TIME-MULTIPLEXED / SUPERIMPOSED PILOT SELECTION FOR MASSIVE MIMO PILOT DECONTAMINATION

*Karthik Upadhya** *Sergiy A. Vorobyov** *Mikko Vehkapera*†

*Department of Signal Processing and Acoustics, Aalto University, Espoo, Finland
†EEE Department, University of Sheffield, Sheffield, United Kingdom
E-mails: karthik.upadhya@aalto.fi, svor@ieee.org, m.vehkapera@sheffield.ac.uk

ABSTRACT

In massive multiple-input multiple-output (MIMO) systems, superimposed (SP) and time-multiplexed (TM) pilots exhibit a complementary behavior, with the former and latter schemes offering a higher throughput in high and low inter-cell interference scenarios, respectively. Based on this observation, in this paper, we propose an algorithm for partitioning users into two disjoint sets comprising users that transmit TM and SP pilots. This selection of user sets is accomplished by minimizing the total inter-cell and intra-cell interference, and since this problem is found to be non-convex, a greedy approach is proposed to perform the partitioning. Based on simulations, it is shown that the proposed method is versatile and offers an improved performance in both high and low-interference scenarios.

***Index Terms*—** Massive MIMO, pilot decontamination, superimposed pilots, hybrid system, pilot selection

1. INTRODUCTION

Channel training and estimation is a critical component of any coherent transceiver. The same holds true for massive multiple-input multiple-output (MIMO) systems, which have been touted as a potential candidate for fifth generation wireless communication systems [1–3]. Existing schemes for channel training in a time-division duplexing (TDD) massive MIMO system employ time-multiplexed pilots and data (henceforth referred to as time-multiplexed (TM) pilots), wherein a subset of the symbols in the uplink (UL) time slot are reserved for pilot transmission. Maintaining high transmission efficiency necessitates the reuse of pilot sequences across cells, which leads to a phenomenon called 'pilot contamination' that limits the UL and downlink (DL) transmission efficiency [1,4,5]. Methods for mitigating pilot contamination utilize additional information about pilot transmissions, such as asymptotic orthogonality of user channels, non-overlapping user angle spread at the base station (BS), coordination between BSs, forward error correction (FEC) code diversity, pilot assignment, power control, and pilot reuse to assign unique signatures to users in order to improve their channel separability [6–15].

Superimposed (SP) pilots have recently been introduced as an alternative pilot structure for massive MIMO [16, 17]. Since superimposed pilots do not require a separate set of symbols for transmitting pilots, they offer a larger set of orthogonal pilots and therefore, do not need to be reused as often as TM pilots. This allows SP pilots to offer superior UL and DL throughput in high interference scenarios, when compared to its TM counterpart [17, 18]. However, in low inter-cell interference scenarios, TM pilots are superior since the interference from data that is transmitted alongside SP pilots results in a ceiling on its throughput.

In this paper[1], we utilize the complementary behavior of TM and SP pilots to develop an approach for selecting the type of pilot that is transmitted by a particular user. In order to perform this selection, we propose a novel framework that is based on minimizing the total inter-cell and intra-cell interference. Based on simulations, we show that the hybrid system offers a performance that is robust to interference when compared to systems that employ only TM or SP pilots.

2. SYSTEM MODEL

We consider a TDD massive MIMO system with L cells and K users per cell. Each cell has a BS with $M \gg K$ antennas. The number of symbols over which the channel is coherent C is divided into C_u and C_d symbols for the UL and DL time slots, respectively. The channel is assumed to be static within the coherence block and realizations are assumed to be independent between coherence blocks. Using the tuple (ℓ, k) to denote user k in cell ℓ, the received signal in the UL at the j'th BS $\mathbf{Y}_j \in \mathbb{C}^{M \times C_u}$ can be written as

$$\mathbf{Y}_j = \sum_{\ell=0}^{L-1} \sum_{k=0}^{K-1} \sqrt{\mu_{\ell,k}} \mathbf{h}_{j,\ell,k} \mathbf{s}_{\ell,k}^T + \mathbf{W}_j \tag{1}$$

