# JAMMING MASSIVE MIMO USING MASSIVE MIMO: ASYMPTOTIC SEPARABILITY RESULTS

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## ABSTRACT

Consider the uplink transmission of a single-cell multi-user multiple-input multiple-output (MIMO) system with $K$ single-antenna users and a base station (BS) equipped with a very large number of antennas denoted by $M$. Consider a jamming device with $N > M$ distributed antennas attempting to deteriorate the communication between the users and the BS. We propose an asymptotic condition on the jamming power under which the jamming-plus-noise subspace overlaps with the signal subspace. Under this condition, existing blind jamming rejection methods, such as the one in [1], fail. The proposed results are based on the application of results from large-dimensional random matrix theory.

***Index Terms*—** Massive MIMO, jamming attacks, random matrix theory, eigenvalue spectrum

## 1. INTRODUCTION

Reciprocity-based Massive (MaMIMO), operating in TDD [2], is currently the most compelling 5G wireless access technology. A sequence of papers [3, 4, 5, 6] have highlighted the susceptibility of MaMIMO to jamming attacks that specifically target the uplink training phase, effectively creating artificial pilot contamination which destroys the channel estimates and thereby, potentially, severely degrades performance in closed-loop operation. (The "reverse" problem, of using MaMIMO technology to jam a conventional wireless link, was studied in [7].) This matter is important, in the light of the foreseen widespread adoption of MaMIMO technology in standards, and amid increasing concerns that intentional jamming represents an increasing threat to wireless infrastructure [8].

In the recent previous work [1], we considered the scenario of MaMIMO with a distributed jammer, potentially consisting of transmitters that create users'-like signals. From a practical perspective, this is a threatening setup as the localization and disarming of such jammers could be very difficult when they smartly adjust their transmission powers. In [1], we proposed a subspace-based algorithm for blind detection and mitigation of such distributed jamming attacks in MaMIMO. This algorithm works particularly well when the signal subspace is separable from the jamming/noise subspaces.

In this paper, we use random matrix theory to establish a fundamental asymptotic condition on the jamming power to prevent the use of blind subspace-based methods for jamming rejection such as the one in [1], which the BS would apply if it knows that jamming exists. This analysis is complementary to our results in [9], derived for a different but related problem. The specific signal model of concern is given in Section 2, and the main result of the paper is Theorem 1. Conclusions are given in Section 7.

*Notations:* The notation $\mathcal{CN}(\mathbf{a}, \mathbf{\Sigma})$ stands for the multivariate complex normal distribution with mean $\mathbf{a}$ and covariance matrix $\mathbf{\Sigma}$. For $x \in \mathbb{R}$, the $(x)^+$ is equal to $\max(x, 0)$. The subscript $(\cdot)^{\mathsf{H}}$ represents the Hermitian transpose of a matrix. The almost sure (a.s.) convergence is denoted by the symbol $\xrightarrow[M\to\infty]{\text{a.s.}}$ (or, equivalently, by $\xrightarrow{\text{a.s.}}$). The notation $\text{supp}(F)$ stands for the support of $F$.

## 2. SYSTEM MODEL

Consider the uplink transmission in a single-cell multi-user MIMO system with $K$ single-antenna users and a BS equipped with $M$ antennas. Consider a massive distributed multi-antenna jamming device equipped with $N$ antennas. This scenario is depicted in Fig. 1.

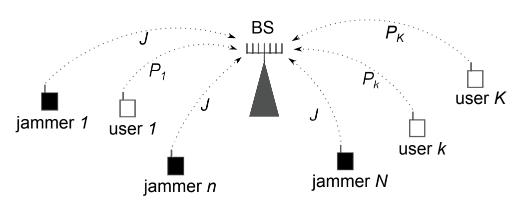


**Fig. 1**. Distributed jamming of the uplink in a single cell containing $K$ single-antenna users and a massive jamming devices with $N$ antennas.

