HYBRID PRECODING USING LONG-TERM CHANNEL STATISTICS FOR MASSIVE MIMO SYSTEMS

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ABSTRACT

Hybrid analog/digital precoding in the downlink of multiuser massive MIMO systems can reduce the number of RF chains hence reducing total cost and improving power efficiency. Having few RF chains, however, makes it difficult for a base station to acquire instantaneous channel state information across all antennas. We develop a hybrid technique that uses only long-term (slowly changing) channel statistics in computing the analog precoding matrix. The proposed analog precoder is designed to maximize signal-to-leakage-plus-noise ratio (SLNR) when combined with a baseband precoder. We also propose a constrained precoder design that reduces the effect of a hardware constraint where the analog precoders are realized with phase shifters.

***Index Terms*—** Hybrid precoding, multiuser, massive MIMO, correlated channel, spatial channel covariance

1. INTRODUCTION

Digitally precoded massive MIMO systems promise to dramatically increase cell spectral efficiency by employing a large number of antennas and transmit-side processing of channel state information. Hybrid analog/digital precoding aims to alleviate the increased cost and power consumption anticipated by such systems by reducing the number of RF chains while maintaining the number of physical antennas. Hybrid precoding divides the linear precoding process between the analog RF and digital baseband parts [1, 2]. Hybrid precoding has been applied to single-user MIMO [3–5] and multiuser MIMO [6–10]. Although most prior work on hybrid precoding for multiuser MIMO systems assumes full channel state information at the transmitter (CSIT), the hybrid structure makes it difficult to estimate the *entire* channel matrix for all antennas since the estimator in the baseband can only see a low(er)-dimensional pre-combined channel through few(er) RF chains.

Employing long-term channel statistics (such as spatial channel covariance) is an attractive alternative to using full CSIT. Since the covariance remains unaffected by short-term fading, the covariance can be efficiently estimated even in the hybrid structure [11, 12]. Limited work has been done on the hybrid precoding using the spatial channel covariance instead of full CSIT for the analog precoder design. In [13, 14], users are divided into groups and all users in a group are required to have the same covariance matrix. In [15], each user's single dominant eigenvector of the covariance matrix constructs the analog precoders while ignoring the interference in the analog part. A multi-layer precoding was introduced in [16] that additionally mitigates inter-cell interference. This multi-layer technique, however, does not consider intra-cell multiuser interference when designing the analog part as [15].

In this paper, we develop a new hybrid precoding technique for multiuser massive MIMO in which the analog precoder depends only on the spatial channel covariance. The proposed analog precoder design is based on finding the best subspace maximizing the SLNR when combined with the baseband precoder. Since the analog precoding is typically realized with phase shifters, we propose an additional stage of baseband precoding, using a compensation matrix, which alleviates the effects of the phase shifters. Simulations show that the proposed approach where all RF chains construct a subspace for all users is superior to the prior work where each RF chain is dedicated to each user. The results also show that the rate loss caused by the phase shifters becomes negligible thanks to the compensation matrix.

2. SYSTEM AND CHANNEL MODEL

Consider a downlink system where a base station (BS) equipped with N antennas and $M(\leq N)$ RF chains communicates with $K(\leq M)$ users with a single antenna. Let $\mathbf{F}_{\mathrm{RF}} \in \mathbb{C}^{N\times M}$, $\mathbf{F}_{\mathrm{BB}} \in \mathbb{C}^{M\times K}$, and $\mathbf{s} \in \mathbb{C}^{K\times 1}$ be an analog RF precoder, a digital baseband precoder, and a signal vector. The received signal is given by

$$\mathbf{y} = \mathbf{H}^{*}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{P}^{\frac{1}{2}}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{P} \in \mathbb{R}^{K\times K}$ is a diagonal matrix to maintain the total transmit power P_{tx}, $\mathbf{n} \in \mathbb{C}^{K\times 1} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$ is circularly

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symmetric complex Gaussian noise, and $\mathbf{H}^* \in \mathbb{C}^{K\times N}$ is a downlink channel matrix for all users. The row of $\mathbf{H}^*$ is each user's channel vector and modeled as $\mathbf{h}_k^* = \mathbf{h}_{w,k}^* \mathbf{R}_K^{\frac{1}{2}}$ where $\mathbf{h}_{w,k}$ has IID complex entries of zero mean and unit variance and $\mathbf{R}_k = \mathbb{E}[\mathbf{h}_k \mathbf{h}_k^*]$ is a spatial channel covariance matrix of user k. We assume that $\mathbf{R}_k$'s have been obtained through covariance estimation for the hybrid structure, see e.g. [11].

