MOVING TARGET LOCALIZATION IN MULTISTATIC SONAR USING TIME DELAYS, DOPPLER SHIFTS AND ARRIVAL ANGLES

Liu Yang[1], *Le Yang*[1], *K. C. Ho*[2]

[1]School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, China
[2]Department of Electrical and Computer Engineering, University of Missouri, MO 65211, USA

ABSTRACT

Identifying the location of a target is a fundamental application in multistatic sonar. Numerous attempts have been made to improve the accuracy, computational efficiency and robustness of target positioning. Previous studies mostly use time delay and angle measurements for localization, or time delays and Doppler shifts if relative motions exist among the transmitters, target and receivers. This paper considers the joint use of time delay, Doppler shift and angle measurements to locate a moving target. We develop an explicit algebraic solution to the problem, and illustrate the benefit of using all three kinds of measurements. The proposed solution is shown by theoretical performance analysis and confirmed by simulations to be able to reach the Cramer-Rao Bound (CRB) accuracy under Gaussian noise, when the noise level is not significant.

***Index Terms*—** Arrival angle, Doppler shift, localization, multistatic sonar, time delay

1. INTRODUCTION

Sonar localization typically refers to determining the position and possibly velocity of a target using a monostatic, bistatic or multistatic sonar. Traditional sonar has both the transmitter and receiver co-located together. Due to the improved precision as well as better system robustness and flexibility, multistatic sonar with distributed deployment of acoustic transmitters and receivers has been developed for submarine detection, multi-target tracking and other applications [1]-[7]. A multistatic system consists of several transmitter-receiver pairs, where each receiver can observe the direct signal from a transmitter, the echo reflected from the target and the arrival angle of the echo. To locate a stationary target, a common approach is to use the time delay between the direct and echo signals [6]-[11] together with the bearing angle [12]-[14]. When the target is moving, Doppler shift between the direct and echo signals is present and it can be exploited for positioning and velocity estimation [14]-[16].

A number of multistatic localization techniques have been proposed in the literature over the years [6]-[17]. Specifically, [7] developed a method based on the Wiener filter. Hanusa *et al.* [16] proposed a maximum likelihood (ML) approach that utilizes Doppler measurements only. After the analysis of three types of active sonar systems, [12] applied the least-squares (LS) algorithm to the weighted measurements including distances and azimuths. While Falcone *et al.* [14] explored the use of range, Doppler and angle measurements in the ML location estimation, Chalise *et al.* [11] took another approach by converting the positioning problem to convex optimization via semi-definite relaxation (SDR). These solutions are iterative and often require good initial guesses close to the actual solution. The SDR location estimate is not able to reach the optimum CRB performance even when the measurements are accurate.

Closed-form solutions have also been derived in the literature. Rui and Ho [8] developed a localization algorithm for a stationary target using arrival time differences and bearing angles. Recently, we obtained in [9] a solution for estimating the position and velocity of a moving target using time delay and Doppler shift measurements and encouraging results were presented. Nevertheless, we have not come across any closed-form solution that is able to exploit all three kinds of measurements, namely time delays, Doppler shifts and arrival angles. It is the purpose of this paper to develop such a solution, with the objective of achieving the CRB performance.

We would like to differentiate the works here from previous. The localization techniques found in the literature mostly use two kinds of measurements, either time delays and arrival angles, or time delays and Doppler shifts. Falcone *et al.* [14] exploited all three kinds of measurements for moving target localization. But the two methods proposed in [14], an LS algorithm assuming identical variances for different kinds of measurements and an ML estimator, are both iterative and require good initialization. We cannot apply the solution from [8] directly due to the presence of the extra Doppler shifts and the additional unknown target velocity. The method presented here is also different from that in [9] which estimates the correction to a preliminary solution to obtain the final localization result.

Throughout the paper, we shall use the common convention that bold lower and upper letters represent column vector and matrix. $\mathbf{I}$ and $\mathbf{O}$ are identity and zero matrices of appropriate size. $\mathbf{1}$ is a column vector of ones. $\mathbf{a}^o$ is the true value of $\mathbf{a}$ and $\Delta\mathbf{a}$ is their difference. $\mathbf{a}(k_1 : k_2)$ is a column vector containing the k_1th to k_2th elements of $\mathbf{a}$. $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix with the elements of $\mathbf{a}$ on the diagonal. $\mathrm{sign}(*)$ is the sign function. $\odot$ and $./$ denote the element-by-element product (Hadamard product) and division.