Let $\tau$ be the length of the channel coherence interval in samples, in which a fixed realization of the channels are obtained. The $M \times 1$ received vector at the BS at time $t = 1, \ldots, \tau$ in a coherence interval is

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t + \widetilde{\mathbf{H}}\widetilde{\mathbf{x}}_t + \mathbf{w}_t \tag{1}$$

where $\mathbf{x}_t \in \mathbb{C}^{K\times 1}$ is the transmitted data vector with independent entries with zero mean and covariance matrix $\mathbf{P} = \mathrm{diag}(P_1,\ldots,P_K)$ with $P_1,\ldots,P_K$ representing the powers (including the corresponding pathlosses) of the transmitted signals in the home cell; $\mathbf{H} \in \mathbb{C}^{M\times K}$ is the channel matrix between the BS and the $K$ users with independent identically distributed (i.i.d.) entries $\mathbf{H}_{m,k} \sim \mathcal{CN}(0,1)$; $\widetilde{\mathbf{x}}_t \in \mathbb{C}^{N\times 1}$ is the jamming signal vector with i.i.d. entries with variance $J$ representing the jamming transmitting power (including the corresponding pathlosses) at each antenna $n$; $\widetilde{\mathbf{H}} \in \mathbb{C}^{M\times N}$ is the channel matrix between the BS and the jammer with entries $\widetilde{\mathbf{H}}_{m,n} \sim \mathcal{CN}(0,1)$; the additive noise is represented by the vector $\mathbf{w}_t \in \mathbb{C}^{M\times 1}$ with $\mathbf{w}_t \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_M)$. We concatenate $\tau$ successive samples of the received vectors given by (1) into the received matrix

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \widetilde{\mathbf{H}}\widetilde{\mathbf{X}} + \mathbf{W} \tag{2}$$

where $\mathbf{Y} = [\mathbf{y}_1,\ldots,\mathbf{y}_\tau] \in \mathbb{C}^{M\times\tau}$, $\mathbf{X} = [\mathbf{x}_1,\ldots,\mathbf{x}_\tau] \in \mathbb{C}^{K\times\tau}$, $\widetilde{\mathbf{X}} = [\widetilde{\mathbf{x}}_1,\ldots,\widetilde{\mathbf{x}}_\tau] \in \mathbb{C}^{N\times\tau}$, and $\mathbf{W} = [\mathbf{w}_1,\ldots,\mathbf{w}_\tau] \in \mathbb{C}^{M\times\tau}$.

We assume in the following that $M < N < \tau$ are large, being in practice of the order of hundreds or thousands. We define the following asymptotic regimes:

- $M/N \to c \in (0,1)$ as $M \to \infty$, $N \to \infty$
- $N/\tau \to \bar{c} \in (0,1)$ as $N \to \infty$, $\tau \to \infty$
- $M/\tau \to \tilde{c} \in (0,1)$ as $M \to \infty$, $\tau \to \infty$.

Further, the number of users $K$ is assumed to be fixed as $M$, $N$, $\tau \to \infty$. The results of this paper are based on asymptotic spectral analysis of the sample covariance matrix of the received signal defined by

$$\widehat{\mathbf{R}} \triangleq \frac{1}{\tau}\mathbf{Y}\mathbf{Y}^{\mathsf{H}}$$

and of the sample covariance matrix of the jamming-plus-noise defined by

$$\mathbf{\Psi} \triangleq \frac{1}{\tau}\left(\widetilde{\mathbf{H}}\widetilde{\mathbf{X}} + \mathbf{W}\right)\left(\widetilde{\mathbf{H}}\widetilde{\mathbf{X}} + \mathbf{W}\right)^{\mathsf{H}}. \tag{3}$$

Note that in random matrix theory the transmission model (2) is described by fixed-rank perturbation models [10] as the signal matrix is of small rank $K$ with probability one. In this paper we derive a condition on the jamming power $J$ under which the signal subspace cannot be separated from the jamming-plus-noise subspace for a given maximum user power. Notice that this condition is derived under the assumption $c \in (0,1)$ for which the support of the limiting spectral distribution (l.s.d.) of $F^{\mathbf{\Psi}}$ is asymptotically composed of one interval.

## 3. MAIN RESULT

The main result of this work is conceptually related to the separability condition described in the work [9]. We recall that the separability condition provided in [9] is given by the minimum user's power required in order to separate the signal subspace from the interference-plus-noise subspace asymptotically. In this work, the aim is to provide the minimum jamming power in order to not be able to separate the jamming-plus-noise subspace from the signal subspace asymptotically.