where $(\cdot)^T$ denotes the transpose, $\mathbf{s}_{\ell,k} \in \mathbb{C}^{C_u \times 1}$ and $\mu_{\ell,k}$ are the transmitted symbol and the UL transmit power, respectively, of user (ℓ, k), $\mathbf{W}_j \in \mathbb{C}^{M \times C_u}$ is the matrix corresponding to additive white Gaussian noise at the BS with each column mutually independent of the other columns and distributed as $\mathcal{CN}\left(\mathbf{0}, \sigma^2 \mathbf{I}_M\right)$, $\mathbf{h}_{j,\ell,k} \in \mathbb{C}^{M \times 1}$ is the channel vector between user (ℓ, k) and BS j. The channel vector $\mathbf{h}_{j,\ell,k}$ is assumed to be distributed as $\mathcal{CN}(0, \beta_{j,\ell,k} \mathbf{I}_M)$ with $\beta_{j,\ell,k}$ denoting the large-scale path-loss coefficient.[2] The parameter $\mu_{\ell,k}$ is chosen using the statistics-aware power control scheme [11], i.e., $\mu_{\ell,k} \triangleq \omega / \beta_{j,\ell,k}$ where ω is a design parameter chosen such that each user satisfies its power constraint. When this power control scheme is employed, the effective path-loss coefficient between the user and the BS can be written as $\bar{\beta}_{j,\ell,k} \triangleq \mu_{\ell,k} \beta_{j,\ell,k}$. In the rest of the paper, we drop the over-bar in $\bar{\beta}_{j,\ell,k}$.

If a matched filter (MF) based precoder is used in the DL and if $d_{\ell,k}$ is the symbol transmitted by BS ℓ to its k'th user, the estimate

[1]This paper is a condensed version of our submitted journal paper [19].

[2]We do not consider shadowing in this paper. However, the framework and the algorithm are valid in the presence of shadowing, provided each user is associated with its strongest BS.

of the data at user (j,m) can be written as

$$\widehat{d}_{j,m} = \frac{1}{M}\left(\sum_{\ell=0}^{L-1} \mathbf{h}_{\ell,j,m}^T \sum_{k=0}^{K-1} \widehat{\mathbf{h}}_{\ell,\ell,k}^* d_{\ell,k} + \eta_{j,m}\right) \tag{2}$$

where $\widehat{\mathbf{h}}_{\ell,\ell,k}$ is an estimate of $\mathbf{h}_{\ell,\ell,k}$, $(\cdot)^*$ denotes the complex conjugate, and $\eta_{j,m}$ is the additive noise at the user terminal that is distributed as $\mathcal{CN}\left(0,\sigma^2\right)$.

3. EXISTING CHANNEL TRAINING SCHEMES

In this section, we briefly review TM and SP pilots and their UL and DL signal-to-interference-plus-noise ratio (SINR) performance.

3.1. Time-multiplexed Pilots

When TM pilots are employed, each user in a cell transmits a $\tau \geq K$ length orthogonal pilot followed by UL data. Assuming that all pilot transmissions are synchronized, the least squares (LS) estimate of the channel in the UL can be found as [1, 17]

$$\widehat{\mathbf{h}}_{j,j,m}^{\mathrm{TM}} = \mathbf{h}_{j,j,m} + \sum_{\substack{\ell \neq j \\ \ell \in \mathcal{L}_j(r)}} \mathbf{h}_{j,\ell,m} + \mathbf{w}_{j,m} \tag{3}$$

where $\mathcal{L}_j(r)$ is the set of cells that use the same pilots as cell j, $\mathbf{w}_{j,m} = \mathbf{W}_j \boldsymbol{\phi}_{j,m}^*/\tau$, $\boldsymbol{\phi}_{j,m}$ is the pilot sequence transmitted by user (j,m), and $(\cdot)^*$ denotes the complex conjugate. From (3), it can be seen that the estimate of the channel is contaminated by the channel vectors of the users in the neighboring cells that use the same pilots. The UL and DL SINRs when this channel estimate is used in an MF-based detector and precoder, respectively, and when $M \to \infty$ can be obtained as [1]

$$\mathrm{SINR}_{j,m}^{\mathrm{TM-ul}} = \frac{\beta_{j,j,m}^2}{\sum_{\substack{\ell \neq j \\ \ell \in \mathcal{L}_j(r)}} \beta_{j,\ell,m}^2} \tag{4}$$

$$\mathrm{SINR}_{j,m}^{\mathrm{TM-dl}} = \frac{\beta_{j,j,m}^2}{\sum_{\substack{\ell \neq j \\ \ell \in \mathcal{L}_j(r)}} \beta_{\ell,j,m}^2} \,. \tag{5}$$

