurement matrix  $\mathbf{\Phi} = \sqrt{E_s}(\mathbf{F}^T \otimes \mathbf{Q}^*)$ , and the dictionary matrix  $\mathbf{\Psi} = (\mathbf{A}_{UE}^c \otimes \mathbf{A}_{BS})$ . The proposed weighted-CBS approach

recovers g (or equivalently  $i^*$ ), by solving the weighted  $\ell_1$ -minimization problem

minimize 
$$\|\mathbf{g}\|_{\mathbf{w},1}$$
  
subject to  $\|\mathbf{y} - \boldsymbol{\Phi} \boldsymbol{\Psi} \mathbf{g}\|_2 \le \epsilon,$  (P1)

where  $\|\mathbf{g}\|_{\mathbf{w},1} = \sum_{i=1}^{M_{\rm UE}M_{\rm BS}} [\mathbf{w}]_i | [\mathbf{g}]_i |$ , and  $\epsilon$  is the upper bound on noise contribution in (6) [10],  $\mathbf{w} \in \mathbb{R}^{M_{\rm UE}M_{\rm BS}}$  is the weighting vector that captures the prior information about non-uniformity in the support of  $\mathbf{g}$ . In absence of prior information about the support set, (P1) can be solved using uniform weights  $\mathbf{w} = \delta \mathbf{1}$ , where  $0 < \delta <= 1$ , i.e., the CBS approach. In the following section, we outline a strategy to extract the weighting information from sub-6 GHz.

#### 4. SUB-6 GHz SYSTEM

We underline all sub-6 GHz variables to distinguish them from mmWave. There are  $\underline{M}_{\rm UE}$  antennas at UE and  $\underline{M}_{\rm BS}$  antennas at the BS. The uplink received signal has the form

$$\underline{\mathbf{r}} = \underline{\mathbf{H}}\underline{\mathbf{f}}\underline{s} + \underline{\mathbf{v}},\tag{7}$$

where all variables are defined analogous to the mmWave case. We proceed by making a simplistic assumption that sub-6 GHz also has P multipaths. The implications of violating this assumption are discussed briefly at the end of this section. The sub-6 GHz channel has P multipaths that are parameterized by  $\{\underline{\alpha}_p, \underline{\theta}_p, \underline{\phi}_p\}$ . In the training phase, UE transmits the symbols  $\underline{s}_m = \sqrt{\underline{E}_s}$  using  $\underline{M}_{\text{UE}}$  orthogonal precoding vectors  $\underline{\mathbf{f}}_m$ . Using the columns of  $\underline{M}_{\text{UE}} \times \underline{M}_{\text{UE}}$  identity matrix as precoding vectors, we collect the  $\underline{M}_{\text{UE}}$  received signals (that is one snapshot of the channel) in an  $\underline{M}_{\text{UE}}\underline{M}_{\text{BS}} \times 1$  vector  $\underline{\tilde{\mathbf{r}}} = [\underline{\mathbf{r}}_1^{\mathsf{T}}, \underline{\mathbf{r}}_2^{\mathsf{T}}, \cdots, \underline{\mathbf{r}}_{\underline{M}_{\text{UE}}}^{\mathsf{T}}]^{\mathsf{T}}$ .

With no analog constraints at sub-6 GHz, the multipath angles can be estimated using typical signal subspace algorithms. In this work, we use the Double Root-MUSIC algorithm (DRM) [15], which can recover  $P \leq \underline{M}_{BS}(\underline{M}_{UE} - 1)$  multipaths, and automatically pairs the AoDs and AoAs. The DRM algorithm requires the channel correlation matrix, which can be estimated by an ensemble average based on a few snapshots of the channel.