**Theorem 1.** *Consider the model* (2) *and let* $P_{\max} = \max(P_1,\ldots,P_K)$. *Define on* $[0,\infty)$ *the following function*

$$\tilde{y}(J) \triangleq \frac{J - P_{\max}M}{(\bar{c}-1)JP_{\max}M + P_{\max}^2M^2}.$$

*Let* $J^\star = \bar{J}/M$ *where* $\bar{J}$ *is the smallest positive solution of the equation*

$$\frac{1-\sigma^4\tilde{c}\tilde{y}(J)^2}{\tilde{y}(J)^2(1-\sigma^2\tilde{c}\tilde{y}(J))^2} = \frac{2\sigma^2\tilde{c}/c}{1-\sigma^2\tilde{c}\tilde{y}(J)}\tilde{I}_1(J) + \frac{1}{c}\tilde{I}_2(J)$$

*where*

$$\tilde{I}_1(J) = \frac{\left(\sqrt{\tilde{b}(J)} - \sqrt{\tilde{a}(J)}\right)^2}{4\bar{c}J\tilde{y}(J)^2},$$

$$\tilde{I}_2(J) = \frac{\left(\sqrt{\tilde{a}(J)\tilde{b}(J)} - 1\right)\left(\sqrt{\tilde{b}(J)} - \sqrt{\tilde{a}(J)}\right)^2}{4\bar{c}J\tilde{y}(J)^3\sqrt{\tilde{a}(J)\tilde{b}(J)}}$$

*with*

$$\tilde{a}(J) = 1 + \sigma^2J\tilde{y}(J)(1-\sqrt{\bar{c}})^2,$$
$$\tilde{b}(J) = 1 + \sigma^2J\tilde{y}(J)(1+\sqrt{\bar{c}})^2.$$

*Let* $\lambda_{\max}$ *be the largest eigenvalue of* $\widehat{\mathbf{R}}$. *Let* $x^\star$ *be the upper bound of the support of the limiting spectral distribution of* $\mathbf{\Psi}$. *Then, for* $J \geq J^\star$,

$$\lambda_{\max} - x^\star \xrightarrow[M\to\infty]{a.s.} 0.$$

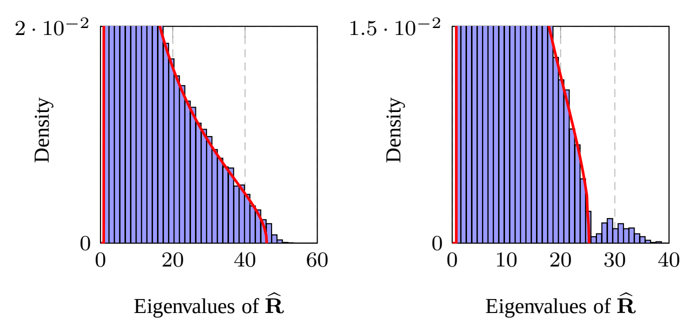


**Fig. 2**. Histogram of the eigenvalues of $\widehat{\mathbf{R}}$ and the l.s.d. of $\mathbf{\Psi}$ (solid line) for $J = J^\star = -14.3$ dB (left) and for $J = -20$ dB (right).

From this theorem, for a given maximum power of the users, we obtain the minimum jamming power required to avoid separability of the signal subspace from the jamming-plus-noise subspace asymptotically. This is depicted in Fig 2, where we illustrate the empirical eigenvalue distributions (e.s.d.) of $\widehat{\mathbf{R}}$ and the l.s.d. of $\mathbf{\Psi}$ under the non-separability condition $J = J^\star$ and under separability condition for $J < J^\star$. The simulations are performed for $K = 2$, $N = 300$, $M = 200$, $\tau = 1000$, $P_1 = P_2 = -10$ dB, and $\sigma^2 = 0$ dB. The plot on the left confirms the result of the above theorem stating that the separability does not happen for $J \geq J^\star$. From the plot on the right, we see that the signal eigenvalues are separated from the jamming-plus-noise eigenvalues, as the jamming power is too small.

The main steps of the proof of Theorem 1 are given in the next section.