Assuming that the analog precoding is composed of phase shifters, a constraint on $\mathbf{F}_{\mathrm{RF}}$ is imposed that all elements in $\mathbf{F}_{\mathrm{RF}}$ have the same amplitude. In our problem, there is another constraint that $\mathbf{F}_{\mathrm{RF}}$ is designed by using only $\mathbf{R}_k$'s, not $\mathbf{h}_k$'s. This is a distinguishing point compared to other prior work [6–10] where $\mathbf{F}_{\mathrm{RF}}$ depends on $\mathbf{h}_k$'s.

3. HYBRID PRECODING USING LONG-TERM CHANNEL STATISTICS

In this section, we develop the unconstrained analog precoding without the phase shifter constraints. In [15, 16], a simple technique for multiuser hybrid precoding was proposed that uses only the covariance to design the analog precoder $\mathbf{F}_{\mathrm{RF}}$. Each column of $\mathbf{F}_{\mathrm{RF}}$ is assigned to one user as

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{v}_{1,\max} & \cdots & \mathbf{v}_{K,\max} \end{bmatrix}, \tag{2}$$

where $\mathbf{v}_{k,\max}$ is a dominant eigenvector of $\mathbf{R}_k$. This approach is similar to [8–10] using full CSIT. The motivation of this technique is to maximize the long-term average power of the desired signal in the analog part. This approach, however, does not consider the interference in the analog part, which results in performance degradation. Moreover, this technique cannot be directly applied when $K < M$.

We propose a different approach that designs $\mathbf{F}_{\mathrm{RF}}$ for helping $\mathbf{F}_{\mathrm{BB}}$ to minimize the interference. We consider the regularized zero-forcing (RZF) [17] with respect to the effective channel $\mathbf{H}^*\mathbf{F}_{\mathrm{RF}}$ in the baseband as

$$\mathbf{F}_{\mathrm{BB}} = \left(\mathbf{F}_{\mathrm{RF}}^* \mathbf{H}\mathbf{H}^* \mathbf{F}_{\mathrm{RF}} + \beta \mathbf{I}_M\right)^{-1} \mathbf{F}_{\mathrm{RF}}^* \mathbf{H}, \tag{3}$$

where β is a regularization parameter and set to $\beta = \frac{K}{\rho}$ as in [17], and $\rho = \frac{P_{\mathrm{tx}}}{\sigma^2}$ denotes the transmit SNR. Considering an equal power strategy that makes each user's power equal after precoding including both $\mathbf{F}_{\mathrm{RF}}$ and $\mathbf{F}_{\mathrm{BB}}$, the k-th diagonal element of $\mathbf{P}$ in (1) is given by

$$p_k = \frac{P_{tx}}{K\|\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{bb},k}\|^2} = \frac{P_{tx}}{K\mathbf{h}_k^*\mathbf{W}^2\mathbf{h}_k}, \tag{4}$$

where $\mathbf{W} = \mathbf{F}_{\mathrm{RF}}\left(\mathbf{F}_{\mathrm{RF}}^*\mathbf{H}\mathbf{H}^*\mathbf{F}_{\mathrm{RF}} + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\mathbf{F}_{\mathrm{RF}}^*$.

Since RZF maximizes SLNR [18], we focus on SLNR in our hybrid precoding design. The SLNR of user k is given by

$$\begin{aligned}\mathrm{SLNR}_k &= \frac{|\mathbf{h}_k^*\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{bb},k}|^2 p_k}{\sum_{i\neq k}|\mathbf{h}_i^*\mathbf{F}_{\mathrm{RF}}\mathbf{f}_{\mathrm{bb},k}|^2 p_k + \sigma^2} \\ &= \frac{\mathbf{h}_k^*\mathbf{W}\mathbf{h}_k\mathbf{h}_k^*\mathbf{W}\mathbf{h}_k}{\mathbf{h}_k^*\mathbf{W}\left(\mathbf{H}\mathbf{H}^* - \mathbf{h}_k\mathbf{h}_k^* + \frac{K}{\rho}\mathbf{I}_N\right)\mathbf{W}\mathbf{h}_k}.\end{aligned} \tag{5}$$

