DETECTION RATE OPTIMIZATION IN SURVEILLANCE RADARS WITH TWO-STEP SEQUENTIAL DETECTION

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ABSTRACT

We consider a surveillance radar that requests a second look of a cell under inspection whenever a reliable decision cannot be made after the first observation. The available degrees of freedom are exploited to maximize the detection rate, defined as the average number of detections from a target per unit of time, under a constraint on the false alarm rate, which is the average number of false alarms per unit of time from the inspected area. A performance comparison with fixed-sample-size detection and alert-and-confirm detection is provided.

***Index Terms*—** Radar surveillance, two-step sequential detection, detection rate, false alarm rate, alert-and-confirm, electronically scanned antennas (ESA).

1. INTRODUCTION

Surveillance radars equipped with an electronically scanned antenna can implement sequential detection procedures to improve their sensitivity compared to traditional fixed-sample-size detection (FXD) [1–5]. Truncated sequential procedures are often employed in practice to reduce complexity and avoid the problem of hanging up in a given beam direction for long time. A common method, called energy-variant sequential detection or alert-and-confirm detection (ACD) [6–11], assumes that the radar searches the whole volume and compares the returns from each resolution cell to a threshold: every time the threshold is crossed (alert), a second look is requested to make a definite decision (confirm). Previous works on ACD have analyzed the probability of detection and the cumulative probability of detection, showing that a sensitivity improvement of several decibel is possible over FXD under a constraint on the probability of false alarm. However, these studies do not account for the fact that ACD and FXD present a different measurement update interval, whereby the improved sensitivity might come at the price of a longer scan time; also, a comparative study with a more intuitive procedure where a second look is requested only if a reliable decision cannot be made after the first observation is missing.

In this work, we consider a two-step sequential detection (TSD) procedure, which subsumes both ACD and FXD as special cases. The statistic from each resolution cell is compared to an upper and a lower threshold and a detection is declared if the upper threshold is crossed; all statistics falling in between these two thresholds are deemed as alerts that must be revisited through a second look in order to make a definite decision. Following [12], dwell time and detection thresholds are selected so as to maximize the detection rate (DR), defined as the average number of detections per unit of time from a target, under a constraint on the false alarm rate (FAR), defined as the average number of false alarms per unit of time from the inspected area. Notice that a larger DR reduces the reaction time of the radar and may facilitate subsequent track-before-detect [13–19] and/or tracking [20–22] algorithms, while FAR has a direct impact on the computational requirement for real-time data processing [23, 24]. Overall, DR and FAR allow a fair comparison among detection strategies with a different measurement update interval. Examples are presented to illustrate the effects of this design philosophy and to compare the performance of TSD, ACD, and FXD.

The remainder of this paper is organized as follows. In Sec. 2, the detection strategy and the signal model are described. In Sec. 3, we discuss the system optimization. In Sec. 4, the performance assessment is provided. Finally, concluding remarks are discussed in Sec. 5.

2. SYSTEM DESCRIPTION

We consider a pulse radar observing an angular sector composed of $N_a \in \mathbb{N}$ azimuth bins and $N_r \in \mathbb{N}$ range bins. The transmitted signal is a pulse train with pulse repetition time T, and the received signal undergoes a standard processing chain, which may involve passband-to-baseband conversion, digitalization, range gating, constant false alarm rate (CFAR) processing, clutter mitigation, and pulse integration.

In TSD, either one or two observations are taken from each resolution cell. We assume that the first observation is based on the elaboration of $M_1 \in \mathbb{N}$ pulses, emitted in each azimuth direction, while the second one comes from the elaboration of $M_2 \in \mathbb{N}$ pulses, emitted only in those azimuth directions containing alerts. Hence, the scan time is $T_s = N_a M_1 T + K M_2 T$, where K is the number of revisited azimuth directions, a random variable taking on values in the set $\{0, 1, \ldots, N_a\}$.