## 4. MAIN STEPS OF THE PROOF OF THEOREM 1

### 4.1. Preliminary results

Let us first consider the jamming covariance matrix $\mathbf{\Gamma} = \frac{1}{\tau}\widetilde{\mathbf{H}}\widetilde{\mathbf{X}}\widetilde{\mathbf{X}}^{\mathsf{H}}\widetilde{\mathbf{H}}^{\mathsf{H}}$. Note that the l.s.d. of $\frac{M}{\tau}\widetilde{\mathbf{X}}\widetilde{\mathbf{X}}^{\mathsf{H}}$ is given by a scaled[1] Marčenko–Pastur (MP) distribution $\bar{F}$ [11] with the support given by the interval $[a, b]$ with $a = \sigma^2 JM(1-\sqrt{\bar{c}})^2$ and $b = \sigma^2 JM(1+\sqrt{\bar{c}})^2$. By applying the results from [12] we can characterize the l.s.d. of $\mathbf{\Gamma}$ given by the following theorem:

**Theorem 2.** *Let* $\mathbf{\Gamma} = \frac{1}{\tau}\widetilde{\mathbf{H}}\widetilde{\mathbf{X}}\widetilde{\mathbf{X}}^{\mathsf{H}}\widetilde{\mathbf{H}}^{\mathsf{H}}$ *where all the matrices are defined as in* (2)*. As* $M \to \infty$*, the e.s.d. of* $\mathbf{\Gamma}$ *converges to* $F^{\mathbf{\Gamma}}$ *with Stieltjes transform (ST)*[2] $m_{\mathbf{\Gamma}}$ *satisfying, for any* $z \in \mathbb{C} \setminus \operatorname{supp}(\mathbf{\Gamma})$,

$$m_{\mathbf{\Gamma}}(z) = \left(z + \frac{1}{c}\int \frac{td\bar{F}}{1+tm_{\mathbf{\Gamma}}(z)}\right)^{-1}$$

*where* $\bar{F}$ *with density given by*

$$\bar{f}(t) = \frac{\sqrt{(t-a)^+(b-t)^+}}{2\pi JM\bar{c}t}$$

*with* $\bar{c} \in (0,1)$.

From the above result and [13], we get the following theorem describing the upper bound of $\operatorname{supp}(\mathbf{\Gamma})$:

**Theorem 3.** *The upper bound of* $\mathbf{\Gamma}$ *is given by*

$$x^o = -\frac{1}{m^o_{\mathbf{\Gamma}}} + \frac{1}{c}\frac{\left(\sqrt{1+bm^o_{\mathbf{\Gamma}}} - \sqrt{1+am^o_{\mathbf{\Gamma}}}\right)^2}{4\bar{c}JM(m^o_{\mathbf{\Gamma}})^2}$$

*where* $m^o_{\mathbf{\Gamma}}$ *is the unique solution in* $(-1/b, 0)$ *of the equation*

$$\int_a^b \left(\frac{m_{\mathbf{\Gamma}}t}{1+m_{\mathbf{\Gamma}}t}\right)^2 d\bar{F} = \frac{1}{c}. \quad (4)$$

From the above theorem we have $m^o_{\mathbf{\Gamma}}$, the ST of $F^{\mathbf{\Gamma}}$ at the point $x^o$, useful in the following.

Recall that as $c \in (0,1)$, $\operatorname{supp}(F^{\mathbf{\Psi}})$ is asymptotically composed of one interval. The following theorem describes the upper bound of the support of $\mathbf{\Psi}$. It is based on the results of [13] and [14].

**Theorem 4.** *Let* $m^o_{\mathbf{\Gamma}}$ *be defined by* (4)*. The upper bound of* $F^{\mathbf{\Psi}}$ *is given by*

$$x^\star = \frac{\sigma^2(1-\tilde{c})m^\star_{\mathbf{\Psi}} - 1}{m^\star_{\mathbf{\Psi}}(1+\sigma^2\tilde{c}m^\star_{\mathbf{\Psi}})} + \frac{1/c}{(1+\sigma^2\tilde{c}m^\star_{\mathbf{\Psi}})^2}\int \frac{td\bar{F}}{1+y(m^\star_{\mathbf{\Psi}})t}$$

*where* $m^\star_{\mathbf{\Psi}}$ *is the unique solution in* $(m^o_{\mathbf{\Gamma}}/(1-\sigma^2\tilde{c}m^o_{\mathbf{\Gamma}}), 0)$ *of the equation*

$$\frac{1-\sigma^4\tilde{c}y(m)^2}{y(m)^2(1-\sigma^2\tilde{c}y(m))^2} = \frac{2\sigma^2\tilde{c}/c}{1-\sigma^2\tilde{c}y(m)}I_1(m) + \frac{1}{c}I_2(m) \quad (5)$$