3.2. Superimposed Pilots

When SP pilots are employed, each user in the cell transmits pilots at a reduced power alongside the data, i.e., $\mathbf{s}_{j,m} = \rho \mathbf{x}_{j,m} + \lambda \mathbf{p}_{j,m}$, where $\mathbf{x}_{j,m} \in \mathbb{C}^{C_u \times 1}$ and $\mathbf{p}_{j,m} \in \mathbb{C}^{C_u \times 1}$ are the data and pilot vectors transmitted by user (j,m), respectively, with transmit powers ρ^2 and λ^2 chosen such that $\rho^2 + \lambda^2 = 1$. The pilot vectors are taken from the columns of an orthogonal matrix $\mathbf{P} \in \mathbb{C}^{C_u \times C_u}$. The LS estimate of the channel when the users transmit SP pilots can be written as [16, 17]

$$\widehat{\mathbf{h}}_{j,\ell,k}^{\mathrm{SP}} = \mathbf{h}_{j,\ell,k} + \frac{\rho}{C_u \lambda} \sum_{n=0}^{L-1} \sum_{p=0}^{K-1} \mathbf{h}_{j,n,p} \mathbf{x}_{n,p}^T \mathbf{p}_{\ell,k}^* - \frac{\mathbf{W}_j \mathbf{p}_{\ell,k}^*}{C_u \lambda} \,. \tag{6}$$

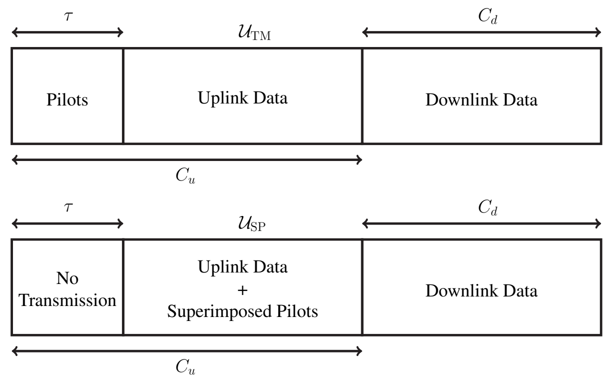

Fig. 1. Frame structure of a hybrid system with users employing TM and SP pilots.

It has been shown in [17] that the values of ρ and λ can be chosen to maximize a lower bound on the UL sum rate as

$$\rho^2 = \left(1 + \sqrt{\frac{M+LK}{C_u}}\right)^{-1} \tag{7}$$

$$\lambda^2 = 1 - \rho^2 = \left(1 + \sqrt{\frac{C_u}{M+LK}}\right)^{-1} \,. \tag{8}$$

Then, the UL and DL SINR of SP pilots, when $M \to \infty$, can be obtained as [17–19]

$$\mathrm{SINR}_{j,m}^{\mathrm{SP-ul}} = \frac{\lambda_{j,m}^2 \rho_{j,m}^2 \beta_{j,j,m}^2}{\frac{1}{C_u} \sum_{\ell=0}^{L-1} \sum_{k=0}^{K-1} \rho_{\ell,k}^2 \mu_{\ell,k} \beta_{j,\ell,k}^2} \tag{9}$$

$$\mathrm{SINR}_{j,m}^{\mathrm{SP-dl}} = \frac{\sqrt{C_u\,(M+LK)}\beta_{j,j,m}^2}{\sum_{\ell=0}^{L-1} \sum_{k=0}^{L-1} \beta_{\ell,j,m}^2} \tag{10}$$

and the corresponding rates in the UL and DL can be written as

$$R_{j,m}^{\mathrm{SP-ul}} = \frac{C_u}{C} \log_2\left(1 + \mathrm{SINR}_{j,m}^{\mathrm{SP-ul}}\right) \tag{11}$$

$$R_{j,m}^{\mathrm{TP-dl}} = \frac{C_d}{C} \log_2\left(1 + \mathrm{SINR}_{j,m}^{\mathrm{SP-dl}}\right) \,. \tag{12}$$

In the rest of the paper, for the sake of clarity and convenience, we use the non-iterative method for channel estimation described in [16, 17]. With suitable modifications, the approach can be extended to the case when iterative methods are used.

