# EMITTER SOURCE LOCALIZATION USING TIME-OF-ARRIVAL MEASUREMENTS FROM SINGLE MOVING RECEIVER

Yanbin Zou and Qun Wan

School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China Email: 201311020327@std.uestc.edu.cn, wanqun@uestc.edu.cn

Abstract—In this paper, we consider using time-of-arrival (TOA) measurements from single moving receiver to locate a stationary source which emits periodical signal. First, we give the TOA measurements model and deduce the Cramér-Rao lower bounds (CRLB). Then, we formulated the maximum likelihood estimation (MLE) problem. We use the semidefinite programming (SDP) method to relax the nonconvex MLE problem into convex problem. It is shown that the original SDP algorithm can not provide a high-quality solution. We jointly add second-order-cone (SOC) constraints and penalty term to improve the tightness of the original SDP algorithm. Besides, we also consider the presence of receiver position errors, and develop the robust localization algorithm. Numerical simulations are conducted to demonstrate the localization performance of the proposed algorithms by comparing with the CRLB.

*Index Term*-Semidefinite programming (SDP), single moving receiver, source localization, time-of-arrival (TOA)

#### I. INTRODUCTION

THE problem of locating a source from time-of-arrival (TOA) measurements or from time-difference-of-arrival (TDOA) measurements using several spatial separated sensors have received significant attention in the signal processing literature because of their importance to many application-s including wireless sensor networks (WSNs) [1], wireless communications [2], intelligent transport [3], and navigation [4]. The TOA measurements require the receiving sensors and source to be synchronized, and the TDOA measurements require the receiving sensors to be synchronized.

For non-cooperative emitter, single platform localization using Doppler and direction-of-arrival (DOA) measurements have been discussed by Becker [5]. In [6], Elad *et al.* propose a stationary source localization method using TOA measurements from single moving receiver. In [6], it is assumed that the signal contains a waveform that appears periodically, besides, the waveform and period are known to the receiver. Using the known signal waveform, the TOA can be estimated by correlating the known signal with the receiving signal; and using the known signal period, the absence of simultaneous measurements collected by multiple stationary stations can be compensated [6]. In this paper, we also consider using single moving receiver to locate a stationary source which emits periodical signal with known signal waveform. However, the signal period is unknown in our research.

The rest of this paper is organized as follows. In Section II, the TOA measurements model is described, and the CRLB is deduced. In Section III, we use the semidefinite programming (SDP) technique to relax the nonconvex maximum likelihood estimator (MLE) problem into convex problem, and jointly use second-order-cone (SOC) constraints and penalty term to improve the tightness of the SDP formulation. In Section IV, we consider the presence of receiver position errors, and then propose two robust algorithms. Simulation results are given in Section V to evaluate the location estimation performance of the proposed algorithms by comparing with CRLB. Finally, we give the conclusions in Section VI.

Notation: Bold uppercase and bold lowercase letters denote matrices and vectors, respectively.  $\mathbf{I}_m$  is the  $m \times m$  identity matrix,  $\mathbf{1}_m$  is the column vector of m ones, and  $\mathbf{0}_{m,n}$  is the  $m \times n$  zero matrix.  $\|\cdot\|$  and  $\|\cdot\|_F$  are the  $l_2$  norm and Frobenius norm, respectively. For arbitrary symmetric matrices of equal size,  $\mathbf{A} \succeq \mathbf{B}$  means that  $\mathbf{A} - \mathbf{B}$  is positive semidefinite.