Our initial goal is to find $\mathbf{F}_{\mathrm{RF}}$ that maximizes the SLNR in (5). Instead of assigning a column of $\mathbf{F}_{\mathrm{RF}}$ to each user as in the prior work, let us try to find a subspace spanned by orthonormal bases $\{\mathbf{v}_1, \ldots, \mathbf{v}_M\}$ where $\mathbf{v}_m \in \mathbb{C}^N$ and $|\mathbf{v}_m| = 1, \forall m = 1, ..., M$. Note that there is no constraint such as $K = M$, so this approach can be applied for the general case of $K \leq M$. Using the bases, let $\mathbf{F}_{\mathrm{RF}}$ be defined as

$$\mathbf{F}_{\mathrm{RF}} = \mathbf{V}\mathbf{A}, \tag{6}$$

where $\mathbf{A} \in \mathbb{C}^{M\times M}$ is an invertible matrix and $\mathbf{V} = \begin{bmatrix}\mathbf{v}_1 & \cdots & \mathbf{v}_M\end{bmatrix} \in \mathbb{U}^{N\times M}$, where $\mathbb{U}^{N\times M}$ is a set of semi-unitary matrices. In the following proposition, we show that the SLNR in (5) has a maximum value when $\mathbf{A}$ is unitary, and thus $\mathbf{F}_{\mathrm{RF}}$ must be semi-unitary.

Proposition 1 *If $\mathbf{V}$ and P_{tx} is given, the SLNR in (5) is maximized when $\mathbf{A}$ is unitary.*

Proof: Let $\tilde{\mathbf{H}}^* = \mathbf{H}^*\mathbf{V}$ and $\tilde{\mathbf{h}}_k^* = \mathbf{h}_k^*\mathbf{V}$. Then, the SLNR in (5) can be rewritten as

$$\begin{aligned}\mathrm{SLNR}_k &= \frac{\tilde{\mathbf{h}}_k^*\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k\tilde{\mathbf{h}}_k^*\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k}{\tilde{\mathbf{h}}_k^*\tilde{\mathbf{W}}_{\mathbf{A}}\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^* - \tilde{\mathbf{h}}_k\tilde{\mathbf{h}}_k^* + \frac{K}{\rho}\mathbf{I}_M\right)\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k} \\ &= \frac{\delta_{\mathbf{A}}}{1-\delta_{\mathbf{A}}},\end{aligned} \tag{7}$$

where $\tilde{\mathbf{W}}_{\mathbf{A}} = \left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^* + \frac{K}{\rho}\left(\mathbf{A}\mathbf{A}^*\right)^{-1}\right)^{-1}$ and

$$\delta_{\mathbf{A}} = \frac{\tilde{\mathbf{h}}_k^*\tilde{\mathbf{W}}_{\mathbf{A}}^*\tilde{\mathbf{h}}_k\tilde{\mathbf{h}}_k^*\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k}{\tilde{\mathbf{h}}_k^*\tilde{\mathbf{W}}_{\mathbf{A}}^*\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^* + \frac{K}{\rho}\mathbf{I}_M\right)\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k}. \tag{8}$$

Note that $0 \leq \delta_{\mathbf{A}} \leq 1$ for any $\rho > 0$ and the SLNR is an increasing function of $\delta_{\mathbf{A}}$. The optimal $\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k$ that maximizes $\delta_{\mathbf{A}}$ has the same direction as the generalized eigenvector of $\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^* + \frac{K}{\rho}\mathbf{I}_M, \tilde{\mathbf{h}}_k\tilde{\mathbf{h}}_k^*\right)$. Since $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^* + \frac{K}{\rho}\mathbf{I}_M$ is invertible, the optimal solution of $\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k$ has a form as

$$\tilde{\mathbf{W}}_{\mathbf{A}}\tilde{\mathbf{h}}_k \propto \left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^* + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\tilde{\mathbf{h}}_k, \tag{9}$$

which implies that $\mathbf{A}\mathbf{A}^* = \mathbf{I}_M$.