Let H_0 and H_1 denote the hypothesis that the target is not present in the range-azimuth cell under test and its alternative, respectively; also, let $y_1 \geq 0$ and $y_2 \geq 0$ be the statistics obtained in the first and second (if any) observation of the cell under test, respectively.[1] Then the test is

$$y_1 \begin{cases} < a, & \text{declare } H_0 \\ \geq b, & \text{declare } H_1 \\ \in [a,b), & \text{take a second observation and} \\ & \quad y_2 \begin{cases} < \eta, & \text{declare } H_0 \\ \geq \eta, & \text{declare } H_1. \end{cases} \end{cases} \tag{1}$$

where $0 \leq a \leq b$ are the boundaries of the sequential test, and $\eta \geq 0$ is the final threshold. Notice that this detection strategy subsumes FXD when $b = a$, since the interval $[a, b)$ degenerates to the empty set and a second look is never requested. When, instead, $b = \infty$, TSD reduces to ACD in [6–10], and the second look is always taken. In the following, we assume that, for ACD, b can be set equal to either a or ∞, so that even ACD subsumes FXD as a special case.

2.1. Signal model

We consider here a simplified signal model, with the goal of capturing the main system trade-offs, while maintaining the problem tractable. We assume that at most one target is present in the inspected area at a random position and that its response is constant over the two observations (whenever the second one is taken), which is usually verified if the second look is taken immediately after the first one, before rotating the antenna beam toward the next azimuth direction. Also, we neglect the cutter[2] and assume that the test statistics in different resolution cells are independent and, under H_0, identically distributed. Finally, for each resolution cell, the test statistics are modeled as

$$y_i = \begin{cases} |\sqrt{M_i \rho_s} s + n_i|^2, & \text{under } H_1 \\ |n_i|^2, & \text{under } H_0 \end{cases} \tag{2}$$

for $i = 1, 2$, where s, n_1 and n_2 are independent unit-variance complex circularly-symmetric Gaussian random variables representing the target response and the noise, respectively, while ρ_s is the signal-to-noise ratio (SNR) per pulse.

3. SYSTEM OPTIMIZATION

The available degrees of freedom for system optimization are the thresholds (a, b, η) and the number of processed pulses (M_1, M_2). Following [12], we propose to maximize DR under a constraint on FAR (computed when no target is present in the whole inspected area), i.e.,

$$\begin{aligned} \max_{\substack{M_1, M_2 \in \mathcal{M} \\ b \geq a \geq 0, \eta \geq 0}} \quad & \mathrm{DR}(a, b, \eta, M_1, M_2) \\ \text{s.t.} \quad & \mathrm{FAR}(a, b, \eta, M_1, M_2) \leq \mathrm{FAR}_{\max} \end{aligned} \tag{3}$$

where $\mathcal{M}$ is the finite set of possible pulse train lengths (tied to the target mobility and/or to the maximum time lag allowed between successive measurements), whose minimum and maximum are denoted $M_{\min}$ and $M_{\max}$, respectively, while $\mathrm{FAR}_{\max} \in \big(0, N_r/(M_{\min}T)\big)$ is the maximum FAR level[3] that can be tolerated (tied to the computational requirement for real-time data processing and/or to the capacity of the human operator to take actions by monitoring the hits visualized on the radar scope).

Next, we provide closed-form expressions of FAR and DR; the derivation is omitted due to the lack of space and can be found in [25]. Let

$$P_{\mathrm{fa},1} = \Pr(y_1 \geq b \mid H_0) = e^{-b} \tag{4a}$$

$$p = \Pr\big(y_1 \in [a,b) \mid H_0\big) = e^{-a} - e^{-b} \tag{4b}$$

$$P_{\mathrm{fa},2} = \Pr\big(y_1 \in [a,b), y_2 \geq \eta \mid H_0\big) = p e^{-\eta} \tag{4c}$$

be the probabilities of having a false alarm in the first look, revisiting a noise-only cell, and having a false alarm in the second look, respectively, and let $P_{\mathrm{fa}} = P_{\mathrm{fa},1} + P_{\mathrm{fa},2}$ be the probability of false alarm in each cell; then, we have