### II. TOA MEASUREMENTS MODEL AND CRLB

Consider a stationary emitter transmitting periodical signal with period  $\tau$ , and the unknown position is  $\mathbf{u} = [x, y]^T$ (for simplicity, we consider the 2-D scenario, the extension to 3-D case is straight forward). Assuming the source begin to transmit periodical signal at  $t_0$ , and the moving receiver receives the signal waveform at  $t_1$ . The position of the moving receiver at  $t_1$  is  $\mathbf{s}_1 = [x_1, y_1]^T$  which is known, for example, from the GPS. After counting N waveforms, the receiver receives the new signal waveform at  $t_2$  and the position of the moving receiver at  $t_2$  is  $\mathbf{s}_2 = [x_2, y_2]^T$ ; so after counting N(M-1) waveforms, the receiver receives the new waveform at  $t_M$  and the position of the moving receiver at  $t_M$  is  $\mathbf{s}_M = [x_M, y_M]^T$ . Consequently, we can write the TOA measurement equations (In this paper, we assume that the TOA is estimated, and our focus is using the estimated TOA to locate the position of emitter)

$$t_i = t_0 + (i-1)N\tau + \frac{\|\mathbf{u} - \mathbf{s}_i\|}{c} + e_i, \ i = 1, 2, \dots, M.$$
(1)

This work was supported in part by the National Natural Science Foundation of China under Grant U1533125, National Science and technology major project under Grant 2016ZX03001022, and the Fundamental Research Funds for the Central Universities under Grant ZYGX2015Z011.

where c is the signal propagation speed, and  $e_i$  is the TOA measurement noise. In order to eliminate ambiguity, it is assumed that  $\|\mathbf{u} - \mathbf{s}_i\| \leq \tau c$  [6]. In (1), setting  $T = N\tau$  and multiplying both sides by c, we can obtain

$$r_i = t_0 c + (i-1)Tc + \|\mathbf{u} - \mathbf{s}_i\| + n_i, \ i = 1, 2, \dots, M.$$
 (2)

where  $r_i$  is pseudo-range measurement, and  $n_i = e_i c$ . For ease of analysis, we assume that  $n_i$  is a zero-mean white Gaussian variable with known variance  $\sigma_i^2$ . From (2),we can write the MLE problem:

$$\min_{\mathbf{u}, t_0, T} \sum_{i=1}^{M} \frac{(r_i - \|\mathbf{u} - \mathbf{s}_i\| - t_0 c - (i-1)Tc)^2}{\sigma_i^2}$$
(3)

where  $\mathbf{u}$ ,  $t_0$  and T are the optimization parameters. The above problem is nonlinear and nonconvex, and the MLE is hard to achieve.

Given the TOA measurement model, the performance of any unbiased estimate of **u** would be limited by the CRLB. In order to deduce the CRLB of the TOA measurement model, we set the unknown parameter vector as  $\boldsymbol{\theta} = [\mathbf{u}^T, t_0, T]^T$ . The Fisher information matrix is calculated as [7]

$$\mathbf{I}(\boldsymbol{\theta}) = \mathbf{H}(\boldsymbol{\theta})^T \mathbf{Q} \mathbf{H}(\boldsymbol{\theta})$$
(4)

where  $\mathbf{Q} = diag([\sigma_1^{-2}, \sigma_2^{-2}, ..., \sigma_M^{-2}])$ , and

$$\mathbf{H}(\boldsymbol{\theta}) = \begin{bmatrix} \frac{x-x_1}{\|\mathbf{u}-\mathbf{s}_1\|} & \frac{y-y_1}{\|\mathbf{u}-\mathbf{s}_1\|} & c & 0\\ \vdots & \vdots & \vdots & \vdots\\ \frac{x-x_M}{\|\mathbf{u}-\mathbf{s}_M\|} & \frac{y-y_M}{\|\mathbf{u}-\mathbf{s}_M\|} & c & (M-1)c \end{bmatrix}$$
(5)

The CRLB of the source position u is computed as

$$Var(\mathbf{u}) \ge [\mathbf{I}^{-1}(\boldsymbol{\theta})]_{1,1} + [\mathbf{I}^{-1}(\boldsymbol{\theta})]_{2,2}$$
(6)  
III. LOCALIZATION ALGORITHM