4. PILOT SELECTION

With the hybrid system, a user transmits either TM or SP pilots. As shown in Fig. 1, users in $\mathcal{U}_{\mathrm{TM}}$ transmit TM pilots for τ symbols followed by UL data. Users in $\mathcal{U}_{\mathrm{SP}}$ maintain radio silence for τ symbols and then transmit SP pilots and data. In this paper, given a set of K users per cell and the path-loss coefficients $\beta_{j,\ell,k}, \forall\, j,\ell,k$, we aim at partitioning the users into disjoint sets $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$ by minimizing the total inter-cell and intra-cell interference.

The received signal at BS j for the proposed pilot system can be written as

$$\mathbf{Y}_j = \mathbf{Y}_j^{\mathrm{TM}} + \mathbf{Y}_j^{\mathrm{SP}} + \mathbf{W}_j \tag{13}$$

where $\mathbf{Y}_j^{\mathrm{TM}}$ and $\mathbf{Y}_j^{\mathrm{SP}}$ are the received signals from the users in $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$, respectively. From Fig. 1, $\mathbf{Y}_j^{\mathrm{TP}}$ and $\mathbf{Y}_j^{\mathrm{SP}}$ can be written as

$$\mathbf{Y}_j^{\mathrm{TM}} \triangleq \sum_{\substack{\ell=0 \\ (\ell,k)\in\mathcal{U}_{\mathrm{TM}}}}^{L-1} \sum_{k=0}^{K-1} \mathbf{h}_{j,\ell,k} \left[\boldsymbol{\phi}_{\ell,k}^T, \sqrt{p_u}\mathbf{x}_{\ell,k}^T\right] \tag{14}$$

$$\mathbf{Y}_j^{\mathrm{SP}} \triangleq \sum_{\substack{\ell=0 \\ (\ell,k)\in\mathcal{U}_{\mathrm{SP}}}}^{L-1} \sum_{k=0}^{K-1} \mathbf{h}_{j,\ell,k} \left[\mathbf{0}_{1\times\tau}, \rho\mathbf{x}_{\ell,k}^T + \lambda\mathbf{p}_{\ell,k}^T\right] \tag{15}$$

where the users in $\mathcal{U}_{\mathrm{TM}}$ transmit data at power p_u. Using the LS estimates of the channels for the users in $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$, which can be obtained similar to (3) and (6), respectively, the UL and DL SINRs of users in $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$, when $M \to \infty$, can be obtained as [1,17,18]

$$\mathrm{SINR}_{j,m}^{\mathrm{TM-ul}} = \frac{\beta_{j,j,m}^2}{\sum\limits_{\substack{\ell\neq j \\ \ell\in\mathcal{L}_j(r) \\ (\ell,m)\in\mathcal{U}_{\mathrm{TM}}}} \beta_{j,\ell,m}^2} \tag{16}$$

$$\mathrm{SINR}_{j,m}^{\mathrm{TM-dl}} = \frac{\beta_{j,j,m}^2}{\sum\limits_{\substack{\ell\neq j \\ \ell\in\mathcal{L}_j(r) \\ (\ell,m)\in\mathcal{U}_{\mathrm{TM}}}} \beta_{\ell,j,m}^2} \tag{17}$$

$$\mathrm{SINR}_{j,m}^{\mathrm{SP-ul}} \approx \frac{\beta_{j,j,m}^2}{\frac{1}{(C_u-\tau)\lambda^2} \sum\limits_{\substack{\ell=0 \\ (\ell,k)\in\mathcal{U}_{\mathrm{SP}}}}^{L-1} \sum\limits_{k=0}^{K-1} \beta_{j,\ell,k}^2} \tag{18}$$