$$\mathrm{FAR} = \begin{cases} \displaystyle\sum_{k=0}^{N_a} \frac{\left(N_r N_a - \frac{k N_r p}{1-(1-p)^{N_r}}\right) \frac{P_{\mathrm{fa},1}}{1-p} + \frac{k N_r P_{\mathrm{fa},2}}{1-(1-p)^{N_r}}}{N_a M_1 T + k M_2 T} \\ \quad \times \binom{N_a}{k} \left(1 - (1-p)^{N_r}\right)^k (1-p)^{N_r(N_a - k)}, & \text{if } p \in (0,1) \\ \dfrac{N_r P_{\mathrm{fa},1}}{M_1 T}, & \text{if } p = 0 \\ \dfrac{N_r P_{\mathrm{fa}}}{(M_1 + M_2) T}, & \text{if } p = 1. \end{cases} \tag{5}$$

Furthermore, let

$$P_{\mathrm{d},1} = \Pr(y_1 \geq b \mid H_1) = e^{\frac{-b}{1+M_1\rho_s}} \tag{6a}$$

$$q = \Pr\big(y_1 \in [a,b) \mid H_1\big) = e^{\frac{-a}{1+M_1\rho_s}} - e^{\frac{-b}{1+M_1\rho_s}} \tag{6b}$$

$$\begin{aligned} P_{\mathrm{d},2} &= \Pr\big(y_1 \in [a,b), y_2 \geq \eta \mid H_1\big) \\ &= \int_0^\infty e^{-\alpha} Q_1\left(\sqrt{2\alpha M_2}, \sqrt{2\eta}\right) \times \end{aligned}$$

[1] We assume here that range/azimuth migration of a prospective target between the two looks can be neglected.

[2] This may be the case of an air-search radar, where the antenna beam points toward the sky and does not collect reflections from the surrounding environment (noise-limited regime).

[3] The lowest FAR is zero, obtained with a probability of false alarm equal to zero, while the largest FAR is $N_r/(M_{\min}T)$, obtained with a probability of false alarm equal to one and $T_s = N_a M_{\min} T$.

$$\left[Q_1\left(\sqrt{2\alpha M_1\rho_s}, \sqrt{2a}\right) - Q_1\left(\sqrt{2\alpha M_1\rho_s}, \sqrt{2b}\right)\right] d\alpha \tag{6c}$$

be the probabilities of detecting the target in the first look, revisiting the resolution bin containing the target, and detecting the target in the second look, respectively, and let $P_{\text{d}} = P_{\text{d},1} + P_{\text{d},2}$ be the probability of detection in each scan; then, we have

$$\text{DR} = \begin{cases} \displaystyle\sum_{k=0}^{N_a-1} \left(\frac{P_{\text{d},1}(1-p)^{N_r-1}}{N_a M_1 T + k M_2 T} \right. \\ \displaystyle\left. + \frac{P_{\text{d},1}\left(1-(1-p)^{N_r-1}\right) + P_{\text{d},2}}{N_a M_1 T + (k+1) M_2 T} \right) \\ \displaystyle\times \binom{N_a-1}{k} \left(1-(1-p)^{N_r}\right)^k (1-p)^{N_r(N_a-1-k)}, \\ \qquad\qquad \text{if } p \in (0,1) \\ \displaystyle\frac{P_{\text{d},1}}{N_a M_1 T} + \frac{P_{\text{d},2}}{N_a M_1 T + M_2 T}, \\ \qquad\qquad \text{if } p = 0 \\ \displaystyle\frac{P_{\text{d},1}}{N_a M_1 T + (N_a-1) M_2} + \frac{P_{\text{d},2}}{N_a (M_1 + M_2) T}, \\ \qquad\qquad \text{if } p = 1 \text{ and } N_r = 1 \\ \displaystyle\frac{P_{\text{d}}}{N_a (M_1 + M_2) T}, \qquad \text{if } p = 1 \text{ and } N_r \geq 2. \end{cases} \tag{7}$$