In this section, we describe how a SDP method can be used a solve problem (3). Note that (3) can be expressed as the

to solve problem (3). Note that (3) can be expressed as the matrix-vector form 
$$T = (T - T)^T$$

$$\min_{\mathbf{u},\mathbf{d},t_0,T} \left(\mathbf{r} - \mathbf{d} - t_0 c \mathbf{1}_M - T c \mathbf{q}\right)^T \mathbf{Q} (\mathbf{r} - \mathbf{d} - t_0 c \mathbf{1}_M - T c \mathbf{q})$$
(7a)

s.t. 
$$d_i = \|\mathbf{u} - \mathbf{s}_i\|, \ i = 1, 2, \dots, M.$$
 (7b)

where  $\mathbf{d} = [d_1, d_2, \dots, d_M]^T$ ,  $\mathbf{r} = [r_1, r_2, \dots, r_M]^T$ ,  $\mathbf{q} = [0, 1, \dots, M - 1]^T$ . Set  $\mathbf{h} = [t_0, T]^T$ , and  $\mathbf{F} = [\mathbf{1}_M, \mathbf{q}]$ . Therefore, (7) can be expressed as

$$\min_{\mathbf{u},\mathbf{d},\mathbf{h}} (\mathbf{r} - \mathbf{d} - c\mathbf{F}\mathbf{h})^T \mathbf{Q} (\mathbf{r} - \mathbf{d} - c\mathbf{F}\mathbf{h})$$
(8a)

s.t. 
$$d_i = \|\mathbf{u} - \mathbf{s}_i\|, \ i = 1, 2, \dots, M.$$
 (8b)

Instead of finding  $\mathbf{u}$ ,  $t_0$  and T jointly, we find the optimum  $t_0$  and T as a dependent function of  $\mathbf{u}$ . Letting the gradient of the objective function in (8a) with respect to  $\mathbf{h}$  to zero then gives

$$-2c\mathbf{F}^T\mathbf{Q}(\mathbf{r}-\mathbf{d}-c\mathbf{F}\mathbf{h}) = \mathbf{0}$$
(9)

As a result, the optimum estimation of h is

$$\hat{\mathbf{h}} = \frac{1}{c} (\mathbf{F}^T \mathbf{Q} \mathbf{F})^{-1} \mathbf{F}^T \mathbf{Q} (\mathbf{r} - \mathbf{d})$$
(10)

Next, substituting  $\hat{\mathbf{h}}$  into (8), we can obtain

$$\min_{\mathbf{u},\mathbf{d}} \ (\mathbf{r} - \mathbf{d})^T \mathbf{G} (\mathbf{r} - \mathbf{d})$$
(11a)

s.t. 
$$d_i = \|\mathbf{u} - \mathbf{s}_i\|, i = 1, 2, \dots, M.$$
 (11b)

where  $\mathbf{G} = (\mathbf{I}_M - \mathbf{F}(\mathbf{F}^T\mathbf{Q}\mathbf{F})^{-1}\mathbf{F}^T\mathbf{Q})^T\mathbf{Q}(\mathbf{I}_M - \mathbf{F}(\mathbf{F}^T\mathbf{Q}\mathbf{F})^{-1}\mathbf{F}^T\mathbf{Q})$ . We can see that (11a) is convex with respect to the unknown variables d. However, (11b) is nonconvex with respect to the unknown variables d and u. Using the semidefinite positive relaxation (SDR) [8] method, we can obtain

$$\min_{\mathbf{d},\mathbf{D},\mathbf{u},y_s} tr(\mathbf{D}\mathbf{G}) - 2\mathbf{r}^T \mathbf{G}\mathbf{d} + \mathbf{r}^T \mathbf{G}\mathbf{r}$$
(12a)