*where*

$$I_1(m) = \int \frac{td\bar{F}}{1+y(m)t}, \;\; I_2(m) = \int \frac{t^2d\bar{F}}{(1+y(m)t)^2}$$

*with*

$$y(m) = \frac{m}{1+\sigma^2\tilde{c}m}.$$

*Proof.* Let $F^{\mathbf{\Gamma}}$ be the l.s.d. of $\mathbf{\Gamma}$ defined as in Theorem 2. Let $\mathbf{\Psi}$ be defined by (3). From [14], as $M \to \infty$, the e.s.d. of $\mathbf{\Psi}$ converges to $F^{\mathbf{\Psi}}$ whose ST $m_{\mathbf{\Psi}}(z)$, for $z \in \mathbb{C}^+$, is the unique solution of

$$m_{\mathbf{\Psi}}(z) = \int \frac{dF^{\mathbf{\Gamma}}(t)}{\frac{t}{1+\sigma^2\tilde{c}m_{\mathbf{\Psi}}(z)} - (1+\sigma^2\tilde{c}m_{\mathbf{\Psi}}(z))\,z + \sigma^2(1-\tilde{c})}$$

such that $m_{\mathbf{\Psi}}(z) \in \mathbb{C}^+$ and $zm_{\mathbf{\Psi}}(z) \in \mathbb{C}^+$. Let $m_{\mathbf{\Psi}}(x)$ be the restriction of $m_{\mathbf{\Psi}}(z)$ to $\mathbb{R}$. Applying the results of [13] and Theorem 2, $x^\star$ coincides with the infimum of the function

$$x(m_{\mathbf{\Psi}}) = \frac{\sigma^2(1-\tilde{c})m_{\mathbf{\Psi}} - 1}{m_{\mathbf{\Psi}}(1+\sigma^2\tilde{c}m_{\mathbf{\Psi}})} + \frac{1/c}{(1+\sigma^2\tilde{c}m_{\mathbf{\Psi}})^2}\int \frac{td\bar{F}}{1+y(m_{\mathbf{\Psi}})t} \quad (6)$$

whose restriction to the interval $(m^\star_{\mathbf{\Psi}}, 0)$ coincides with the inverse with respect to composition of the restriction of $m_{\mathbf{\Psi}}(x)$ to $(x^\star, 0)$. By deriving (6) after some calculations we get the result. □

Note that from the above theorem $m^\star_{\mathbf{\Psi}}$ is the ST at the point $x^\star$ expressed as $m^\star_{\mathbf{\Psi}} = \lim_{x \to x^\star} m_{\mathbf{\Psi}}(x)$.

[1] Note that $J$ is order of magnitude of $1/M$ so that the interval $[a, b]$ is compact.

[2] The ST $m_F$ of a spectral distribution $F$ with support in $\mathbb{R}$ denoted by $\operatorname{supp}(F)$ is defined by $m_F(z) = \int (t-z)^{-1}dF(t)$. It completely characterizes the spectral distribution $F$.

### 4.2. Core of the proof

The following result provides a non-separability condition of the jamming-plus-noise and the signal subspaces, given a signal power $P_{\max} = \max(P_1, \ldots, P_K)$. The proof is based on the results [9] involving $x^\star$ and $m^\star_{\Psi}$ given by Theorem 4.

**Theorem 5.** *Consider the model* (2) *and let* $m^\star_{\Psi}$ *be defined as in* (5)*. Define*

$$y^\star \triangleq \frac{m^\star_{\Psi}}{1 + \sigma^2 \tilde{c} m^\star_{\Psi}}.$$

*Define*

$$J^\star \triangleq \frac{P^2_{\max} M^2 y^\star + P_{\max} M}{P_{\max} M (1 - \bar{c}) y^\star + 1}.$$

*Let* $J \geq J^\star$ *and let* $\lambda_{\max}$ *be the largest eigenvalue of* $\widehat{\mathbf{R}}$*. Then,*

$$\lambda_{\max} \xrightarrow[M \to \infty]{a.s.} x^\star.$$

The proof is finalized using the result of the above theorem by defining $\tilde{y}(J)$ as a function of $J$ and injecting it into the expression (5) (replacing $y(m)$ by $\tilde{y}(J)$).