$$\mathrm{SINR}_{j,m}^{\mathrm{SP-dl}} \approx \frac{\beta_{j,j,m}^2}{\frac{\rho^2}{(C_u-\tau)\lambda^2} \sum\limits_{\substack{\ell=0 \\ (\ell,k)\in\mathcal{U}_{\mathrm{SP}}}}^{L-1} \sum\limits_{k=0}^{K-1} \beta_{\ell,j,m}^2} \tag{19}$$

where the approximations in (18) and (19) have been made assuming that the users in $\mathcal{U}_{\mathrm{TM}}$ do not interfere with the users in $\mathcal{U}_{\mathrm{SP}}$.[3] [4] Since, by design, the users in $\mathcal{U}_{\mathrm{SP}}$ do not interfere with the transmission of $\mathcal{U}_{\mathrm{TM}}$, the transmissions of both sets of users can be considered to be independent of each other. In addition, for the sake of simplicity, it is assumed that M is large enough such that the above expressions are valid.

In order to obtain an approach to partition the users into $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$, we define the following terms. Let $I_{j,m}^{\mathrm{TM-ul}}$ and $I_{j,m}^{\mathrm{TM-dl}}$ be the total interference in the UL and DL, respectively, caused by user (j,m) when assigned to $\mathcal{U}_{\mathrm{TM}}$. Similarly, let $I_{j,m}^{\mathrm{SP-ul}}$ and $I_{j,m}^{\mathrm{SP-dl}}$ be the total interference in the UL and DL, respectively, caused by user (j,m) when assigned to $\mathcal{U}_{\mathrm{SP}}$. Then, from the denominators of (16), (17), (18), and (19), $I_{j,m}^{\mathrm{TM-ul}}$, $I_{j,m}^{\mathrm{TM-dl}}$, $I_{j,m}^{\mathrm{SP-ul}}$, and $I_{j,m}^{\mathrm{SP-dl}}$ can be obtained as [19, Section IV.A]

$$I_{j,m}^{\mathrm{TM-ul}} = \sum_{\substack{\ell\neq j \\ \ell\in\mathcal{L}_j(r) \\ (\ell,k)\in\mathcal{U}_{\mathrm{TM}}}} \sum_{k=0}^{K-1} \beta_{\ell,j,k}^2 \delta_{m,k} = \sum_{\substack{\ell\neq j \\ \ell\in\mathcal{L}_j(r) \\ (\ell,m)\in\mathcal{U}_{\mathrm{TM}}}} \beta_{\ell,j,m}^2 \tag{20}$$

$$I_{j,m}^{\mathrm{TM-dl}} = \sum_{\substack{n\neq j \\ n,\ell\in\mathcal{L}_j(r) \\ (n,k)\in\mathcal{U}_{\mathrm{TM}}}} \sum_{\ell} \sum_{k=0}^{K-1} \beta_{n,\ell,k}^2 \delta_{j,\ell}\delta_{m,k} = \sum_{\substack{n\neq j \\ n\in\mathcal{L}_j(r) \\ (n,m)\in\mathcal{U}_{\mathrm{TM}}}} \beta_{n,j,m}^2 \tag{21}$$

$$I_{j,m}^{\mathrm{SP-ul}} = \frac{1}{(C_u-\tau)\lambda^2} \sum_{\substack{\ell=0 \\ (\ell,k)\in\mathcal{U}_{\mathrm{SP}}}}^{L-1} \sum_{k=0}^{K-1} \beta_{\ell,j,m}^2 \tag{22}$$

$$I_{j,m}^{\mathrm{SP-dl}} = \frac{\rho^2}{(C_u-\tau)\lambda^2} \sum_{\substack{\ell=0 \\ (\ell,k)\in\mathcal{U}_{\mathrm{SP}}}}^{L-1} \sum_{k=0}^{K-1} \beta_{\ell,j,m}^2 = \rho^2 I_{j,m}^{\mathrm{SP-ul}}\,. \tag{23}$$