s.t. 
$$\mathbf{D}_{i,i} = \mathbf{y}_s - 2\mathbf{u}^T \mathbf{s}_i + \mathbf{s}_i^T \mathbf{s}_i, i = 1, 2, \dots, M.$$
 (12b)  
 $\mathbf{D}_{i,j} \ge |\mathbf{y}_s - \mathbf{u}^T (\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T \mathbf{s}_j|, 1 \le i < j \le M.$  (12c)

$$\begin{bmatrix} \mathbf{1} & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0}_{M+1,M+1}$$
(12d)

$$\begin{bmatrix} \mathbf{I}_2 & \mathbf{u} \\ \mathbf{u}^T & y_s \end{bmatrix} \succeq \mathbf{0}_{3,3} \tag{12e}$$

where (12c) is deduced by the Cauchy-Schwartz inequality. The above convex optimization formulation is not tight. The reason is that **G** is singular [9]. In order to improve the tightness of (12), similar to [9], we jointly add SOC constraints and penalty term in (12). First, adding the penalty term  $\eta tr(\mathbf{D})$  in the objective function can let  $(\mathbf{G} + \eta \mathbf{I})$  be definite positive. Second, adding SOC constraints [10]

$$\|\mathbf{u} - \mathbf{s}_i\| \le d_i, \ 1 \le i \le M.$$

$$(13)$$

that make the whole constraints be tight.

**1**Т

Finally, we obtain an improved SDP algorithm

$$\min_{\mathbf{d},\mathbf{D},\mathbf{u},\mathbf{y}_s} tr(\mathbf{D}\mathbf{G}) - 2\mathbf{r}^T \mathbf{G}\mathbf{d} + \mathbf{r}^T \mathbf{G}\mathbf{r} + \eta tr(\mathbf{D})$$
(14a)

s.t. 
$$\mathbf{D}_{i,i} = \mathbf{y}_s - 2\mathbf{u}^T \mathbf{s}_i + \mathbf{s}_i^T, \ i = 1, 2, \dots, M.$$
 (14b)

$$\|\mathbf{u} - \mathbf{s}_i\| \le d_i, \ i = 1, 2, \dots, M.$$
(14c)  
$$\mathbf{D}_{i,j} \ge |\mathbf{y}_s - \mathbf{u}^T(s_i + s_j) + \mathbf{s}_i^T \mathbf{s}_j|, \ 1 \le i < j \le M.$$

$$\begin{bmatrix} \mathbf{1} & \mathbf{a}^{T} \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0}_{M+1,M+1}$$
(14e)

$$\begin{bmatrix} \mathbf{I}_2 & \mathbf{u} \\ \mathbf{u}^T & y_s \end{bmatrix} \succeq \mathbf{0}_{3,3} \tag{14f}$$

where  $\eta$  is a regularization parameter. A suitable selection of  $\eta$  is important to achieve a good estimation. However, it is not easy to obtain the opatimal  $\eta$  [9]. As a result, similar to [9], we take K constant values  $\eta_k$ , k = 1, 2, ..., K to compute the (14), and then use the estimated results  $\hat{\mathbf{u}}_k$ , k = 1, 2, ..., K to select the optimal  $\hat{\mathbf{u}}$  that give the minimum  $J_k$ 

$$J_k = (\mathbf{r} - \hat{\mathbf{d}}_k)^T \mathbf{G} (\mathbf{r} - \hat{\mathbf{d}}_k), \ k = 1, 2, \dots, K.$$
(15)

where  $\hat{\mathbf{d}}_k = [\hat{d}_{k1}, \hat{d}_{k2}, \dots, \hat{d}_{kM}]^T$ , and  $\hat{d}_{ki} = \|\hat{\mathbf{u}}_k - \mathbf{s}_i\|$ ,  $i = 1, 2, \dots, M$ .

#### IV. ROBUST LOCALIZATION ALGORITHMS WITH RECEIVER POSITION ERRORS

In the preceding development of (12) and (14), we assume that the positions of receiver are accurate. However, in practice the receiver positions may not be exact because of the imperfection of navigation system. In this section, we will focus on developing robust localization algorithm under the presence of receiver position errors.