We shall first introduce the problem and measurement models in Section 2. Section 3 develops the proposed closed-form solution. Section 4 presents the theoretical performance analysis. Section 5 contains the simulations and Section 6 concludes the paper.

2. PROBLEM STATEMENT

We are interested in determining the position of a moving target $\mathbf{u}^o = [x^o, y^o]^T \in \mathcal{R}^2$ and its velocity $\dot{\mathbf{u}}^o = [\dot{x}^o, \dot{y}^o]^T \in \mathcal{R}^2$ on the 2-D plane using a multistatic sonar system having M transmitters $\mathbf{t}_i = [x_{\mathbf{t}_i}, y_{\mathbf{t}_i}]^T \in \mathcal{R}^2, i = 1, 2, \ldots, M$, and N receivers $\mathbf{s}_j = [x_{\mathbf{s}_j}, y_{\mathbf{s}_j}]^T \in \mathcal{R}^2, j = 1, 2, \ldots, N$. The transmitters and receivers are all static and their accurate positions are available. Fig. 1 illustrates the localization scenario in consideration.

Signals from different transmitters are assumed disjoint in time

This work was supported by the Natural Science Foundation of China under Grants 61304264 and 61305017, and by the Natural Science Foundation of Jiangsu Province under Grants BK20130154 and BK20140166.

and/or in frequency. Receivers observe the signal of each transmitter from the direct propagation and the reflection of the moving target. Maximizing the cross-ambiguity function between the direct and reflected signals provides a time delay and Doppler shift measurements, where the latter comes from target motion. Also, each receiver has the ability to estimate the bearing angle of the target from the arrival direction of the echo signal [12]. Let $r^o_{i,j}$ be the true time delay multiplied with the signal propagation speed. It is related to the distances among the transmitter i, receiver j and the target by

$$r^o_{i,j} = \|\mathbf{u}^o - \mathbf{t}_i\| + \|\mathbf{u}^o - \mathbf{s}_j\| - \|\mathbf{t}_i - \mathbf{s}_j\|\,. \tag{1}$$

Here, $\| * \|$ represents the Euclidean norm. For each transmitter-receive pair, (1) defines an ellipse on which the target lies.

The time derivative of (1) gives the Doppler shift model

$$\dot{r}^o_{i,j} = \boldsymbol{\rho}^T_{\mathbf{u}^o,\mathbf{t}_i}\dot{\mathbf{u}}^o + \boldsymbol{\rho}^T_{\mathbf{u}^o,\mathbf{s}_j}\dot{\mathbf{u}}^o \tag{2}$$

where the true range rate $\dot{r}^o_{i,j}$ is equal to the Doppler shift between transmitter i and receiver j divided by the transmitter carrier frequency and multiplied by the signal propagation speed. The notation $\boldsymbol{\rho}_{\mathbf{a},\mathbf{b}} = (\mathbf{a}-\mathbf{b})/\|\mathbf{a}-\mathbf{b}\|$ denotes a unit vector from $\mathbf{b}$ to $\mathbf{a}$. The bearing angle of the target observed at receiver j is [12][13]

$$\theta^o_j = \tan^{-1}\left(\frac{y^o - y_{\mathbf{s}_j}}{x^o - x_{\mathbf{s}_j}}\right) \tag{3}$$

where $\theta^o_j \in (0^o, 360^o)$.

The observed measurements are $r_{i,j} = r^o_{i,j} + \Delta r_{i,j}$, $\dot{r}_{i,j} = \dot{r}^o_{i,j} + \Delta\dot{r}_{i,j}$ and $\theta_j = \theta^o_j + \Delta\theta_j$. Grouping distance, range rate and angle measurements give the column vectors $\mathbf{r}$, $\dot{\mathbf{r}}$ and $\boldsymbol{\theta}$. The measurement vector is $\mathbf{m} = [\mathbf{r}^T, \dot{\mathbf{r}}^T, \boldsymbol{\theta}^T]^T = \mathbf{m}^o + \Delta\mathbf{m}$. The additive noise vector $\Delta\mathbf{m} = [\Delta\mathbf{r}^T, \Delta\dot{\mathbf{r}}^T, \Delta\boldsymbol{\theta}^T]^T$ is zero-mean Gaussian with covariance matrix $\mathbf{Q_m}$. As the three kinds of measurements are in general uncorrelated with one another [14], $\mathbf{Q_m} = \text{diag}(\mathbf{Q_r}, \mathbf{Q_{\dot{r}}}, \mathbf{Q}_{\boldsymbol{\theta}})$ is block diagonal with diagonal blocks $\mathbf{Q_r}$, $\mathbf{Q_{\dot{r}}}$ and $\mathbf{Q}_{\boldsymbol{\theta}}$ being the covariance matrices of $\mathbf{r}$, $\dot{\mathbf{r}}$ and $\boldsymbol{\theta}$.

