COMPRESSIVE PULSE-DOPPLER RADAR SENSING VIA 1-BIT SAMPLING WITH TIME-VARYING THRESHOLD

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ABSTRACT

This paper proposes a compressive pulse-Doppler radar that works through one-bit quantization of the received noisy signal. The one-bit quantization is performed by comparing the signal with a time-varying reference level. Considering the sparsity of the targets in the range-Doppler domain, the problem is dealt with by a sparse recovery method. The proposed method leads to an optimization problem that can be tackled by a convex approximation. Numerical examples show that the proposed method has a promising performance in the detection/estimation of the target parameters. Moreover, it is seen that in low signal to noise ratio, increasing the sampling rate at the receiver side is a compensating factor that effectively improves the performance.

Index Terms— One-bit quantization, compressive sampling, pulse-Doppler radar

1. INTRODUCTION

Signal quantization is a key task in digital signal processing applications. The most ideal case of quantization in terms of signal amplitude resolution is to have infinite precision samples. In practice however, the amplitude quantization precision (or equivalently the quantization bit-depth) is in tradeoff with the sampling rate, cost, and energy consumption.

From the bit-depth point of view, the most extreme form of quantization is to reduce the signal to one bit per sample, which can be performed simply by comparing the signal to a known reference level. This way, one-bit sampling is in fact to treat the quantized measurements as sign values instead of their real values. A main advantage of one-bit quantization is that it allows very high sampling rates, at low cost and with low energy consumption. In some applications, the power consumption of one-bit sampling at a rate of 240 gigasamples per second, is only about 10 milliwatts which is less than 5 percent of the power that a conventional analog-to-digital converter (ADC) typically consumes [1,2]. This energy efficiency is one of the motivating factors for using one-bit sampling in millimeter wave communications and massive multiple-input multiple-output communications systems [4]. Moreover, the conventional ADC is rather expensive[1]; this is while the one-bit sampling is extremely cheap, allowing for a totally affordable system.

One-bit sampling has so far been studied in the literature from different perspectives. Some papers have looked into the topic in a classical statistical framework [5–9]. The topic has also been studied from a sampling/reconstruction viewpoint in works such as [10] and more recently in [11].

Most of the recent works on one-bit sampling however, study the problem from a compressive sensing viewpoint [12–23]. It has been shown that sparse signals can be recovered with high accuracy from a sufficiently large record of one-bit measurements [15]. The early works in compressive one-bit sampling share a common limiting feature, which is considering a fixed quantization threshold (usually zero). Indeed, as argued in [21], with this limitation it is not possible to determine the actual energy of the unknown signal. Some of the recent papers in contrast, have considered random time-varying thresholds [21–23]. Specifically in [23], the problem of estimating signal parameters after quantization to single bit samples is considered where the one-bit samples are captured by comparing the signal to a time-varying reference level.

The authors of the present paper have recently proposed the idea of a radar sensing via one-bit compressive sampling in [24]. There, it has been shown that by quantizing the received noisy signal to one bit (using time-varying thresholds), it is possible to perform the radar sensing for stationary targets. In this paper, we move on to a more practical (and of course more demanding) case of radar sensing for moving targets which adds the Doppler effect to the scenario. Specifically, we propose a compressive pulse-Doppler radar

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[1]Conventional ADCs can typically cost thousands of dollars, even at sampling rates of 2 or 3 gigasamples per second [3].

that works through one-bit quantization of the received noisy signal, which is performed by comparing the signal with a time-varying threshold. Since the targets are sparse in the range-Doppler domain, by using a sparse recovery method, the radar sensing objective is expressed as an optimization problem that can be tackled numerically. Simulation results illustrate that the proposed method has a promising performance in the sensing of the targets. It is further seen that in low signal to noise ratio (SNR), increasing the sampling rate at the receiver compensates for the SNR, hence improving the performance.

The remainder of the paper is organized as follows. In Section 2, we derive our model for the one-bit compressive pulse-Doppler radar and the relevant sensing problem is formulated. Then we propose a solution which employs a norm-based sparse framework. Numerical examples are then presented in Section 3 to show the accuracy of the proposed method. Finally, Section 4 concludes the paper.