Under the condition of receiver position errors, the receiver positions can be expressed as

$$\mathbf{a}_i = \mathbf{s}_i + \boldsymbol{\beta}_i, \ i = 1, 2, \dots, M.$$
(16)

where  $\beta_i$  denotes the zero-mean white Gaussian vector with known covariance  $\delta_i^2 \mathbf{I}_2$ . Besides, we assume that  $n_i$  and  $\beta_i$  are mutually independent. We can write the MLE problem:

$$\min_{\mathbf{u},\mathbf{s}_{i},t_{0},T} \sum_{i=1}^{M} \frac{(r_{i} - \|\mathbf{u} - \mathbf{s}_{i}\| - t_{0}c - (i-1)Tc)^{2}}{\sigma_{i}^{2}} + \sum_{i=1}^{M} \frac{\|\mathbf{a}_{i} - \mathbf{s}_{i}\|^{2}}{\delta_{i}^{2}}$$
(17a)

where  $\mathbf{u}, \mathbf{s}_i, t_0$  and T are the optimization parameters.

Next, we will deduce the CRLB under receiver position errors. Set the unknown parameter vector as  $\boldsymbol{\kappa} = [\mathbf{u}^T, t_0, T, \mathbf{s}_1^T, \mathbf{s}_2^T, \dots, \mathbf{s}_M^T]^T$ . The Fisher information matrix of  $\boldsymbol{\kappa}$  is calculated as [7]

$$\mathbf{I}(\boldsymbol{\kappa}) = \mathbf{I}_1(\boldsymbol{\kappa}) + \mathbf{I}_2(\boldsymbol{\kappa}) \tag{18}$$

where  $\mathbf{I}_1(\boldsymbol{\kappa}) = \mathbf{P}_1(\boldsymbol{\kappa})\mathbf{Q}\mathbf{P}_1^T(\boldsymbol{\kappa}), \ \mathbf{I}_2(\boldsymbol{\kappa}) = \mathbf{P}_2(\boldsymbol{\kappa})\mathbf{W}\mathbf{P}_2^T(\boldsymbol{\kappa}), \\ \mathbf{W} = diag([\delta_1^{-2}, \delta_1^{-2}, \delta_2^{-2}, \delta_2^{-2}, \dots, \delta_M^{-2}, \delta_M^{-2}]),$ 

$$\mathbf{P}_{1}(\boldsymbol{\kappa}) = \begin{bmatrix} \frac{\mathbf{u}-\mathbf{s}_{1}}{\|\mathbf{u}-\mathbf{s}_{1}\|} & \frac{\mathbf{u}-\mathbf{s}_{2}}{\|\mathbf{u}-\mathbf{s}_{2}\|} & \cdots & \frac{\mathbf{u}-\mathbf{s}_{M}}{\|\mathbf{u}-\mathbf{s}_{M}\|} \\ c & c & \cdots & c \\ 0 & c & \cdots & (M-1)c \\ -\frac{\mathbf{u}-\mathbf{s}_{1}}{\|\mathbf{u}-\mathbf{s}_{1}\|} & \mathbf{0}_{2,1} & \cdots & \mathbf{0}_{2,1} \\ \mathbf{0}_{2,1} & -\frac{\mathbf{u}-\mathbf{s}_{2}}{\|\mathbf{u}-\mathbf{s}_{2}\|} & \cdots & \mathbf{0}_{2,1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{2,1} & \mathbf{0}_{2,1} & \cdots & -\frac{\mathbf{u}-\mathbf{s}_{M}}{\|\mathbf{u}-\mathbf{s}_{M}\|} \end{bmatrix}$$
(19)