For FXD, exploiting the results in [12], it can be verified that problem (3) has the following solution

$$M_1^* = \begin{cases} M_{\text{min}}, \text{ if } x^* \leq M_{\text{min}} \\ M_{\text{max}}, \text{ if } x^* \geq M_{\text{max}} \\ \displaystyle\underset{M_1 \in \{[x^*]_l, [x^*]_r\}}{\arg\max} \frac{1}{M_1} \left(\frac{\text{FAR}_{\text{max}} M_1 T}{N_r} \right)^{\frac{1}{1+M_1\rho_s}}, \text{ otherwise} \end{cases} \tag{8a}$$

$$b^* = -\ln\left(\frac{\text{FAR}_{\text{max}} M_1^* T}{N_r} \right) \tag{8b}$$

where $x^* \in \left(0, N_r/(\text{FAR}_{\text{max}} e T)\right)$ is the unique solution to[4]

$$\left(\frac{\text{FAR}_{\text{max}} x T}{N_r} \right)^{\frac{1}{1+x\rho_s}} = \frac{1}{e} \tag{9}$$

and $[x^*]_l$ and $[x^*]_r$ are the left and right nearest neighbors of x^* in $\mathcal{M}$, respectively. It is seen from (8) that the optimal detection threshold must meet the FAR constraint with the equality sign, while the optimal number of integrated pulses attempts to provide a value of P_{d} approximately equal to $\frac{1}{e}$. Clearly, if the pair $(\text{FAR}_{\text{max}}, \rho_s)$ requires a value of x^* smaller than M_{min} or larger than M_{max} to (approximately) meet (9), then the value of P_{d} is necessarily larger or smaller than $\frac{1}{e}$, respectively. For $b > a$, obtaining a closed-form expression for the solution of (3) appears unfeasible; hence, we resort to numerical evaluation.

[4]The left hand side of (9) is P_{d} evaluated at $b = a = b^*$, once the integer variable M_1 is relaxed and replaced by the continuous variable x.

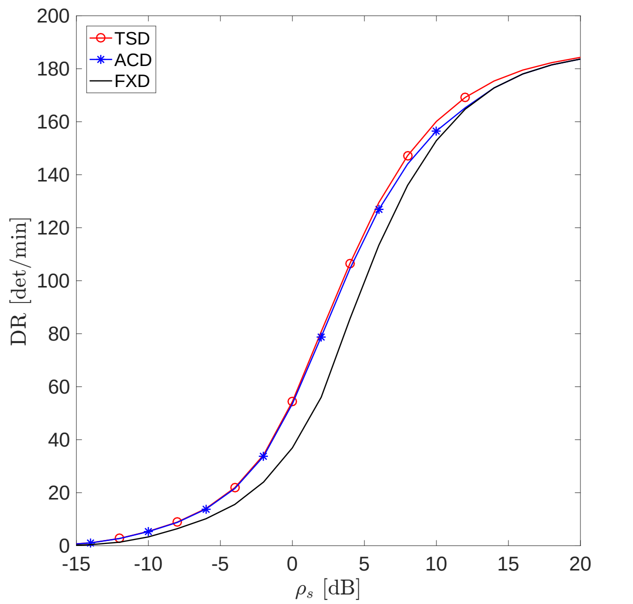

Fig. 1. DR versus ρ_s when $T = 1$ ms, $\mathcal{M} = \{8m\}_{m=1}^{8}$, $N_a = 40$, $N_r = 1000$, and $\text{FAR}_{\text{max}} = 0.5$ fa/min.

4. ANALYSIS

In the following, we set $T = 1$ ms and $\mathcal{M} = \{8m\}_{m=1}^{8}$, and analyze the optimized DR and the corresponding system parameters. DR and FAR are expressed in detections per minute (det/min) and false alarms per minute (fa/min), respectively.