2. PROBLEM STATEMENT AND THE PROPOSED SOLUTION

2.1. Problem Formulation

Considering a pulse-Doppler radar under a single-input single-output setup, the transmitted multi-pulse intra-coded signal (shown in Fig. 1) is given by

$$s_T(t) = \sum_{\ell=0}^{L-1}\sum_{n=0}^{N-1} c_n p(t - n\tau_0 - \ell T_p), \tag{1}$$

where $p(\cdot)$ is the basic sub-pulse with width τ_0, N is the number of sub-pulses in each pulse, L is the total number of pulses, and $\{c_n\}_{n=0}^{N-1}$ is the code sequence for all pulses, i.e.,

$$\forall \ell = 0, \ldots, L-1, \qquad c_{n,\ell} = c_n. \tag{2}$$

Now suppose that the range and Doppler domain of the radar are grided into K_r and K_d bins, respectively. Then, at the receiver we have

$$\begin{aligned} s_{Rec}(t) &= \sum_{k_\mathrm{r}=1}^{K_\mathrm{r}}\sum_{k_\mathrm{d}=1}^{K_\mathrm{d}} \alpha_{k_\mathrm{r}k_\mathrm{d}} s_T(t-\tau_{k_\mathrm{r}}) e^{j\omega_{k_\mathrm{d}}t} + \epsilon(t) \\ &= \sum_{k_\mathrm{r},k_\mathrm{d}}\sum_{\ell,n} \alpha_{k_\mathrm{r}k_\mathrm{d}} c_n p(t-n\tau_0-\ell T_p-\tau_{k_\mathrm{r}}) e^{j\omega_{k_\mathrm{d}}t} + \epsilon(t), \end{aligned}$$

where τ_{k_r}, ω_{k_d} and $\alpha_{k_\mathrm{r}k_\mathrm{d}}$ are the time delay, Doppler frequency, and the (complex-valued) gain associated with the target with the index pair $(k_\mathrm{r}, k_\mathrm{d})$ in the range/Doppler domain, and $\epsilon(t)$ is the additive noise. Define the gain matrix

$$\boldsymbol{\alpha} \triangleq \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1K_\mathrm{d}} \\ \vdots & \ddots & \vdots \\ \alpha_{K_\mathrm{r}1} & \cdots & \alpha_{K_\mathrm{r}K_\mathrm{d}} \end{bmatrix}, \tag{3}$$

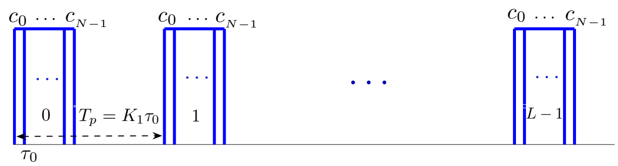

Fig. 1. The transmitted multi-pulse intra-coded signal.

and let

$$\mathbf{f}(t) \triangleq \begin{bmatrix} \sum_{\ell=0}^{L-1}\sum_{n=0}^{N-1} c_n p(t-n\tau_0-\ell T_p-\tau_1) \\ \vdots \\ \sum_{\ell=0}^{L-1}\sum_{n=0}^{N-1} c_n p(t-n\tau_0-\ell T_p-\tau_{K_\mathrm{r}}) \end{bmatrix}, \tag{4}$$

where $T_p = K_\mathrm{r}\tau_0$. Defining

$$\boldsymbol{\phi}(t) \triangleq [e^{j\omega_1 t}, \ \cdots, \ e^{j\omega_{K_\mathrm{d}} t}]^T, \tag{5}$$

the received signal can be restated as follows

$$s_{Rec}(t) = \mathbf{f}^T(t)\boldsymbol{\alpha}\boldsymbol{\phi}(t) + \epsilon(t), \tag{6}$$

where $(\cdot)^T$ denotes the transpose.