Once the angle estimates are available, we use Algorithm 1 for calculating w. The main idea behind weighting in (P1) is to heavily penalize the entries in g that are likely to be zero, and vice versa. The proposed algorithm obtains such weights by incorporating, (i) the mismatch in sub-6 GHz and mmWave multipaths, and (ii) the success probability of the DRM algorithm. If we define the set of mmWave multipath angles as  $\mathcal{A} = \{(\omega_1, \nu_1), \cdots, (\omega_P, \nu_P)\}$ , and the set of sub-6 GHz multipath angles as  $\mathcal{A} = \{(\omega_1, \nu_1), \cdots, (\omega_P, \nu_P)\}$ , then the probability of mismatch  $\rho_{mis}$  is

$$\rho_{\rm mis} = 1 - \frac{1}{P} \mathbb{E} \left[ |\mathcal{A} \cap \underline{\mathcal{A}}| \right],\tag{8}$$

and the success probability of the DRM algorithm is

$$\rho_{\rm sdrm} = \mathbb{E}\left[\mathbb{1}\left\{|\underline{\omega}_p - \underline{\hat{\omega}}_p| < M_{\rm UE}^{-1}, |\underline{\nu}_p - \underline{\hat{\nu}}_p| < M_{\rm BS}^{-1}\right\}\right].$$
(9)

In Algorithm 1,  $\mathcal{P}$  is the index set for the entries that are likely to be nonzero. For the ideal case (no mismatch and 100% success rate of DRM),  $\rho_{\rm sdrm}(1-\rho_{\rm mis}) = 1$ , and entries indexed by  $\mathcal{P}$  are not penalized. Further, when  $\rho_{\rm sdrm}(1-\rho_{\rm mis}) = 1 - \rho_{\rm sdrm}(1-\rho_{\rm mis}) = 0.5$ , weighted-CBS reduces to CBS. We assume that the accurate estimates of  $\rho_{\rm mis}$ and  $\rho_{\rm sdrm}$  are available, and leave the empirical estimation of these probabilities to a future work.

Algorithm 1 Weight vector calculation

**Input:** The multipath angle estimates  $(\underline{\hat{\omega}}_p, \underline{\hat{\nu}}_p)$ ,  $p = 1, 2, \cdots, P$ , success probability of DRM  $\rho_{\text{sdrm}}$ , and mismatch probability  $\rho_{\text{mis}}$ .

Output: The weight vector, w.

- 1: Use  $(\underline{\hat{\omega}}_p, \underline{\hat{\nu}}_p)$  with  $\hat{\alpha}_p = 1$  in (3) to get  $\hat{\mathbf{H}}$ .
- 2: Calculate initial weights  $\mathbf{w}_{\text{init}} = |\Psi^* \operatorname{vec}(\hat{\mathbf{H}})|$ .
- 3: Populate the index set  $\mathcal{P}$ , with the indices of P largest elements of  $\mathbf{w}_{init}$ .
- 4: Calculate weight vector as  $[\mathbf{w}]_{\mathcal{P}} = 1 \rho_{\text{sdrm}}(1 \rho_{\text{mis}})$ and  $[\mathbf{w}]_{\{1,2,\cdots,M_{\text{UE}} \times M_{\text{BS}}\} \setminus \mathcal{P}} = \rho_{\text{sdrm}}(1 - \rho_{\text{mis}}).$

The sub-6 GHz channel is expected to have more multipaths compared to the mmWave channel, in part due to lower pathloss and better propagation conditions. In this case  $\rho_{sdrm}$  will typically decrease and so will the gains of the weighted-CBS. That said, if the weighting is more than 50% accurate, then the weighted-CBS approach will perform better than the CBS approach [10]. For clustered multipaths, the mean angle and angle spread can be estimated, e.g., using [16], instead of estimating all paths individually. In this case, the proposed weighted-CBS approach can be used with appropriate modifications in weight vector calculation.

## 5. SPATIAL CONGRUENCE

Some mismatch between sub-6 GHz and mmWave characteristics is expected, e.g., the delay spread varies with frequency [17]. The spatial characteristics, however, are more consistent. In [18], the spatial characteristics of 5.8 GHz, 14.8 GHz, and 58.7 GHz channels were reported to be almost identical. The measurement results in [7] also confirm the value of sub-6 GHz angular information for mmWave link establishment. As such, we expect sufficient (albeit not perfect) congruence between sub-6 GHz and mmWave.