and

$$\mathbf{P}_{2}(\boldsymbol{\kappa}) = \begin{bmatrix} \mathbf{0}_{2,2} & \mathbf{0}_{2,2} & \dots & \mathbf{0}_{2,2} \\ \mathbf{0}_{1,2} & \mathbf{0}_{1,2} & \dots & \mathbf{0}_{1,2} \\ \mathbf{0}_{1,2} & \mathbf{0}_{1,2} & \dots & \mathbf{0}_{1,2} \\ \mathbf{I}_{2} & \mathbf{0}_{2,2} & \dots & \mathbf{0}_{2,2} \\ \mathbf{0}_{2,2} & \mathbf{I}_{2} & \dots & \mathbf{0}_{2,2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{2,2} & \mathbf{0}_{2,2} & \dots & \mathbf{I}_{2} \end{bmatrix}$$
(20)

The CRLB of the source position u is computed as

$$Var(\mathbf{u}) \ge [\mathbf{I}^{-1}(\boldsymbol{\kappa})]_{1,1} + [\mathbf{I}^{-1}(\boldsymbol{\kappa})]_{2,2}$$
(21)

Note that (17) can be written in the matrix-vector form

$$\min_{\mathbf{u},\mathbf{s}_i,t_0,T} (\mathbf{r} - \mathbf{d})^T \mathbf{G}(\mathbf{r} - \mathbf{d}) + \left\| (\mathbf{A} - \mathbf{X}(:, 2: M+1)) \mathbf{W}_1^{\frac{1}{2}} \right\|_{H}^{\frac{1}{2}}$$
(22a)

s.t. 
$$d_i = \|\mathbf{X}(:, 1) - \mathbf{X}(:, i+1)\|, \ i = 1, 2, \dots, M.$$
(22b)

where  $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_M]$ ,  $\mathbf{X} = [\mathbf{u}, \mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_M]$ ,  $\mathbf{W}_1 = diag([\delta_1^{-2}, \delta_2^{-2}, \dots, \delta_M^{-2}])$ . Using the SDR [8] method, we can obtain

$$\min_{\mathbf{d},\mathbf{D},\mathbf{X},\mathbf{Y}} tr(\mathbf{D}\mathbf{G}) - 2\mathbf{r}^{T}\mathbf{G}\mathbf{d} - 2tr(\mathbf{W}_{1}\mathbf{A}^{T}\mathbf{X}(:, 2: M+1)) + tr(\mathbf{W}_{1}\mathbf{Y}(2: M+1, 2: M+1))$$
(23a)  
s.t.  $\mathbf{D}_{i,i} = \mathbf{Y}(1, 1) - 2\mathbf{Y}(1, i+1) + \mathbf{Y}(i+1, i+1),$ 

$$i = 1, 2, \dots, M.$$
 (23b)

$$D_{i,j} \ge |\mathbf{Y}(1,1) - \mathbf{Y}(1,i+1) - \mathbf{Y}(1,j+1) + \mathbf{Y}(i+1,j+1)|, \ 1 \le i \le j \le M.$$
(23c)

$$\mathbf{D} \succeq \mathbf{d} \mathbf{d}^T \tag{23d}$$

$$\mathbf{Y} \succeq \mathbf{X}^T \mathbf{X}$$
(23e)

$$\min_{\mathbf{D},\mathbf{X},\mathbf{Y}} tr(\mathbf{D}\mathbf{G}) - 2\mathbf{r}^{T}\mathbf{G}\mathbf{d} - 2tr(\mathbf{W}\mathbf{A}^{T}\mathbf{X}(:, 2: M + 1)) + tr(\mathbf{W}\mathbf{Y}(2: M + 1, 2: M + 1)) + \eta tr(\mathbf{D})$$
(24a)  
s.t.  $\mathbf{D}_{i,i} = \mathbf{Y}(1, 1) - 2\mathbf{Y}(1, i + 1) + \mathbf{Y}(i + 1, i + 1),$   
 $i = 1, 2, \dots, M.$  (24b)  
 $\|\mathbf{Y}(\cdot, 1) - \mathbf{Y}(: i + 1)\| \leq d \quad i = 1, 2, \dots, M$ 