□

When $\mathbf{A}$ is unitary, $\delta_{\mathbf{A}}$ has the maximal value of $\delta_{\mathbf{A}} = \tilde{\mathbf{h}}_k^*\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^* + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\tilde{\mathbf{h}}_k$ and the SLNR in (7) is

$$\begin{aligned}\mathrm{SLNR}_k &= \frac{\mathbf{h}_k^*\mathbf{V}\left(\mathbf{V}^*\mathbf{H}\mathbf{H}^*\mathbf{V} + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\mathbf{V}^*\mathbf{h}_k}{1 - \mathbf{h}_k^*\mathbf{V}\left(\mathbf{V}^*\mathbf{H}\mathbf{H}^*\mathbf{V} + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\mathbf{V}^*\mathbf{h}_k} \\ &= \mathbf{h}_k^*\mathbf{V}\left(\mathbf{V}^*\left(\sum_{i\neq k}^{K}\mathbf{h}_i\mathbf{h}_i^*\right)\mathbf{V} + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\mathbf{V}^*\mathbf{h}_k,\end{aligned} \tag{10}$$

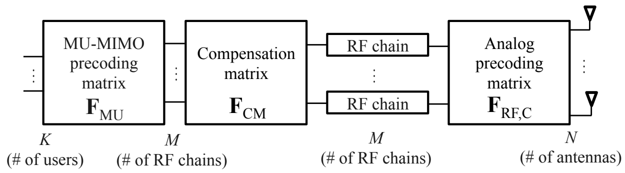

Fig. 1. Proposed constrained hybrid precoding structure.

where the equality comes from the matrix inversion lemma.

Let $\mathbf{T}_k = \mathbf{V}^*\mathbf{R}_k^{\frac{1}{2}}$ and $\mathbf{g}_k = \frac{1}{\sqrt{N}}\mathbf{h}_{w,k}$. If we assume that $\mathbf{R}_k$ has uniformly bounded spectral norm, the SLNR in the large antenna regime becomes

$$\begin{aligned}\mathrm{SLNR}_k &= N\mathbf{g}_k^H\mathbf{T}_k^*\left(N\sum_{i\neq k}^{K}\mathbf{T}_k\mathbf{g}_i\mathbf{g}_i^*\mathbf{T}_k^* + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\mathbf{T}_k\mathbf{g}_k \\ &\xrightarrow{a.s.} \mathrm{Tr}\left(\mathbf{T}_k^*\left(N\sum_{i=1}^{K}\mathbf{T}_k\mathbf{g}_i\mathbf{g}_i^*\mathbf{T}_k^* + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\mathbf{T}_k\right),\end{aligned} \tag{11}$$

where the almost sure property comes from the trace lemma and the rank-1 perturbation lemma [19]. By [20, Theorem 1], the SLNR_k in (11) converges to a deterministic value γ_k, where $\gamma_1, ..., \gamma_K$ are the unique nonnegative solution of

$$\gamma_k = \mathrm{Tr}\left(\mathbf{V}^*\mathbf{R}_k\mathbf{V}\left(\sum_{j=1}^{K}\frac{\mathbf{V}^*\mathbf{R}_j\mathbf{V}}{1+\gamma_j} + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\right). \tag{12}$$

Let us consider maximizing the average asymptotic SLNR as $\max \frac{1}{K}\sum_{k=1}^{K}\gamma_k$. Since this is difficult to solve directly due to K fixed point equations in (12), to relax the problem, let us assume that $\gamma_1 = \cdots = \gamma_K = \gamma = \frac{1}{K}\sum_{k=1}^{K}\gamma_k$. Then, the optimization problem becomes

$$\begin{aligned}&\max_{\mathbf{V}\in\mathbb{U}^{N\times M}} \gamma \quad \text{s.t.} \\ &\gamma = \mathrm{Tr}\left(\mathbf{V}^*\mathbf{R}_{\mathrm{tot}}\mathbf{V}\left(\frac{K\mathbf{V}^*\mathbf{R}_{\mathrm{tot}}\mathbf{V}}{1+\gamma} + \frac{K}{\rho}\mathbf{I}_M\right)^{-1}\right),\end{aligned} \tag{13}$$

where $\mathbf{R}_{\mathrm{tot}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{R}_k$. Let $\mathbf{V}^*\mathbf{R}_{\mathrm{tot}}\mathbf{V} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^*$ by eigenvalue decomposition and let $\lambda_1, ..., \lambda_M$ denote its eigenvalues in descending order. Then, the constraint in (13) becomes