In order to quantize the received signal, it is compared to a time-varying threshold $h(t) \triangleq h_R(t) + ih_I(t) \in \mathbb{C}$, and the sign of the resulting difference is observed for the real and imaginary parts. Let $y(t) \triangleq y_R(t) + iy_I(t)$ denote the observed data at time t, i.e.,

$$\begin{aligned} y_R(t) &= \operatorname{sgn}(\operatorname{Re}[s_{Rec}(t)] - h_R(t)) \\ &= \operatorname{sgn}\left(\operatorname{Re}\left[\mathbf{f}^T(t)\boldsymbol{\alpha}\boldsymbol{\phi}(t) + \epsilon(t)\right] - h_R(t)\right), \end{aligned} \tag{7}$$

and

$$\begin{aligned} y_I(t) &= \operatorname{sgn}(\operatorname{Im}[s_{Rec}(t)] - h_I(t)) \\ &= \operatorname{sgn}\left(\operatorname{Im}\left[\mathbf{f}^T(t)\boldsymbol{\alpha}\boldsymbol{\phi}(t) + \epsilon(t)\right] - h_I(t)\right), \end{aligned} \tag{8}$$

in which $\operatorname{Re}[\cdot]$ and $\operatorname{Im}[\cdot]$ denote the real and imaginary parts, respectively, and

$$\operatorname{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases} \tag{9}$$

Next, assume that M samples are captured at times $t_1, \ldots, t_M$, with a rate of r/τ_0 ($r \in \mathbb{Z}^+$) samples-per-second according to

$$t_m = m\frac{\tau_0}{r}, \qquad m = 1, \ldots, M. \tag{10}$$

Let $\mathbf{h} \triangleq \mathbf{h}_R + i\mathbf{h}_I$, $\boldsymbol{\epsilon}$, and $\mathbf{y} \triangleq \mathbf{y}_R + i\mathbf{y}_I$ respectively denote the vector of the thresholds, the vector of the additive noise samples, and the vector of the quantized observed data i.e.,

$$\begin{cases} \mathbf{h} \triangleq [h(t_1), \ldots, h(t_M)]^T, \\ \boldsymbol{\epsilon} \triangleq [\epsilon(t_1), \ldots, \epsilon(t_M)]^T, \\ \mathbf{y} \triangleq [y(t_1), \ldots, y(t_M)]^T. \end{cases} \tag{11}$$

Defining

$$\mathbf{F} \triangleq \left[\mathbf{f}(t_1)\middle|\mathbf{f}(t_2)\middle|\cdots\middle|\mathbf{f}(t_M)\right]^T, \tag{12}$$

and

$$\mathbf{\Phi} \triangleq [\boldsymbol{\phi}(t_1)|\boldsymbol{\phi}(t_2)|\cdots|\boldsymbol{\phi}(t_M)], \tag{13}$$

the observed data can be expressed compactly as follows

$$\begin{cases} \mathbf{y}_R = \operatorname{sgn}(\operatorname{Re}[\operatorname{Diag}\{\mathbf{F}\boldsymbol{\alpha}\mathbf{\Phi}\} + \boldsymbol{\epsilon}] - \mathbf{h}_R), \\ \mathbf{y}_I = \operatorname{sgn}(\operatorname{Im}[\operatorname{Diag}\{\mathbf{F}\boldsymbol{\alpha}\mathbf{\Phi}\} + \boldsymbol{\epsilon}] - \mathbf{h}_I) \end{cases} \tag{14}$$

where $\operatorname{Diag}\{\cdot\}$ gives the diagonal elements of a matrix.

Now, the problem is to estimate the matrix $\boldsymbol{\alpha}$ from the quantized observed data, i.e., $\mathbf{y}$.

2.2. The Proposed Solution

To deal with the aforementioned problem, we begin by vectorizing the matrix $\boldsymbol{\alpha}$ (column by column) as follows

$$\widetilde{\boldsymbol{\alpha}} \triangleq [\alpha_{11}, \cdots, \alpha_{K_{\mathrm{r}}1} \;\; \alpha_{12}, \cdots, \alpha_{K_{\mathrm{r}}2} \;\; \cdots \;\; \alpha_{1K_{\mathrm{d}}} \cdots \alpha_{K_{\mathrm{r}}K_{\mathrm{d}}}]^T \tag{15}$$