If ξ^{ul} and ξ^{dl} are weights such that $\xi^{\mathrm{ul}} + \xi^{\mathrm{dl}} = 1$, then the total cost due to inter-cell and intra-cell interference can be expressed as

$$\begin{aligned} I\left(\mathcal{U}_{\mathrm{TM}}, \mathcal{U}_{\mathrm{SP}}\right) \triangleq \sum_{\ell=0}^{L-1} \sum_{k=0}^{K-1} \Big(& T_{\ell,k}^{\mathrm{TM}} \mathbf{1}_{\{(\ell,k)\in\mathcal{U}_{\mathrm{TM}}\}} \\ & + T_{\ell,k}^{\mathrm{SP}} \mathbf{1}_{\{(\ell,k)\in\mathcal{U}_{\mathrm{SP}}\}} \Big) \end{aligned} \tag{24}$$

where $T_{\ell,k}^{\mathrm{TM}}$ and $T_{\ell,k}^{\mathrm{SP}}$ are the total costs incurred when user (ℓ,k) is assigned to $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$, respectively, and can be written as

$$T_{\ell,k}^{\mathrm{TM}} \triangleq \xi^{\mathrm{ul}} I_{\ell,k}^{\mathrm{TM-ul}} + \xi^{\mathrm{dl}} I_{\ell,k}^{\mathrm{TM-dl}} \tag{25}$$

$$T_{\ell,k}^{\mathrm{SP}} \triangleq \xi^{\mathrm{ul}} I_{\ell,k}^{\mathrm{SP-ul}} + \xi^{\mathrm{dl}} I_{\ell,k}^{\mathrm{SP-dl}}\,. \tag{26}$$

Using (24) as the objective function, the sets $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$ can be obtained as the solution of the following optimization problem

$$\begin{aligned} (\mathcal{U}_{\mathrm{TM}}, \mathcal{U}_{\mathrm{SP}}) = \arg &\min_{\substack{\mathcal{U}_{\mathrm{TM}}\subseteq\mathcal{U} \\ \mathcal{U}_{\mathrm{SP}}\subseteq\mathcal{U}}} I\left(\mathcal{U}_{\mathrm{TM}}, \mathcal{U}_{\mathrm{SP}}\right) \\ \text{subject to} \quad & \mathcal{U}_{\mathrm{TM}} \cup \mathcal{U}_{\mathrm{SP}} = \mathcal{U} \\ & \mathcal{U}_{\mathrm{TM}} \cap \mathcal{U}_{\mathrm{SP}} = \varnothing \end{aligned} \tag{27}$$

where $\mathcal{U}$ is the set of all users in the system and $\varnothing$ is the null set. However, this optimization problem is combinatorial in nature and requires a search over $2^{|\mathcal{U}|}$ combinations. Alternatively, a greedy approach can be used to partition $\mathcal{U}$ into $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$. At each step, given $\mathcal{U}_{\mathrm{TM}}$ and $\mathcal{U}_{\mathrm{SP}}$, a user $\left(\tilde{\ell}, \tilde{k}\right)$ in $\mathcal{U}_{\mathrm{TM}}$ is chosen as

$$\left(\tilde{\ell}, \tilde{k}\right) = \arg \max_{(\ell,k)\in\mathcal{U}_{\mathrm{TM}}} T_{\ell,k}^{\mathrm{TM}}\,. \tag{28}$$

This user is added to $\mathcal{U}_{\mathrm{SP}}$ if

$$I\left(\mathcal{U}'_{\mathrm{TM}}, \mathcal{U}'_{\mathrm{SP}}\right) \leq I\left(\mathcal{U}_{\mathrm{TM}}, \mathcal{U}_{\mathrm{SP}}\right) \tag{29}$$

where $\mathcal{U}'_{\mathrm{TM}} = \mathcal{U}_{\mathrm{TM}} \backslash \left(\tilde{\ell}, \tilde{k}\right)$ and $\mathcal{U}'_{\mathrm{SP}} = \mathcal{U}_{\mathrm{SP}} \cup \left(\tilde{\ell}, \tilde{k}\right)$. The algorithm is initialized with $\mathcal{U}_{\mathrm{TM}} = \mathcal{U}$ and is terminated when either $\mathcal{U}_{\mathrm{TM}}$ is empty or when (29) is no longer satisfied. The approach detailed above is summarized in Algorithm 1.