The objective is to estimate the unknown target location vector $\boldsymbol{\gamma}^o = [\mathbf{u}^{oT}, \dot{\mathbf{u}}^{oT}]^T$ using the measurements $\mathbf{m}$, under the assumption that the time variations of the target reflected signals from different transmitters can be neglected. Such an assumption is valid when the signal propagation speed is much larger than the target velocity [2]. Generalizing the proposed technique to the 3-D scenario is straightforward.

3. SOLUTION

To solve the problem in Section 2, we shall follow the approach from [8] by introducing nuisance variables and obtaining the solution through successive stages. The solution from [8] is not directly applicable here because of the extra Doppler shift measurements and the additional unknown target velocity. The nuisance parameters are used to transform the nonlinear measurement equations into pseudo-linear ones that enable the application of linear estimation techniques. Different stages reduce successively the number of nuisance variables to reach the final solution.

3.1. Stage-1

We start with expressing the time delay equation (1) as $r^o_{i,j} - \|\mathbf{u}^o - \mathbf{t}_i\| + \|\mathbf{t}_i - \mathbf{s}_j\| = \|\mathbf{u}^o - \mathbf{s}_j\|$. Squaring both sides and substituting $r^o_{i,j} = r_{i,j} - \Delta r_{i,j}$, we arrive at the distance solution equation

$$\begin{aligned} &2\|\mathbf{u}^o - \mathbf{s}_j\|\Delta r_{i,j} \approx \\ &\quad -2(\mathbf{t}_i - \mathbf{s}_j)^T\mathbf{u}^o - 2(r_{i,j} + \|\mathbf{t}_i - \mathbf{s}_j\|)\|\mathbf{u}^o - \mathbf{t}_i\| \\ &\quad + r^2_{i,j} + 2r_{i,j}\|\mathbf{t}_i - \mathbf{s}_j\| + 2\mathbf{t}^T_i(\mathbf{t}_i - \mathbf{s}_j) \end{aligned} \tag{4}$$

where the squared error term $\Delta r^2_{i,j}$ has been dropped. Taking the time derivative of (4) gives the Doppler solution equation

$$\begin{aligned} &\|\mathbf{u}^o - \mathbf{s}_j\|\Delta\dot{r}_{i,j} + \boldsymbol{\rho}^T_{\mathbf{u}^o,\mathbf{s}_j}\dot{\mathbf{u}}^o\Delta r_{i,j} \approx \\ &-(\mathbf{t}_i - \mathbf{s}_j)^T\dot{\mathbf{u}}^o - (r_{i,j} + \|\mathbf{t}_i - \mathbf{s}_j\|)\boldsymbol{\rho}^T_{\mathbf{u}^o,\mathbf{t}_i}\dot{\mathbf{u}}^o \\ &-\dot{r}_{i,j}\|\mathbf{u}^o - \mathbf{t}_i\| + r_{i,j}\dot{r}_{i,j} + \dot{r}_{i,j}\|\mathbf{t}_i - \mathbf{s}_j\| \end{aligned} \tag{5}$$

where the approximation comes from ignoring $\Delta r_{i,j}\Delta\dot{r}_{i,j}$. Besides, we have from (3) that $\sin\theta^o_j(x^o - x_{\mathbf{s}_j}) = \cos\theta^o_j(y^o - y_{\mathbf{s}_j})$. The angle noise $\Delta\theta_j$ is typically quite less than θ_j. Hence, we have [13]

$$\sin(\theta_j - \Delta\theta_j) \approx \sin\theta_j - \Delta\theta_j\cos\theta_j \tag{6a}$$

$$\cos(\theta_j - \Delta\theta_j) \approx \cos\theta_j + \Delta\theta_j\sin\theta_j\,. \tag{6b}$$

As a result, the angle solution equation is

$$\|\mathbf{u}^o - \mathbf{s}_j\|\Delta\theta_j \approx \mathbf{p}^T_j\mathbf{u}^o - \mathbf{p}^T_j\mathbf{s}_j \tag{7}$$

where $\mathbf{p}_j = [\sin\theta_j, -\cos\theta_j]^T$. We now try to solve for $\boldsymbol{\gamma}^o$ from (4), (5) and (7).