Figure 1 shows the optimized DR as a function of ρ_s when $N_a = 40$, $N_r = 1000$, and $\text{FAR}_{\text{max}} = 0.5$ fa/min. As expected, TSD outperforms FXD, while ACD remains in between. All curves present an S-shaped monotonic growth and converge to the asymptotic value $1/(N_a M_{\text{min}} T)$ for $\rho_s \to +\infty$, as a consequence of the fact that $P_{\text{d}} \to 1$. Also, all curves are lower bounded by $\text{FAR}_{\text{max}}/(N_r N_a)$, which is the asymptotic DR value for FXD in the limit that $\rho_s \to 0$. Interestingly, TSD is competitive with respect FXD in the steep region of the S-shaped curve, which corresponds to ρ_s values in the range $[-10, 15]$ dB for the scenario considered here, while it provides a negligible advantage outside this region.

Figure 2 shows the values of P_{d}, P_{fa}, M_1, and M_2, which yield the optimized DR in Figure 1, as a function of ρ_s. We also include the average scan time (AST) when no target in present in the inspected area. Notice that P_d remains close to $1/e$ for $\rho_s \in [-8, 0]$ dB, not only for FXD, as predicted by (8), but also for TSD; in this region, AST progressively re-

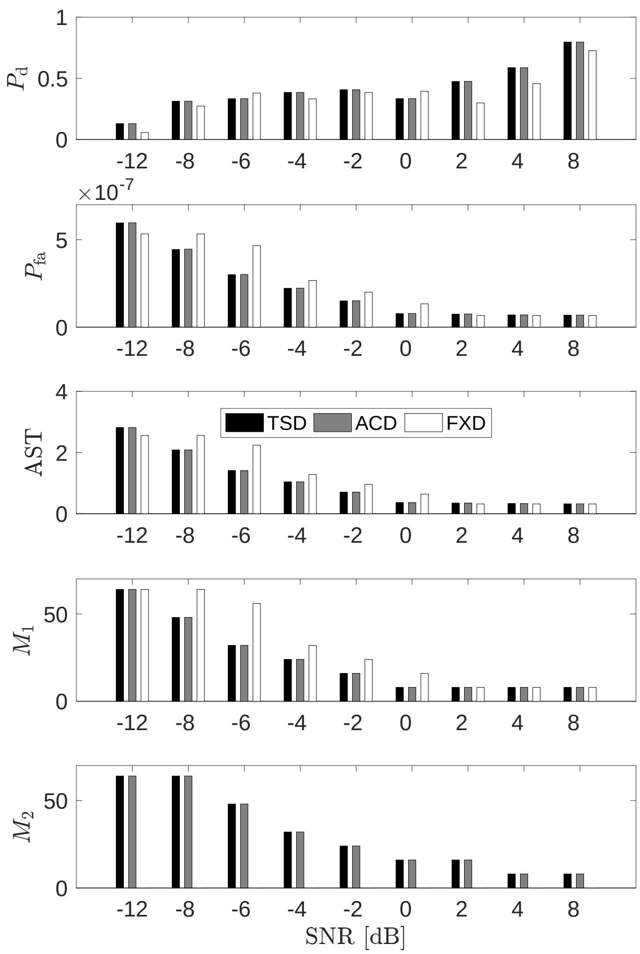

Fig. 2. P_d, P_fa, $\mathrm{AST}_{H_0^*}$, M_1, and M_2 versus ρ_s when $T = 1$ ms, $\mathcal{M} = \{8m\}_{m=1}^{8}$, $N_a = 40$, $N_r = 1000$, and $\mathrm{FAR}_\mathrm{max} = 0.5$ fa/min.

duces as ρ_s increases for all strategies: this is a consequence of the fact that stronger targets can be detected in a shorter time. Interestingly, for similar P_d values, TSD provides a shorter AST with respect to FXD, thus granting a DR gain: this is achieved by shortening the dwell time for each azimuth direction during the first observation and by investing part of the saved time for the revisits. As expected, if AST is reduced, P_fa must be lowered to maintain the same FAR level. If $\rho_s < -8$ dB, P_d gets progressively smaller for all detection strategies, since the dwell time is upper bounded. In this regime, both TSD and FXD use M_max pulses per azimuth direction in the first look, while TSD may take advantage of the second look to possibly achieve a larger P_d: however, as ρ_s gets smaller, the improvement of P_d obtainable at the price of a longer AST becomes less and less rewarding. If $\rho_s > 0$ dB, instead, P_d gets progressively larger for all detection strategies. In this regime, both TSD and FXD use M_min pulses per azimuth direction in the first look, while TSD may take advantage of the second look to possibly achieve a larger P_d at