$$\|\mathbf{X}(:,1) - \mathbf{X}(:,i+1)\| \le d_i, \ i = 1, 2, \dots, M.$$
(24c)

$$\mathbf{D}_{i,j} \ge |\mathbf{Y}(1,1) - \mathbf{Y}(1,i+1) - \mathbf{Y}(1,j+1)|$$

$$+ \mathbf{Y}(i+1, j+1)|, \ 1 \le i < j \le M.$$
(24d)

$$\begin{bmatrix} \mathbf{l} & \mathbf{d}^{T} \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0}_{M+1,M+1}$$
(24e)

$$\begin{bmatrix} \mathbf{1}_2 & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}_{M+3,M+3}$$
 (24f)

Similar to [9], we take K constant values  $\eta_k$ , k = 1, 2, ..., K to compute the (24), and then use the estimated results  $\hat{\mathbf{X}}_k$ , k = 1, 2, ..., K to select the optimal  $\hat{\mathbf{u}} = \hat{\mathbf{X}}(:, 1)$  that give the minimum  $J_k$ 

$$J_k = (\mathbf{r} - \hat{\mathbf{d}}_k)^T \mathbf{G}(\mathbf{r} - \hat{\mathbf{d}}_k), \ k = 1, 2, \dots, K.$$
(25)

where 
$$\hat{\mathbf{d}}_{k} = [\hat{d}_{k1}, \hat{d}_{k2}, \dots, \hat{d}_{kM}]^{T}$$
, and  $\hat{d}_{ki} = \|\hat{\mathbf{X}}_{k}(:, 1) - \mathbf{a}_{i}\|, i = 1, 2, \dots, M.$ 

## V. SIMULATION RESULTS

In this section, we conduct several numerical simulations to demonstrate the performance of the four proposed SDP algorithms: (12) (label as Proposed-1), (14) (label as Proposed-2), (23) (label as Proposed-3) and (24) (label as Proposed-4). The four SDP algorithms are implemented by CVX toolbox [11] using SDPT3 as a solver [12]. 1000 Monte Carlo realizations were done in the following simulations.

 $\mathbf{d}$ 

We let the receiver moving in a 2-dimensional plane, the signal period  $\tau$  is set to 1ms, and the number Nbetween two TOA measurements is set to 4000, so the corresponding  $T = N \times \tau$  is 4s. The emitter source starting transmission time  $t_0$  is drawn from an uniform distribution [0, 1]s. The receiver moving trajectory is a count-clockwise circle, and the positions of receiver at different measurement times are:  $[400, 0]^T m$ ,  $[200, 200\sqrt{3}]^T m$ ,  $[-200, 200\sqrt{3}]^T m$ ,  $[-400, 0]^T m$ ,  $[-200, -200\sqrt{3}]^T m$ ,  $[200, -200\sqrt{3}]^T m$ . The variance of noise are assumed to be identical, i.e.,  $\sigma_i^2 = \sigma^2$ ,  $\delta_i^2 = \delta^2$ ,  $i = 1, 2, \ldots, M$ . In the following simulations, we set  $\eta_1 = 10^{-7}$ ,  $\eta_2 = 10^{-6}$ ,  $\eta_3 = 10^{-5}$ ,  $\eta_4 = 10^{-4}$ ,  $\eta_5 = 10^{-3}$ for the computation of (14) and (24).



Fig. 1. RMSE vs.  $\sigma$ ,  $\mathbf{u} = [100, -52]^T m$ .



Fig. 2. RMSE vs.  $\sigma$ ,  $\mathbf{u} = [600, 594]^T m$ .