Note that (4) is a nonlinear equation in $\mathbf{u}^o$ but appears to be linearly related to $\mathbf{u}^o$ and $\|\mathbf{u}^o - \mathbf{t}_i\|$. Similarly, the nonlinear equation (5) is linear with respect to $\dot{\mathbf{u}}^o$, $\boldsymbol{\rho}^T_{\mathbf{u}^o,\mathbf{t}_i}\dot{\mathbf{u}}^o$ and $\|\mathbf{u}^o - \mathbf{t}_i\|$. The idea here is to introduce the nuisance parameters

$$\alpha^o_i = ||\mathbf{u}^o - \mathbf{t}_i|| \quad \text{and} \quad \beta^o_i = \boldsymbol{\rho}^T_{\mathbf{u}^o,\mathbf{t}_i}\dot{\mathbf{u}}^o \tag{8}$$

assume they are independent of $\mathbf{u}^o$ and $\dot{\mathbf{u}}^o$, and apply the weighted least-squares (WLS) minimization to obtain a solution.

Let $\boldsymbol{\alpha}^o = [\alpha^o_1, \alpha^o_2, ..., \alpha^o_M]^T$ and $\boldsymbol{\beta}^o = [\beta^o_1, \beta^o_2, ...\beta^o_M]$ be the collections of nuisance parameters, where M is the number of transmitters. Ignoring the approximation errors, stacking (4), (5) and (7) over transmitters and receivers yield the matrix equation

$$\mathbf{B}_1\Delta\mathbf{m} = \mathbf{h}_1 - \mathbf{G}_1\boldsymbol{\varphi}^o_1 \tag{9}$$

where $\boldsymbol{\varphi}^o_1 = [\mathbf{u}^{oT}, \boldsymbol{\alpha}^{oT}, \dot{\mathbf{u}}^{oT}, \boldsymbol{\beta}^{oT}]^T$ is the Stage-1 unknown vector, the vector $\mathbf{h}_1$ contains the terms of known values, the matrix $\mathbf{G}_1$ contains the factors multiplied with $\boldsymbol{\varphi}^o_1$ and $\mathbf{B}_1$ is a sparse matrix containing factors multiplied with the measurement noises.

The linear form of (9) enables us to obtain the WLS solution

$$\boldsymbol{\varphi}_1 = [\mathbf{u}^T_1, \boldsymbol{\alpha}^T, \dot{\mathbf{u}}^T_1, \boldsymbol{\beta}^T]^T = (\mathbf{G}^T_1\mathbf{W}_1\mathbf{G}_1)^{-1}\mathbf{G}^T_1\mathbf{W}_1\mathbf{h}_1 \tag{10}$$

where the weighting matrix is

$$\mathbf{W}_1 = \mathbf{B}^{-T}_1\mathbf{Q}^{-1}_m\mathbf{B}^{-1}_1\,. \tag{11}$$

Note that $\mathbf{B}_1$ depends on the unknowns $\mathbf{u}^o$ and $\dot{\mathbf{u}}^o$. A simple approach to handle this problem is to replace $\mathbf{B}_1$ with an identity matrix to generate an initial solution from which $\mathbf{W}_1$ can be better approximated, which can then be used to produce a more accurate estimate of $\boldsymbol{\varphi}^o_1$.

Under small measurement noise conditions (see Section 4), the bias of $\boldsymbol{\varphi}_1$ is negligible and its covariance matrix is approximately equal to [19]

$$\text{cov}(\boldsymbol{\varphi}_1) \approx (\mathbf{G}^T_1\mathbf{W}_1\mathbf{G}_1)^{-1}. \tag{12}$$

$\mathbf{u}_1$ and $\dot{\mathbf{u}}_1$ are what we are interested in. Nevertheless, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are related to the target location parameters $\boldsymbol{\gamma}^o$ as well but their correlations with them have been ignored to allow for a simple solution. Next, we shall explore the functional relationships between the nuisance parameters and $\boldsymbol{\gamma}^o$ to refine the localization result.