5. SIMULATION RESULTS

In this section, we compare the bit error rate (BER) and throughput of the hybrid system with systems employing TM and SP pilots. The simulations are performed with hexagonal cells of 1km diameter in two scenarios (i) *Scenario* 1: The users are uniformly distributed in the cells; (ii) *Scenario* 2: the users in both the reference and interfering cells are in a fixed configuration and are equally spaced on a

[3] In this paper, we assume for the sake of simplicity that p_u is small enough with respect to the transmit powers of the users in $\mathcal{U}_{\mathrm{SP}}$. This does not affect the throughput of the users in $\mathcal{U}_{\mathrm{TM}}$, since their UL and DL SINRs are independent of p_u. In the absence of this assumption, the BS will have to estimate and remove $\mathbf{Y}_j^{\mathrm{TP}}$ before estimating the channels of users in $\mathcal{U}_{\mathrm{TM}}$.

[4] Moreover, it can be seen from the simulation results that the hybrid system outperforms the existing schemes despite using these approximate expressions when selecting the pilots.

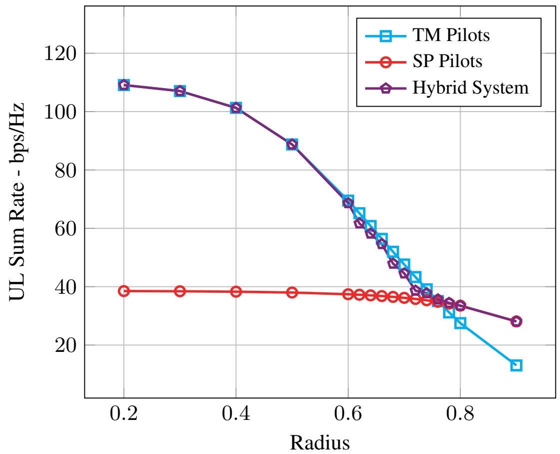

Fig. 2. Sum Rate in the UL over users in the first tier of cells vs. user radius in Scenario 2

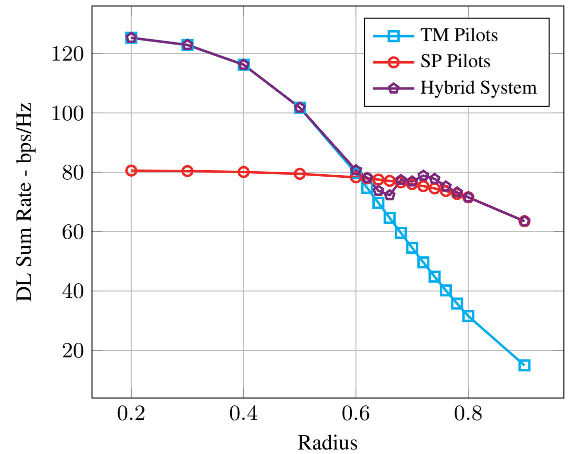

Fig. 3. Sum Rate in the DL over users in the first tier of cells vs. user radius in Scenario 2

Algorithm 1 Greedy algorithm to select $\mathcal{U}_{\rm TM}$ and $\mathcal{U}_{\rm SP}$

Data: $\beta_{j,\ell,k},\ \forall j, \ell = 0,\ldots,L-1,\ \ k=0,\ldots,K-1$
Initialize: $\mathcal{U}_{\rm TM} \leftarrow \mathcal{U}, \mathcal{U}_{\rm SP} \leftarrow \varnothing$
1: Compute $\left(\tilde{\ell},\tilde{k}\right)$ as in (28)
2: Set $\mathcal{U}'_{\rm TM} \leftarrow \mathcal{U}_{\rm TM} \backslash \left(\tilde{\ell},\tilde{k}\right)$ and $\mathcal{U}'_{\rm SP} \leftarrow \mathcal{U}_{\rm SP} \cup \left(\tilde{\ell},\tilde{k}\right)$
3: **if** $\mathcal{U}_{\rm TM} \neq \varnothing$ **and if** $I\left(\mathcal{U}'_{\rm TM},\mathcal{U}'_{\rm SP}\right) \leq I\left(\mathcal{U}_{\rm TM},\mathcal{U}_{\rm SP}\right)$ **then**
4: $\quad\mathcal{U}_{\rm TM} := \mathcal{U}'_{\rm TM}, \mathcal{U}_{\rm SP} := \mathcal{U}'_{\rm SP}$
5: Return to Step (1).
6: **else**
7: STOP
8: **end if**