## 5. PERFORMANCE ANALYSIS

The performance under the system model (2) is analyzed in terms of an achievable net spectral efficiency under linear detection. We denote by $\tau_p \geq K$ the length of the pilot sequence. A spectral efficiency [15] for user $k = 1, \ldots, K$ is given by

$$\mathcal{S}_k = \left(1 - \frac{\tau_p}{\tau}\right) \log_2 \left(1 + \frac{\left|\mathbb{E}\left[\mathbf{b}_k^{\mathsf{H}} \mathbf{h}_k\right]\right|^2 P_k}{\frac{\mathbb{E}\left[\|\bar{\mathbf{y}}_k\|^2\right]}{\tau - \tau_p} - \left|\mathbb{E}\left[\mathbf{b}_k^{\mathsf{H}} \mathbf{h}_k\right]\right|^2 P_k}\right) \quad (7)$$

where $\mathbf{b}_k \in \mathbb{C}^{K \times 1}$ is the detection vector, $\mathbf{h}_k \in \mathbb{C}^{K \times 1}$ is the $k$th column of $\mathbf{H}$, and $\bar{\mathbf{y}}_k = \mathbf{b}_k^{\mathsf{H}} \mathbf{Y} \in \mathbb{C}^{K \times \tau_d}$ is the filtered received vector for user $k$, with $\tau_d = \tau - \tau_p$. An approximate minimum mean square error (MMSE) detection filter for user $k = 1, \ldots, K$ is given by

$$\mathbf{b}_k^{\text{MMSE}} = \hat{\mathbf{h}}_k^{\mathsf{H}} \mathbf{P}^{\frac{1}{2}} \left(\mathbf{P}^{\frac{1}{2}} \hat{\mathbf{h}}_k^{\mathsf{H}} \hat{\mathbf{h}}_k \mathbf{P}^{\frac{1}{2}} + \sigma^2 \mathbf{I}_K\right)^{-1}$$

where $\hat{\mathbf{h}}_k \in \mathbb{C}^{K \times 1}$ is the $k$th column of the least square estimator $\widehat{\mathbf{H}}$ given by

$$\widehat{\mathbf{H}} = \frac{1}{\tau_p} \mathbf{Y}_p \mathbf{X}_p^{\mathsf{H}} \mathbf{P}^{-\frac{1}{2}}$$

with $\mathbf{X}_p \in \mathbb{C}^{K \times \tau_p}$ the matrix with orthogonal user pilots and $\mathbf{Y}_p \in \mathbb{C}^{M \times \tau_p}$ the received pilot matrix.

## 6. NUMERICAL RESULTS

We provide simulation results for $K = 2$, $M = 32$, $N = 64$, $\tau = 256$, $\tau_p = 2$. The normalized received user powers are equal to $P_1 = P_2 = P = 0$ dB and the noise variance is equal to $\sigma^2 = 0$ dB. For this scenario, by applying the results of Theorem 1, we find $J^\star = -4.5$ dB. In Fig. 3, the performance of the jamming (Jamming) is drawn in terms of achievable net spectral efficiency (bit/s/Hz) given by (7) versus jamming power $J$ in dB for user 1 (equivalent to user 2 as the users have the same power). The results are compared to the achievable net spectral efficiency for the scenario without jamming (Jamming free) and for a blind jamming rejection method (Jamming rejection) similar to the algorithm of [1] consisting in projecting the received data onto the signal subspace generated by $K$ largest eigenvalues of $\widehat{\mathbf{R}}$. We observe that the jamming rejection approach leads to an almost zero spectral efficiency for $J > J^\star$, confirming the rejection algorithm is bound to fail when $J$ exceeds $J^\star$. Moreover, it is shown that for $J$ close to $J^\star$, it is better to ignore jamming than to use blind rejection algorithms since the signal and jamming subspaces cannot be separated. Notice that for low values of $J$, the jamming rejection algorithm outperforms the other methods as it eliminates not only the jamming but also the noise by projecting the received signal to the signal subspace (see [1] for discussions).