It is verified that $\operatorname{Diag}\{\mathbf{F}\boldsymbol{\alpha}\mathbf{\Phi}\}$ can be recast as

$$\begin{aligned} &\begin{bmatrix} e^{j\omega_1 t_1}\mathbf{f}^T(t_1) & \cdots & e^{j\omega_{K_{\mathrm{d}}} t_1}\mathbf{f}^T(t_1) \\ e^{j\omega_1 t_2}\mathbf{f}^T(t_2) & \cdots & e^{j\omega_{K_{\mathrm{d}}} t_2}\mathbf{f}^T(t_2) \\ \vdots & \vdots & \vdots \\ e^{j\omega_1 t_M}\mathbf{f}^T(t_M) & \cdots & e^{j\omega_{K_{\mathrm{d}}} t_M}\mathbf{f}^T(t_M) \end{bmatrix} \widetilde{\boldsymbol{\alpha}} \\ &= (\mathbf{1}_{1\times K_{\mathrm{d}}} \otimes \mathbf{F}) \odot (\mathbf{\Phi}^T \otimes \mathbf{1}_{1\times K_{\mathrm{r}}})\widetilde{\boldsymbol{\alpha}}, \end{aligned} \tag{16}$$

where $\otimes$ and $\odot$ are the Kronecker and the Hadamard products, respectively, and $\mathbf{1}$ is a matrix of all ones.

Thus, the observed quantized data can be expressed as

$$\begin{cases} \mathbf{y}_R = \operatorname{sgn}\left(\operatorname{Re}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \boldsymbol{\epsilon}] - \mathbf{h}_R\right), \\ \mathbf{y}_I = \operatorname{sgn}\left(\operatorname{Im}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \boldsymbol{\epsilon}] - \mathbf{h}_I\right), \end{cases} \tag{17}$$

in which

$$\widetilde{\mathbf{F}} \triangleq (\mathbf{1}_{1\times K_{\mathrm{d}}} \otimes \mathbf{F}) \odot (\mathbf{\Phi}^T \otimes \mathbf{1}_{1\times K_{\mathrm{r}}}), \tag{18}$$

and $\widetilde{\boldsymbol{\alpha}}$ has been defined in (15). Noting the sparsity of the targets in the range and Doppler domains, it is expected that many of the components of the vector $\widetilde{\boldsymbol{\alpha}}$ will be zero. Thus, the problem is to find a vector $\widetilde{\boldsymbol{\alpha}}$ which is sparse, consistent with the measurement with small fitting error. The problem is therefore expressed in a form similar to the one in [23] and [24], and it thus leads to the following optimization problem

$$\begin{aligned} \min_{\widetilde{\boldsymbol{\alpha}},\mathbf{z}} \quad & \| \mathbf{z} \|_2 + \lambda \| \widetilde{\boldsymbol{\alpha}} \|_0 \\ \text{s.t.} \quad & \mathbf{y}_R \odot (\operatorname{Re}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \mathbf{z}] - \mathbf{h}_R) \geq 0 \\ & \mathbf{y}_I \odot (\operatorname{Im}[\widetilde{\mathbf{F}}\widetilde{\boldsymbol{\alpha}} + \mathbf{z}] - \mathbf{h}_I) \geq 0 \end{aligned} \tag{19}$$

where $\| \cdot \|$ is the zero norm, $\mathbf{z}$ is the fitting error, and λ is a user parameter which adjusts the sparsity of the result. Note that the zero norm (with which the problem is hard to solve) can be well approximated by the ℓ_1-Norm ($\| \cdot \|_1$) that makes the problem convex and tractable [25].