3.2. Stage-2

If we express α_i^o as $\alpha_i^o = \alpha_i - \Delta\alpha_i$, where $\Delta\alpha_i$ is the estimation error, and square both sides, (8) can be written as

$$2\alpha_i\Delta\alpha_i \approx \alpha_i^2 - \mathbf{t}_i^T\mathbf{t}_i + 2\mathbf{t}_i^T\mathbf{u}^o - \mathbf{u}^{oT}\mathbf{u}^o. \tag{13}$$

The approximation comes from dropping $\Delta\alpha_i^2$ that is relatively s-mall with respect to $2\alpha_i\Delta\alpha_i$. (13) contains a total of M equations that relate the target position $\mathbf{u}^o$ with $\boldsymbol{\alpha}$.

Next, evaluating the product $\alpha_i^o\beta_i^o = (\alpha_i - \Delta\alpha_i)(\beta_i - \Delta\beta_i)$, substituting (8) and dropping the second-order error $\Delta\alpha_i\Delta\beta_i$ give

$$\beta_i\Delta\alpha_i + \alpha_i\Delta\beta_i \approx \alpha_i\beta_i + \mathbf{t}_i^T\dot{\mathbf{u}}^o - \mathbf{u}^{oT}\dot{\mathbf{u}}^o. \tag{14}$$

(14) gives another M equations that relate the target position and velocity with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

To allow for a simple solution, we introduce in this stage two nuisance variables

$$\zeta^o = \mathbf{u}^{oT}\mathbf{u}^o \quad \text{and} \quad \eta^o = \mathbf{u}^{oT}\dot{\mathbf{u}}^o \tag{15}$$

and define the unknown vector as

$$\boldsymbol{\varphi}_2^o = [\mathbf{u}^{oT}, \zeta^o, \dot{\mathbf{u}}^{oT}, \eta^o]^T . \tag{16}$$

The collection of (13) and (14) for $i = 1, 2, \ldots, M$ together with $\Delta\mathbf{u}_1 = \mathbf{u}_1 - \mathbf{u}^o$ and $\Delta\dot{\mathbf{u}}_1 = \dot{\mathbf{u}}_1 - \dot{\mathbf{u}}^o$ form the matrix equation with $(2M+2)$ rows,

$$\mathbf{B}_2\Delta\boldsymbol{\varphi}_1 = \mathbf{h}_2 - \mathbf{G}_2\boldsymbol{\varphi}_2^o. \tag{17}$$

where $\Delta\boldsymbol{\varphi}_1 = [\Delta\mathbf{u}_1^T, \Delta\boldsymbol{\alpha}^T, \Delta\dot{\mathbf{u}}_1, \Delta\boldsymbol{\beta}^T]^T$ and we have neglected the approximation errors. In (17), $\mathbf{B}_2$ and $\mathbf{h}_2$ are from the values of the Stage-1 solution $\boldsymbol{\varphi}_1$ and $\mathbf{G}_2$ contains only unity and $\mathbf{t}_i$. The WLS solution to (17) is

$$\boldsymbol{\varphi}_2 = [\mathbf{u}_2^T, \zeta, \dot{\mathbf{u}}_2^T, \eta]^T = (\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2)^{-1}\mathbf{G}_2^T\mathbf{W}_2\mathbf{h}_2 \tag{18}$$

where the weighting matrix is

$$\mathbf{W}_2 = \mathbf{B}_2^{-T}\text{cov}(\boldsymbol{\varphi}_1)^{-1}\mathbf{B}_2^{-1} . \tag{19}$$

The bias of $\boldsymbol{\varphi}_2$ is linearly proportional to that of $\boldsymbol{\varphi}_1$ which is assumed to be relatively small compared to variance over the small error region. The covariance matrix of $\boldsymbol{\varphi}_2^o$ is

$$\text{cov}(\boldsymbol{\varphi}_2) \approx (\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2)^{-1} . \tag{20}$$

3.3. Stage-3

We would like to further improve the target location estimate by exploiting the values of the nuisance variables ζ and η. Using Hadamard product (element-by-element multiplication), we have from (15)

$$\Delta\zeta = \zeta - \mathbf{1}^T(\mathbf{u}^o \odot \mathbf{u}^o) \tag{21}$$

$$\Delta\eta = \eta - \mathbf{1}^T(\mathbf{u}^o \odot \dot{\mathbf{u}}^o) . \tag{22}$$