In Fig. 1, we test the performance of the Proposed-1 and Proposed-2 algorithms to the varying of  $\sigma$  when the source position is located at  $\mathbf{u} = [100, -52]^T m$  which is inside the receiver moving trajectory. From Fig. 1, it can be seen that the Proposed-2 algorithm is superior to the Proposed-1 algorithm, which is according with the study in Section III. And the Proposed-2 algorithm can reach the CRLB.

In Fig. 2, the source position is located outside the receiver moving trajectory, i.e.,  $\mathbf{u} = [600, 594]^T m$ . From the figure, it can be seen that the Proposed-2 algorithm is also superior to the Proposed-1 algorithm, and Proposed-2 algorithm is very close to the CRLB.



Fig. 3. RMSE vs.  $\sigma$ , **u** is uniformly and randomly chosen inside a circle region:  $\|\mathbf{u}\| \leq 1000m$ .



Fig. 4. RMSE vs.  $\delta$  when  $\sigma = 1m$ , **u** is uniformly and randomly chosen inside a circle region:  $||\mathbf{u}|| \le 1000m$ .

In Fig. 3, the source position is uniformly and randomly chosen inside a circle region. From Fig. 3, it can be seen that the location estimation performance of the Proposed-2 is also superior to Proposed-1.

In Fig. 4, we test the performance of the Proposed-3 and Proposed-4 algorithms when consider the receiver position errors. The source position is uniformly and randomly chosen inside a circle region. From Fig. 4, it can be seen that the Proposed-4 algorithm is superior to the Proposed-3 algorithm, and is close to the CRLB.

#### VI. CONCLUSIONS

We present a single moving receiver location model based on TOA measurement in this paper. First, we give the measurements model and deduce the CRLB. Second, we use the SDP technique to relax the nonconvex MLE problem into convex problem, and obtain the original SDP localization algorithm. More importantly, we jointly use the the SOC constraints and penalty term to improve the tightness of the original SDP algorithm. Besides, we also consider the presence of receiver position errors, and we develop the robust localization algorithm for position errors. Finally, from the simulation results, it is shown that the modified SDP algorithms are superior to the original SDP algorithms both for accurate and non-accurate receiver position.

#### REFERENCES

- A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 24–40, July 2005.
- [2] J. J. Caffery Jr, Wireless Location in CDMA Cellular Radio Systems. Springer, 2006.
- [3] Y. Zhao, "Mobile phone location determination and its impact on intelligent transportation systems," *IEEE Trans. Intell. Transp. Syst.*, vol. 1, no. 1, pp. 55–64, Mar. 2000.
- [4] J. B.-Y. Tsui, Fundamentals of Global Positioning System Receivers. New York: Wiley, 2000.
- [5] K. Becker, "An efficient method of passive emitter location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 4, pp. 1091–1104, Oct. 1992.
  [6] E. Tzoreff, B. Z. Bobrovsky, and A. J. Weiss, "Single receiver emitter
- [6] E. Izoreff, B. Z. Bobrovsky, and A. J. Weiss, "Single receiver emitter geolocation based on signal periodicity with oscillator instability," *IEEE Trans. Signal Process.*, vol. 62, no. 6, pp. 1377–1385, Mar. 2014.

- [7] S. M. Kay, Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory. Prentice Hall, 1993.
- [8] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.
- [9] Y. Zou and Q. Wan, "Asynchronous time-of-arrival-based source localization with sensor position uncertainties," DOI: 10.1109/LCOM-M.2016.2589930, IEEE Commun. Lett., 2016.
- [10] K. Yang, G. Wang, and Z.-Q. Luo, "Efficient convex relaxation methods for robust target localization by a sensor network using time differences of arrivals," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2775–2784, July 2009.
- [11] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.
- [12] K.-C. Toh, M. J. Todd, and R. H. Tütüncü, "Sdpt3-a matlab software package for semidefinite programming, version 1.3," *Optimization meth*ods and software, vol. 11, no. 1-4, pp. 545–581, 1999.