$$\begin{aligned}\gamma &= \frac{1}{K}\mathrm{Tr}\left(\mathbf{U}\mathbf{\Lambda}\mathbf{U}^*\left(\frac{\mathbf{U}\mathbf{\Lambda}\mathbf{U}^*}{1+\gamma} + \frac{1}{\rho}\mathbf{I}_M\right)^{-1}\right) \\ &= \frac{1}{K}\sum_{m=1}^{M}\frac{1}{\frac{1}{1+\gamma} + \frac{1}{\rho\lambda_m}},\end{aligned} \tag{14}$$

and the solution to (13) is given in the following proposition.

Algorithm 1 Find $\mathbf{F}_{\mathrm{RF,C}}$

Input: $\mathbf{F}_{\mathrm{RF,UC}}$
Initialization: $\mathbf{F}_{(0)} = \measuredangle(\mathbf{F}_{\mathrm{RF,UC}})$, $n = 0$
repeat
  $n \leftarrow n + 1$
  $\mathbf{F}_{(n)} = \measuredangle(\mathbf{F}_{\mathrm{RF,UC}}\mathbf{F}_{\mathrm{RF,UC}}^*\mathbf{F}_{(n-1)})$
until $\|\mathbf{F}_{\mathrm{RF,UC}}\mathbf{F}_{\mathrm{RF,UC}}^*\mathbf{F}_{(n-1)} - \mathbf{F}_{(n)}\|_F$ converges
Output: $\mathbf{F}_{\mathrm{RF,C}} = \mathbf{F}_{(n)}$

Proposition 2 *The optimal solution to* (13) *is the matrix composed of M dominant eigenvectors of* $\mathbf{R}_{\mathrm{tot}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{R}_k$.

Proof: Let $\mu_1, ..., \mu_N$ be the eigenvalues of $\mathbf{R}_{\mathrm{tot}}$ in descending order. By Cauchy's interlacing theorem [21] and the semi-unitary property of $\mathbf{V}$, the eigenvalues of $\mathbf{V}^*\mathbf{R}_{\mathrm{tot}}\mathbf{V}$ have the interlacing property such as

$$\mu_{N-M+i} \leq \lambda_i \leq \mu_i, \quad \text{for } i = 1, ..., M. \tag{15}$$

Consequently, the constraint in (13) becomes

$$\gamma = \frac{1}{K}\sum_{m=1}^{M}\frac{1}{\frac{1}{1+\gamma} + \frac{1}{\rho\lambda_m}} \leq \frac{1}{K}\sum_{m=1}^{M}\frac{1}{\frac{1}{1+\gamma} + \frac{1}{\rho\mu_m}}, \tag{16}$$

where the equality holds if $\mathbf{V}$ is composed of M dominant eigenvectors of $\mathbf{R}_{\mathrm{tot}}$. Since the solution of the fixed point equation with respect to γ has the maximum value if the equality holds, the proof is completed. □

4. HYBRID PRECODING UNDER PHASE SHIFTER CONSTRAINTS

In this section, we develop the constrained analog precoding under the phase shifter constraints. We will refer to the unconstrained version derived in Section 3 as $\mathbf{F}_{\mathrm{RF,UC}}$ and its constrained version as $\mathbf{F}_{\mathrm{RF,C}}$.

The goal is to make $\mathbf{F}_{\mathrm{RF,C}}$ as similar to $\mathbf{F}_{\mathrm{RF,UC}}$ as possible. A simple way to find the most similar $\mathbf{F}_{\mathrm{RF,C}}$ is [8]

$$\min_{\mathbf{F}_{\mathrm{RF,C}}} \|\mathbf{F}_{\mathrm{RF,UC}} - \mathbf{F}_{\mathrm{RF,C}}\|_{\mathrm{F}}^2 \quad \text{s.t.} \quad |[\mathbf{F}_{\mathrm{RF,C}}]_{i,j}| = 1, \tag{17}$$

which is known as a reasonable approximation of $\mathbf{F}_{\mathrm{RF,UC}}$ [3]. The solution of (17) is given by $[\mathbf{F}_{\mathrm{RF,C}}^{(\mathrm{opt})}]_{i,j} = e^{j\measuredangle\left([\mathbf{F}_{\mathrm{RF,UC}}]_{i,j}\right)}$, where $\measuredangle(\alpha)$ denotes the phase of a complex number α. The problem of this approach is that $\mathbf{F}_{\mathrm{RF,C}}$ loses the orthogonality that $\mathbf{F}_{\mathrm{RF,UC}}$ retains. Note that $\mathbf{F}_{\mathrm{RF,UC}}$ needs to be semi-unitary according to Proposition 1.