In addition, expanding $\mathbf{u}_2 \odot \mathbf{u}_2 = (\mathbf{u}^o + \Delta\mathbf{u}_2) \odot (\mathbf{u}^o + \Delta\mathbf{u}_2)$ and $\mathbf{u}_2 \odot \dot{\mathbf{u}}_2 = (\mathbf{u}^o + \Delta\mathbf{u}_2) \odot (\dot{\mathbf{u}}^o + \Delta\dot{\mathbf{u}}_2)$, and dropping the second order errors $\Delta\mathbf{u}_2 \odot \Delta\mathbf{u}_2$ and $\Delta\mathbf{u}_2 \odot \Delta\dot{\mathbf{u}}_2$ yield

$$2\mathbf{u}_2 \odot \Delta\mathbf{u}_2 \approx \mathbf{u}_2 \odot \mathbf{u}_2 - \mathbf{u}^o \odot \mathbf{u}^o \tag{23a}$$

$$\dot{\mathbf{u}}_2 \odot \Delta\mathbf{u}_2 + \mathbf{u}_2 \odot \Delta\dot{\mathbf{u}}_2 \approx \mathbf{u}_2 \odot \dot{\mathbf{u}}_2 - \mathbf{u}^o \odot \dot{\mathbf{u}}^o . \tag{23b}$$

We define the unknowns in Stage-3 as

$$\boldsymbol{\varphi}_3^o = [(\mathbf{u}^o \odot \mathbf{u}^o)^T, (\mathbf{u}^o \odot \dot{\mathbf{u}}^o)^T]^T \tag{24}$$

to take advantage of the linear forms in (21)–(23b). Collecting the four equations together and ignoring the approximation errors yield

$$\mathbf{B}_3\Delta\boldsymbol{\varphi}_2 = \mathbf{h}_3 - \mathbf{G}_3\boldsymbol{\varphi}_3^o \tag{25}$$

where $\Delta\boldsymbol{\varphi}_2 = [\Delta\mathbf{u}_2^T, \Delta\zeta, \Delta\dot{\mathbf{u}}_2^T, \Delta\eta]^T$. The matrices $\mathbf{B}_3$ and $\mathbf{G}_3$ and the vector $\mathbf{h}_3$ should be clear from (21)–(23).

The WLS solution for $\boldsymbol{\varphi}_3^o$ is readily available as

$$\boldsymbol{\varphi}_3 = (\mathbf{G}_3^T\mathbf{W}_3\mathbf{G}_3)^{-1}\mathbf{G}_3^T\mathbf{W}_3\mathbf{h}_3 \tag{26}$$

and the weighting matrix is

$$\mathbf{W}_3 = \mathbf{B}_3^{-T}\text{cov}(\boldsymbol{\varphi}_2)^{-1}\mathbf{B}_3^{-1} . \tag{27}$$

The estimate $\boldsymbol{\varphi}_3$ has negligible bias when the noise level is s-mall, and its covariance matrix is

$$\text{cov}(\boldsymbol{\varphi}_3) = (\mathbf{G}_3^T\mathbf{W}_3\mathbf{G}_3)^{-1} . \tag{28}$$

3.4. Stage-4

The last processing stage maps $\boldsymbol{\varphi}_3$ back to the target position and velocity. From (24) and to eliminate the sign ambiguity, the final estimate of the target location vector $\boldsymbol{\gamma}^o$ is $\boldsymbol{\gamma} = [\mathbf{u}^T, \dot{\mathbf{u}}^T]^T$, where

$$\mathbf{u} = \text{diag}(\text{sign}(\mathbf{u}_1)) \cdot \sqrt{\boldsymbol{\varphi}_3(1:2)} \tag{29a}$$

$$\dot{\mathbf{u}} = \boldsymbol{\varphi}_3(3:4)./\mathbf{u}. \tag{29b}$$

The operation in (29b) assumes that none of the coordinates in the target position estimate is near zero. If this happens, applying a change in the coordinate origin before processing for localization can eliminate this problem.

Taking the differential of (24), the localization error is

$$\Delta\boldsymbol{\gamma} = \begin{bmatrix}\Delta\mathbf{u} \\ \Delta\dot{\mathbf{u}}\end{bmatrix} = \mathbf{B}_4^{-1}\Delta\boldsymbol{\varphi}_3 \tag{30}$$

where

$$\mathbf{B}_4 = \begin{bmatrix}\text{diag}(2\mathbf{u}^o) & \mathbf{O} \\ \text{diag}(\dot{\mathbf{u}}^o) & \text{diag}(\mathbf{u}^o)\end{bmatrix} \tag{31}$$

and $\Delta\boldsymbol{\varphi}_3 = \boldsymbol{\varphi}_3 - \boldsymbol{\varphi}_3^o$ is the Stage-3 estimation error. As a consequence, the proposed algorithm has an estimation covariance matrix

$$\text{cov}(\boldsymbol{\gamma}) = \mathbf{B}_4^{-1}\text{cov}(\boldsymbol{\varphi}_3)\mathbf{B}_4^{-T}. \tag{32}$$

Remark: The algorithm development applies approximations by neglecting the second-order error terms (see (4), (5), (13), (14) and (23)). Ignoring them leads to some performance degradation in the localization result, however it is relatively insignificant (see e.g., [18] for more discussions). It can also cause the estimation performance to deviate from the CRB earlier as the noise level increases. (see Section 4).