#### 6. SIMULATION RESULTS

The simulation parameters are given in Table 1. The transmission power for both sub-6 GHz and mmWave is 37 dBm and the UE-BS separation is 400 m. The bandwidth of the sub-6 GHz and mmWave system is 15 MHz and 85 MHz respectively. The pathloss is calculated based on center frequencies of sub-6 GHz and mmWave, with pathloss coefficients 2.5 and 3, respectively. We use  $\alpha_p \sim C\mathcal{N}(0, \sigma_{\alpha}^2)$ . The sub-6 GHz correlation is estimated using 100 snapshots of the channel and the problem (P1) is implemented in CVX [19]. We

Table 1. Simulation I arameters	Table 1	: Simu	lation I	Parameters
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Parameter	Value	Parameter	Value
$M_{\rm UE}$	16	$M_{\rm BS}$	32
$\underline{M}_{\rm UE}$	2	$\underline{M}_{\rm BS}$	4
f	$28~\mathrm{GHz}$	f	$3.5~\mathrm{GHz}$
Δ	1/2	$\underline{\Delta}$	1/2
$D_{\rm UE}$	4	$D_{\rm BS}$	5

use effective achievable rate  $R_{\rm eff}$  as the performance metric, which is defined as

$$R_{\rm eff} = \frac{1}{T} \sum_{t=1}^{T} \eta \log_2(1 + |\mathbf{a}_{\rm BS}^*(\bar{\nu}_{\hat{n}})\mathbf{H}\mathbf{a}_{\rm UE}(\bar{\omega}_{\hat{m}})|^2 \text{SNR}), \quad (10)$$

where  $\eta \triangleq \max(0, 1 - (N_{\rm UE} \times N_{\rm BS})/L_{\rm H})$ ,  $L_{\rm H}$  is the channel coherence time,  $\hat{n}$  and  $\hat{m}$  are the estimated transmit and receive codeword indices, and T = 1000 is the number of independent trials for ensemble averaging.

The rate results as a function of the number of measurements are plotted in Fig. 2, for three channel coherence values and  $\rho_{\rm mis} = 0.2$ . We do not assume any loss in the rate of weighted-CBS for correlation construction, as channel correlation can be constructed in the data transmission phase. For  $L_{\rm H} = 1000$  case, the CBS obtains its highest rate with 98 measurements, whereas weighted-CBS attains a better rate with 32 measurements, implying a 3x training overhead reduction. Further, for the same  $L_{\rm H} = 1000$  case, weighted-CBS outperforms exhaustive search with only 4 measurements, implying a 128x training overhead reduction.



Fig. 2: The effective achievable rate of weighted-CBS against the number of measurements. ( $\rho_{mis} = 0.2$ ).

The weighted-CBS approach relies on the weighting information. Hence, we assess the degree of spatial congruence, between sub-6 GHz and mmWave, required for the success of weighted-CBS. In Fig. 3, we plot the rate as a function of the probability of mismatch  $\rho_{\rm mis}$ . It is observed that in practice for  $\rho_{\rm mis}$  larger than 0.43, the weighting has a detrimental effect on the recovery. With  $\rho_{\rm sdrm}$  factored in (which is empirically estimated to be 0.89), the weight vector entries for  $\rho_{\rm mis} = 0.43$  are  $\mathbf{w}_{\mathcal{P}} = 0.4927 \approx 0.5$  and  $\mathbf{w}_{\{1,2,\cdots,M_{\rm UE}M_{\rm BS}\}\setminus\mathcal{P}} = 0.5073 \approx 0.5$ , which is close to uniform weighting, i.e., CBS. If perfect sub-6 GHz angle estimation is assumed, then weighted-CBS performs better than CBS with  $\rho_{\rm mis} < 0.5$ , which is consistent with earlier theoretical findings [10].



Fig. 3: The effective achievable rate of weighted-CBS against mismatch probability. ( $N_{\rm UE} \times N_{\rm BS} = 36, L_{\rm H} = \infty$ ).

# 7. CONCLUSION AND FUTURE WORK

We proposed weighted compressed beam-selection for mmWave systems. For co-located and aligned (sub-6 GHz and mmWave) arrays of comparable aperture, the proposed approach exploits the spatial information of sub-6 GHz channel (obtained via Double Root-MUSIC) for mmWave compressed beam-selection. The rate results showed that with channel coherence of 1000 symbols, the proposed approach reduces the training overhead of exhaustive search and compressed beam-selection by 128x and 3x, respectively. The proposed approach provides benefit over compressed beam-selection if the reliability of weighting information is more than 50%. The directions for future work include off-the-grid AoDs and AoAs, extensions to wideband channels, and incorporating angle spread.

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