This problem can be solved by applying a compensation matrix in the baseband part to restore the orthogonality lost in the analog part as shown in Fig. 1. Let us define the compensation matrix $\mathbf{F}_{\mathrm{CM}}$ as

$$\mathbf{F}_{\mathrm{CM}} = \left(\mathbf{F}_{\mathrm{RF,C}}^*\mathbf{F}_{\mathrm{RF,C}}\right)^{-\frac{1}{2}}, \tag{18}$$

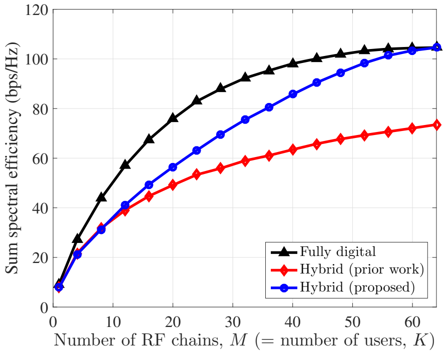

Fig. 2. Sum spectral efficiency vs. M when $K = M$, $N = 64$, $L = 5$, $\sigma_{\mathrm{AS}} = 10$, and SNR$=$ 10dB. Hybrid techniques are performed without the phase shifter constraints.

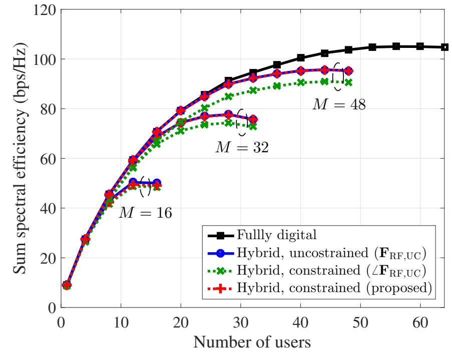

Fig. 3. Sum spectral efficiency vs. K when $N = 64$, $M = \{16, 32, 48\}$, $L = 5$, $\sigma_{\mathrm{AS}} = 10$, and SNR$=$ 10dB. The green dotted curve indicates the solution of (17).

which makes $\mathbf{F}_{\mathrm{RF,C}}\mathbf{F}_{\mathrm{CM}}$ semi-unitary as $\mathbf{F}_{\mathrm{RF,UC}}$ [22].

One benefit from using this compensation matrix is that it is not necessary to make $\mathbf{F}_{\mathrm{RF,C}}$ semi-unitary. Let $\mathbf{A}$ be an arbitrary invertible matrix, which is decomposed by SVD as $\mathbf{U}_{\mathbf{A}}\mathbf{D}_{\mathbf{A}}\mathbf{V}_{\mathbf{A}}^*$. If $\mathbf{F}_{\mathrm{RF,UC}}\mathbf{A}$ is used as the analog precoding matrix, then the compensated analog precoding matrix becomes $\mathbf{F}_{\mathrm{RF,UC}}\mathbf{A}\mathbf{F}_{\mathrm{CM}} = \mathbf{F}_{\mathrm{RF,UC}}\mathbf{U}_{\mathbf{A}}$, which is also an optimal unconstrained analog precoding. Therefore, the optimal unconstrained analog precoding $\mathbf{F}_{\mathrm{RF,UC}}$ can be replaced by $\mathbf{F}_{\mathrm{RF,UC}}\mathbf{A}$ for any invertible matrix $\mathbf{A}$ without any performance loss, if the compensation matrix in (18) is used. Using this feature, the problem in (17) can be replaced by

$$\min_{\mathbf{F}_{\mathrm{RF,C}},\mathbf{A}} \|\mathbf{F}_{\mathrm{RF,UC}}\mathbf{A} - \mathbf{F}_{\mathrm{RF,C}}\|_{\mathrm{F}}^2 \quad \text{s.t.} \quad |[\mathbf{F}_{\mathrm{RF,C}}]_{i,j}| = 1. \tag{19}$$