In the sequel, (without loss of generality) assume that the Doppler grids are equally spaced, i.e.,

$$\omega_{k_{\mathrm{d}}} = -\omega_D + (k_{\mathrm{d}} - 1)\Delta, \quad \text{for } \; k_{\mathrm{d}} = 1, \ldots, K_{\mathrm{d}} \tag{20}$$

where ω_D is the maximum possible Doppler frequency of the potential target, and $\Delta = \frac{2\omega_D}{K_{\mathrm{d}}-1}$. In this way, we have

$$\boldsymbol{\phi}(t) = [e^{-j\omega_D t}, \; e^{-j(\omega_D - \Delta)t}, \cdots, e^{j(\omega_D-\Delta)t}, \; e^{j\omega_D t}]^T. \tag{21}$$

Then, from (21) and (10) it can be seen that $\mathbf{\Phi}$ is a Vandermonde matrix given by

$$\mathbf{\Phi} = \left[\boldsymbol{\phi}\middle|\boldsymbol{\phi}^{(2)}\middle|\cdots\middle|\boldsymbol{\phi}^{(M)}\right] \;, \tag{22}$$

where and $\boldsymbol{\phi}^{(i)}$ denotes the i'th Hadamard-power of $\boldsymbol{\phi}$.

$$\boldsymbol{\phi} = \left[e^{-j\omega_D \frac{\tau_0}{r}}, \; e^{-j(\omega_D - \Delta)\frac{\tau_0}{r}}, \cdots, \; e^{j\omega_D \frac{\tau_0}{r}}\right]^T, \tag{23}$$

Similarly we assume that the range grids are also equally spaced such that they represent delay steps equal to τ_0/r, i.e.,

$$\tau_{k_{\mathrm{r}}} = k_{\mathrm{r}}\frac{\tau_0}{r}, \quad k_{\mathrm{r}} = 1, \ldots, K_{\mathrm{r}} \tag{24}$$

For $n = 0, \ldots, N-1$, defining $\mathbf{c}_n \triangleq c_n\mathbf{1}_{r\times 1}$, and assuming reasonably that $K_r > rN$, from (2), (4), (10), (12), and (24) it can be seen that the matrix $\mathbf{F}$ is given by

$$\mathbf{F} = \left[\begin{array}{cccc} \mathbf{c}_0 & & & \\ \vdots & \ddots & & \\ \mathbf{c}_{N-1} & \ddots & \ddots & \\ & \ddots & \ddots & \mathbf{c}_0 \\ & & \ddots & \vdots \\ & & & \mathbf{c}_{N-1} \\ \mathbf{c}_0 & & & \\ \vdots & \ddots & & \\ \mathbf{c}_{N-1} & \ddots & \ddots & \\ & \ddots & \ddots & \mathbf{c}_0 \\ & & \ddots & \vdots \\ & & & \mathbf{c}_{N-1} \\ & \vdots & & \\ & & \vdots & \\ \mathbf{c}_0 & & & \\ \vdots & \ddots & & \\ \mathbf{c}_{N-1} & \ddots & \ddots & \\ & \ddots & \ddots & \mathbf{c}_0 \\ & & \ddots & \vdots \\ & & & \mathbf{c}_{N-1} \end{array}\right] \begin{array}{l} 1 \\ \\ r(N-1)+1 \\ K_r \\ \\ K_r + rN - r \\ rK_r + 1 \\ \\ \\ \\ \\ \\ \\ \\ (L-1)rK_r + 1 \\ \\ \\ \\ \\ \\ \end{array} \tag{25}$$

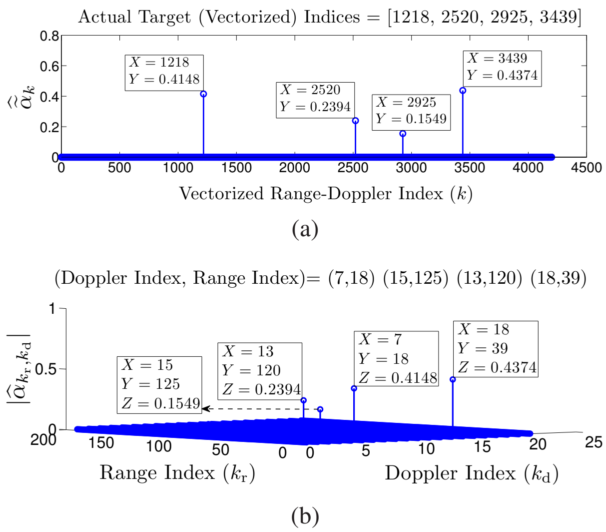

Fig. 2. The sensing performance of the proposed radar system for Example 1: (a) The estimated vectorized target indices $\widehat{\widetilde{\boldsymbol{\alpha}}}$; (b) Range-Doppler representation of the targets.

in which the entries outside the shown repetitive diagonal boxes are all zero, and the column on the right side of the matrix is the starting row index for the relevant inline entry[2].