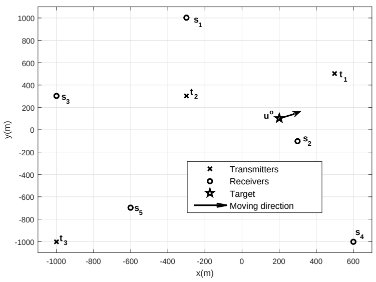

Fig. 1. Localization scenario.

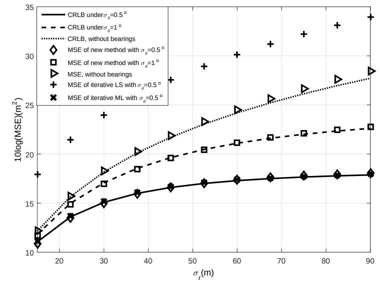

Fig. 2. Position estimation performance as a function of $\sigma_{\mathbf{r}}$ and $\sigma_{\boldsymbol{\theta}}$.

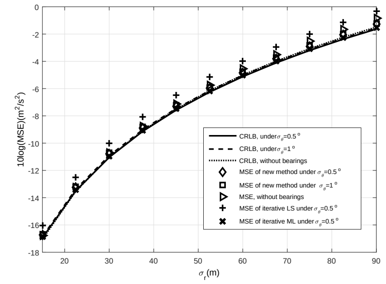

Fig. 3. Velocity estimation performance as a function of $\sigma_{\mathbf{r}}$ and $\sigma_{\boldsymbol{\theta}}$.

4. PERFORMANCE ANALYSIS

The CRB for estimating the unknown $\boldsymbol{\gamma}^o$ is equal to the inverse of the Fisher Information Matrix (FIM). It is given by [6] [19]

$$\mathrm{CRLB}(\boldsymbol{\gamma}^o) = \left(\boldsymbol{\nabla}^T \mathbf{Q}_{\mathbf{m}}^{-1} \boldsymbol{\nabla}\right)^{-1} \tag{33}$$

where

$$\boldsymbol{\nabla} = \left[\frac{\partial \mathbf{m}^o}{\partial \mathbf{u}^o}\,,\ \frac{\partial \mathbf{m}^o}{\partial \dot{\mathbf{u}}^o}\right]. \tag{34}$$

We next evaluate the covariance matrix of the proposed location estimator cov($\boldsymbol{\gamma}$) and show that under the following two small measurement noise conditions, we are able to reach the CRB accuracy,

(C1) $|\Delta r_{i,j}| \ll r^o_{i,j},\ |\Delta \dot{r}_{i,j}| \ll \dot{r}^o_{i,j}\ ,\ \ |\Delta\theta_j| \ll |\theta^o_j|$,

(C2) $|\Delta r_{i,j}| \ll \|\mathbf{u}^o - \mathbf{s}_j\|,\ |\Delta \dot{r}_{i,j}| \ll \|\mathbf{u}^o - \mathbf{s}_j\|,\ |\Delta\theta_j| \ll \|\mathbf{u}^o - \mathbf{s}_j\|$.

The first condition requires the measurement noise be small relative to the actual value. The second condition is satisfied with the target not close to any receiver.

The proof starts by taking the inverse on both sides of (32) and substituting successively the expressions of cov($\boldsymbol{\varphi}_3$), $\mathbf{W}_3$, cov($\boldsymbol{\varphi}_2$), $\mathbf{W}_2$, cov($\boldsymbol{\varphi}_1$) and $\mathbf{W}_1$, which results in

$$\mathrm{cov}(\boldsymbol{\gamma})^{-1} = \mathbf{G}_4^T \mathbf{Q}_{\mathbf{m}}^{-1} \mathbf{G}_4 \tag{35}$$