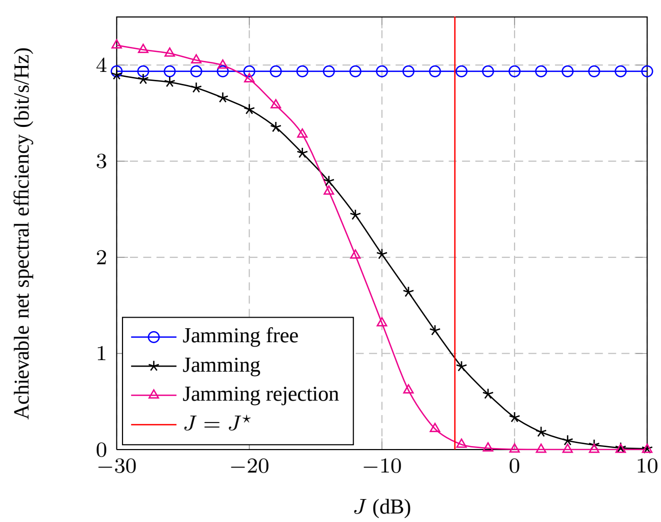


**Fig. 3.** Achievable net spectral efficiency of user 1 (or 2) versus $J$ (dB).

## 7. CONCLUSIONS

We provided an asymptotic condition on the jamming power under which the signal subspace cannot be separated from the jamming-plus-noise subspace. Numerical results indicated that this condition can be used to predict the regime in which blind jamming mitigation schemes, such as the one in [1], can be successfully applied.

## 8. REFERENCES


[1] J. Vinogradova, E. Björnson, and E. G. Larsson, "Detection and mitigation of jamming attacks in massive MIMO systems using random matrix theory," in *IEEE SPAWC*, pp. 1–5, 2016.

[2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Comm. Mag.*, vol. 52, no. 2, pp. 186–195, 2014.

[3] D. Kapetanovic, G. Zheng, and F. Rusek, "Physical layer security for massive MIMO: An overview on passive eavesdropping and active attacks," *IEEE Comm. Mag.*, vol. 53, pp. 21–27, 2015.

[4] Y. O. Basciftci, C. E. Koksal, and A. Ashikhmin, "Securing massive MIMO at the physical layer," in *IEEE CNS*, pp. 272–280, 2015.

[5] J. K. Tugnait, "Self-contamination for detection of pilot contamination sttack in multiple antenna systems," *IEEE Wireless Communications Letters*, vol. 4, no. 5, pp. 525–528, 2015.

[6] H. Pirzadeh, S. M. Razavizadeh, and E. Björnson, "Subverting massive MIMO by smart jamming," *Wireless Comm. Letters*, vol. 5, no. 1, pp. 20–23, 2016.

[7] M. Karlsson and E. G. Larsson, "Massive MIMO as a cyber-weapon," in *48th Asilomar Conference on Signals, Systems and Computers, ACSSC 2014, Pacific Grove, CA, USA, November 2-5, 2014*, pp. 661–665, 2014.

[8] J. H. Reed and M. Lichtman, "Virginia Techs response to firstnet NOI," 2012.

[9] J. Vinogradova, E. Björnson, and E. G. Larsson, "On the separability of the signal from intereference-plus-noise subspaces in blind pilot decontamination," in *IEEE ICASSP*, pp. 3421–3425, 2016.

[10] J. Baik and J. W. Silverstein, "Eigenvalues of large sample covariance matrices of spiked population models," *J. Multivariate Anal.*, vol. 97, no. 6, pp. 1382–1408, 2006.

[11] V. A. Marčenko and L. A. Pastur, "Distribution of eigenvalues for some sets of random matrices," *Mathematics of the USSR-Sbornik*, vol. 1, no. 4, pp. 457–483, 1967.

[12] J. W. Silverstein and Z. D. Bai, "On the empirical distribution of eigenvalues of a class of large-dimensional random matrices," *J. Multivariate Anal.*, vol. 54, no. 2, pp. 175–192, 1995.

[13] J. W. Silverstein and S. Choi, "Analysis of the limiting spectral distribution of large-dimensional random matrices," *J. Multivariate Anal.*, vol. 54, no. 2, pp. 295–309, 1995.

[14] R. B. Dozier and J. W. Silverstein, "Analysis of the limiting spectral distribution of large dimensional information-plus-noise type matrices," *Journal of Multivariate Analysis*, vol. 98, no. 6, pp. 1099–1122, 2007.

[15] J. Jose, A. E. Ashikhmin, T. L. Marzetta, and S. Vishwanath, "Pilot contamination and precoding in multi-cell TDD systems," *IEEE Trans. on Wireless Comm.*, vol. 10, no. 8, pp. 2640–2651, 2011.