Due to the increased degrees of freedom, the constrained analog precoding $\mathbf{F}_{\mathrm{RF,C}}$ can be made further closer to the optimal constrained analog precoding. The solution to (19) can be obtained by an alternating minimization technique. The algorithm first finds the optimal $\mathbf{A}$ assuming $\mathbf{F}_{\mathrm{RF,C}}$ is fixed. Given a fixed $\mathbf{F}_{\mathrm{RF,C}}$, the optimal $\mathbf{A}$ is given by

$$\mathbf{A}^{(\mathrm{opt})} = \mathbf{F}_{\mathrm{RF,UC}}^*\mathbf{F}_{\mathrm{RF,C}}. \tag{20}$$

Then, assuming $\mathbf{A}$ is fixed, the optimal $\mathbf{F}_{\mathrm{RF,C}}$ is given by

$$\mathbf{F}_{\mathrm{RF,C}}^{(\mathrm{opt})} = \measuredangle(\mathbf{F}_{\mathrm{RF,UC}}\mathbf{A}), \tag{21}$$

where $\measuredangle(\mathbf{X})$ is a matrix whose (i,j)-th element is $e^{j\measuredangle([\mathbf{X}]_{i,j})}$. Using (20) and (21), the optimal $\mathbf{F}_{\mathrm{RF,C}}$ is obtained from Algorithm 1. Once $\mathbf{F}_{\mathrm{RF,C}}$ is obtained, $\mathbf{F}_{\mathrm{CM}}$ is given by (18).

The overall constrained hybrid precoding is constructed as $\mathbf{F}_{\mathrm{RF,C}}\mathbf{F}_{\mathrm{CM}}\mathbf{F}_{\mathrm{MU}}$ as shown in Fig. 1 where $\mathbf{F}_{\mathrm{MU}}$ is the RZF precoder with respect to the effective channel $\mathbf{H}^*\mathbf{F}_{\mathrm{RF,C}}\mathbf{F}_{\mathrm{CM}}$.

5. SIMULATION RESULTS

We use a geometric channel model to provide a realistic performance evaluation. There are L channel paths between a user and a BS equipped with a uniform linear array. Let α_ℓ and ϕ_ℓ denote the ℓ-th complex path gain and angle of departure (AoD), respectively. The channel vector of user k is expressed as $\mathbf{h}_k = \sum_{\ell=1}^{L} \alpha_\ell \mathbf{a}(\phi_\ell)$ where $\mathbf{a}(\phi_\ell)$ is the array response vector. We assume that ϕ_ℓ's are Laplacian distributed with angle spread σ_{AS}, and $\alpha_\ell \sim \mathcal{CN}(0, \sigma_{\alpha_\ell}^2)$ where $\sigma_{\alpha_\ell}^2$'s are randomly generated from a chi-square distribution with two degrees of freedom and normalized such that $\sum_{\ell=1}^{L} \sigma_{\alpha_\ell}^2 = 1$.

Fig. 2 compares the proposed hybrid precoding with the prior technique in (2) when $N = 64$, $L = 5$, $\sigma_{\mathrm{AS}} = 10$, and SNR$=$ 10dB. Since the prior work is targeted at the case of $M = K$, both are compared in that case for fair comparison, and the simulation is performed without the phase shifter constraints. Fig. 2 shows that the proposed work considerably outperforms the prior work as K and M are not small. In addition, the prior work cannot approach to the fully digital precoding case even when M becomes N.

In Fig. 3, the case of $K < M$ is evaluated, and the proposed constrained hybrid precoding under the phase shifter constraints is compared to the unconstrained case. Fig. 3 shows that the proposed constrained hybrid precoding has almost the same sum spectral efficiency as the unconstrained case, and significantly outperforms the prior work in (17).

6. CONCLUSIONS

In this paper, we proposed a hybrid precoding technique that uses long-term channel statistics when computing analog precoders. Under phase shifter constraints, we proposed a compensation matrix that minimizes the rate loss caused by using phase shifters. Simulation results showed that the proposed hybrid precoding outperforms the prior work in sum spectral efficiency, and the gain becomes higher as the number of users increases. The results also showed that the proposed constrained precoding solution combined with the compensation matrix performs close to the unconstrained case.

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