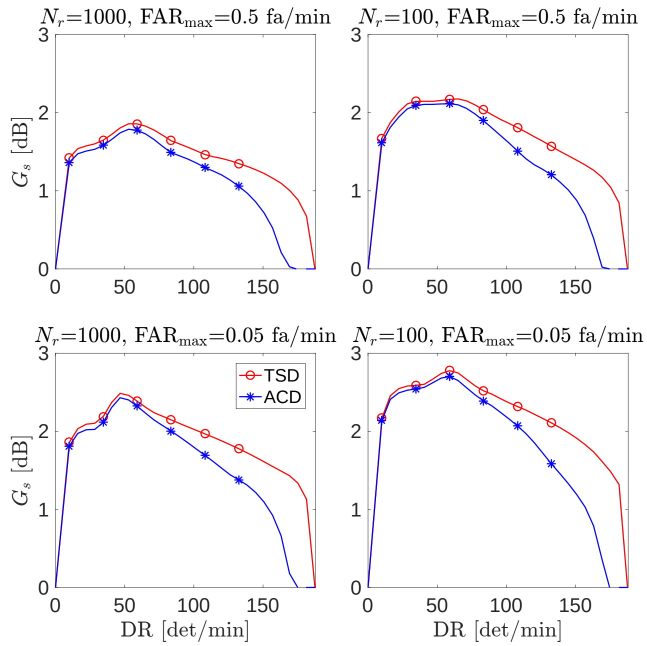

Fig. 3. Sensitivity gain granted by two-step sequential detection (TSD) and alert-and-confirm detection (ACD) with respect to fixed-sample-size detection versus DR when $N_a = 40$, $N_r = 100$ or 1000 and $\mathrm{FAR}_\mathrm{max} = 0.5$ or 0.05 fa/min.

the price of increasing AST: however, using the second look becomes less and less rewarding as ρ_s gets larger.

Finally, Figure 3 reports the sensitivity gain of TSD and ACD with respect to FXD as a function of DR. The sensitivity gain of a procedure A (either TSD or ACD) with respect to FXD at $\mathrm{DR} = x$ is defined as $G_s = \frac{\rho_s|_{\mathrm{DR}=x \text{ for FXD}}}{\rho_s|_{\mathrm{DR}=x \text{ for } A}}$. Four system configurations are considered: $\mathrm{FAR}_\mathrm{max} = 0.5$ fa/min and $N_r = 1000$ (top-left), $\mathrm{FAR}_\mathrm{max} = 0.5$ fa/min and $N_r = 100$ (top-right), $\mathrm{FAR}_\mathrm{max} = 0.05$ fa/min and $N_r = 1000$ (bottom-left), and $\mathrm{FAR}_\mathrm{max} = 0.05$ fa/min and $N_r = 100$ (bottom-right). It is seen by inspection that G_s gets larger as N_r and/or $\mathrm{FAR}_\mathrm{max}$ are decreased; in the two extreme scenarios (top-left and bottom-right) a gain of at most 1.8 and 2.8 dB, respectively, can be achieved by TSD over a wide DR range.

5. CONCLUSIONS

In this work, we have investigated the value of two-step sequential detection when time becomes a resource at stake and new figures of merit accounting for its cost are employed for system optimization and performance assessment. Specifically, we have selected the dwell time and the detection thresholds in order to maximize the detection rate (DR) under a constraint on the false alarm rate (FAR). Results indicate that a second look is unnecessary for extremely small and extremely large signal-to-noise ratios. Also, two-step sequential detection becomes less and less rewarding as the FAR constraint is relaxed or the number of range cells is increased.

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