Having characterized $\mathbf{F}$ and $\mathbf{\Phi}$, we are now ready to evaluate the performance of the proposed method, by numerically solving the minimization problem in (19).

3. NUMERICAL EXAMPLES

In this section, we provide numerical examples to evaluate the performance of the proposed method. In all examples, the CAN algorithm [26] is used to produce the complex code $c_0, \ldots, c_{N-1}$. We consider $\mathbf{h}_R$ and $\mathbf{h}_I$ as independent vectors of i.i.d. random variables uniformly distributed over $[-1,\ 1]$ (which is the range for the normalized amplitude of the reflected signals). Using ℓ_1-norm approximation in (19) and by setting $\lambda = 1$, we use Matlab's CVX toolbox [27] to find $\widetilde{\boldsymbol{\alpha}}$.

In the first two examples, the targets are randomly located by producing random i.i.d. delay and Doppler frequency indices. In Example 3 however, we have used the same target indices as those in Example 2, for the sake of comparison.

Example 1: Consider the scenario with parameters set according to Table 1, in which N_t is the number of targets and the SNR is measured with respect to the weakest reflected signal. As it can be seen in Fig. 2, the proposed method has been able to detect all four targets accurately.

Example 2: Consider the same setup of Example 1 with a lower SNR of 2 dB. As it can be seen in Fig. 3, by decreasing the SNR, an error has occurred in the estimation of the Doppler index for one of the targets. As shown by the red dashed boxes, the Doppler index of the third target has been estimated incorrectly as 18, instead of the actual index 13.

Example 3: Consider the same scenario in Example 2, but with a higher sampling rate of $r = 3$. As it can be observed in Fig. 4, increasing the sampling rate compensates for the low SNR and improves the sensing performance, such that all four targets are identified accurately.

[2]Considering $K_r > rN$, the only case in which the diagonal boxes overlap is with $r = 1$ for which $rK_r + 1 < K_r + rN$.

Table 1. The parameters in Example 1.

N_t	SNR	N	r	K_{r}	K_{d}	L	τ_0	ω_D	λ
4	6 dB	16	2	200	21	5	0.01 ms	2000π rad/sec	1

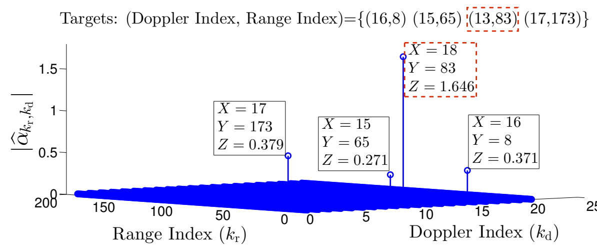

Fig. 3. The performance of the proposed method for Example 2; The erroneous estimated target is shown in dashed box.

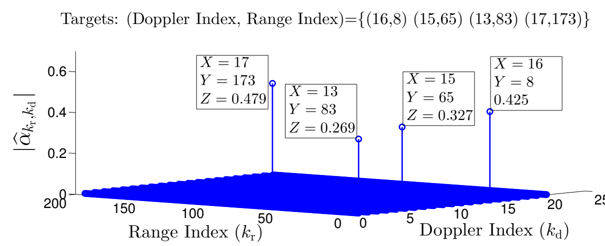

Fig. 4. The sensing performance of the proposed method for Example 3.

4. CONCLUSION

A compressive pulse-Doppler radar based on one-bit quantization of the received noisy signal was proposed. Due to the sparsity of the targets in the range-Doppler domain, the problem was approached by a sparse recovery method which leads to an optimization problem that could be tackled numerically. Numerical examples showed that the proposed sensing method has a promising performance. It was further seen that increasing the sampling rate at the receiver can compensate the performance loss of low SNR. This feature makes the proposed method even more favorable, knowing that one-bit quantization allows for high sampling rates at a low cost.

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