Table 1. UL and DL performance of TM, SP, and hybrid systems in Scenario 1

	UL Sum Rate	DL Sum Rate	Total Rate	BER in the UL	BER in the DL
Hybrid System	47.72	86.46	134.19	1.31×10^{-2}	1.52×10^{-5}
TM Pilots	51.06	66.30	117.36	2.96×10^{-2}	3.66×10^{-2}
SP Pilots	35.40	75.60	111.00	2.69×10^{-2}	4.77×10^{-5}

circle with the BS in the center. The radius of this circle is varied from 0.2 to 0.9km in the simulation. The number of cells in the system is set to $L = 19$ cells with $M = 600$ antennas and $K = 5$ users per cell. However, the optimization is performed over 7 cells which consists of the central and first tier of cells. In addition, the BER and throughput is measured over the users in the central and first tier of cells. The number of symbols in the UL and DL, i.e., C_u and C_d are set to 40 symbols. The values of ρ and λ are computed from (7) and (8), respectively. The signal-to-noise ratios (SNRs) in the UL and DL, i.e., ω/σ^2 are set to 10dB, where ω is the design parameter in the statistics-aware power control scheme and is set to 1. In addition p_u for the users in $\mathcal{U}_{\rm TM}$ is set to 0.1. For the hybrid system, the parameters $\xi^{\rm ul}$ and $\xi^{\rm dl}$ are both set to 0.5. The results in Scenario 1 are generated by averaging over 10^3 realizations of user locations. For each realization of user locations, the throughput and BER is averaged over 100 realizations of channel and data vectors. The results in Scenario 2 are obtained by averaging over 10^4 realizations of channel and data. Gaussian signaling and 4-quadrature amplitude modulation (QAM) are used to compute the throughput and BER, respectively.

In Figs. 2 and 3, the UL and DL sum rates, respectively, for the systems employing TM and SP pilots, and the hybrid system are plotted against the user radius in the cell. As can be observed from the figures, the UL and DL throughputs of TM pilots are higher than that of SP pilots in the range of radius $[0.2, 0.6]$. Similarly, the UL and DL throughputs of SP pilots are higher than that of TM pilots in the range $[0.8, 1]$. Therefore, in these two ranges, there is a clear choice of $\mathcal{U}_{\rm TM}$ and $\mathcal{U}_{\rm SP}$ for the partitioning algorithm. However, in the range $[0.6, 0.8]$, the behavior of the greedy algorithm is dependent on the parameters $\xi^{\rm ul}$ and $\xi^{\rm dl}$, and since both parameters are equal,the algorithm attempts to strike a balance between the UL and DL sum rates and offers a performance that is in between TM and SP pilots. In addition, since Algorithm 1, is greedy, the UL and DL performance of the resulting partition is non-smooth across different user radius.

In Table 1, the throughput and BER in Scenario 1 are detailed for TM and SP pilot-based systems as well as the proposed hybrid system. The proposed hybrid system offers roughly 14.34% higher total throughput when compared to the existing methods. In addition, the proposed method offers a higher throughput in the DL than TM pilot-based methods. However, this improved DL performance comes at a cost of lower throughput in the UL, but the hybrid system allows the DL rate to be traded-off against the UL rate through the parameters $\xi^{\rm ul}$ and $\xi^{\rm dl}$.

6. CONCLUSION

We have proposed an algorithm, for TDD massive MIMO systems, that minimizes the total inter-cell and intra-cell interference by selecting the type of pilot that a user transmits. By means of simulations, it is shown that the proposed scheme offers a performance that is robust with respect to the user location in the cell. However, the objective function, that is described in this paper, is non-convex and requires cooperation between BSs. Obtaining a distributed solution to solve this optimization problem is a potential direction for future research.

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