where $\mathbf{G}_4 = \mathbf{B}_1^{-1}\mathbf{G}_1\mathbf{B}_2^{-1}\mathbf{G}_2\mathbf{B}_3^{-1}\mathbf{G}_3\mathbf{B}_4$. After putting the definitions of $\mathbf{B}_1$, $\mathbf{G}_1$, $\mathbf{B}_2$, $\mathbf{G}_2$, $\mathbf{B}_3$, $\mathbf{G}_3$ and $\mathbf{B}_4$, and applying some algebraic manipulations, $\mathbf{G}_4$ can be expressed in the following partitioned form

$$\mathbf{G}_4 = \begin{bmatrix} \mathbf{G}_{4,\mathbf{r}} & \mathbf{O} \\ \mathbf{G}_{4,\dot{\mathbf{r}}}^{(1)} & \mathbf{G}_{4,\dot{\mathbf{r}}}^{(2)} \\ \mathbf{G}_{4,\boldsymbol{\theta}} & \mathbf{O} \end{bmatrix}. \tag{36}$$

It can be shown that under (C1) and (C2),

$$\begin{aligned} &\mathbf{G}_{4,\mathbf{r}} \approx \partial\mathbf{r}^o/\partial\mathbf{u}^o, \quad \mathbf{G}_{4,\dot{\mathbf{r}}}^{(1)} \approx \partial\dot{\mathbf{r}}^o/\partial\mathbf{u}^o, \\ &\mathbf{G}_{4,\dot{\mathbf{r}}}^{(2)} \approx \partial\dot{\mathbf{r}}^o/\partial\dot{\mathbf{u}}^o, \quad \mathbf{G}_{4,\boldsymbol{\theta}} \approx \partial\boldsymbol{\theta}^o/\partial\mathbf{u}^o. \end{aligned} \tag{37}$$

In other words,

$$\mathbf{G}_4 \approx \boldsymbol{\nabla}\,. \tag{38}$$

Comparing (35) with (33) completes the analysis that the proposed algorithm can reach the CRB performance under conditions (C1) and (C2).

5. SIMULATIONS

The simulation scenario is shown in Fig. 1. The target is located at $\mathbf{u}^o = [200, 100]^T$m and has a velocity of $\dot{\mathbf{u}}^o = [20, 10]^T$m/s. The sonar system has $M = 3$ transmitters and $N = 5$ receivers, and their positions are $\mathbf{t}_1 = [500, 500]^T$m, $\mathbf{t}_2 = [-300, 300]^T$m, $\mathbf{t}_3 = [-1000, -1000]^T$m, $\mathbf{s}_1 = [-300, 1000]^T$m, $\mathbf{s}_2 = [300, -100]^T$m, $\mathbf{s}_3 = [-1000, 300]^T$m, $\mathbf{s}_4 = [600, -1000]^T$m and $\mathbf{s}_5 = [-600, -700]^T$m. The transmitters have carrier frequencies $f_{c,1} = 10$kHz, $f_{c,2} = 12$kHz and $f_{c,3} = 14$kHz. The signal propagation speed is $1500m/s$. The covariance matrices of the time delay and arrival angle measurements are $\mathbf{Q}_{\mathbf{r}} = \sigma_{\mathbf{r}}^2\mathbf{I}$ and $\mathbf{Q}_{\boldsymbol{\theta}} = \sigma_{\boldsymbol{\theta}}^2\mathbf{I}$. It is $\mathbf{Q}_{\dot{\mathbf{r}}} = 10^5\sigma_r^2\mathrm{diag}(\mathbf{1}^T/f_{c,1}^2, \mathbf{1}^T/f_{c,2}^2, \mathbf{1}^T/f_{c,3}^2)$ for range rates.

We conduct Monte Carlo simulations of $L = 10^4$ ensemble runs for the proposed solution, the method from [9] without using bearing measurements and the two iterative algorithms from [14]. The localization performance is evaluated by the mean square error (MSE): MSE($\mathbf{a}$)=$\sum_{k=1}^{L}\|\mathbf{a}^{(k)} - \mathbf{a}^o\|^2/L$. Fig. 2 summarizes the results for position estimation as a function of $\sigma_{\mathbf{r}}$ and $\sigma_{\boldsymbol{\theta}}$. First, comparing with the results from [9] reveals that incorporating the angle measurements can significantly improve the positioning accuracy, especially when the time delay and Doppler noise levels become large. Second, the MSE of the proposed solution matches the CRB well. Third, the iterative LS solution from [14] has poor performance by not accounting for the noise powers of different kinds of measurements. Before the thresholding effect, the proposed method performs close to the ML estimator from [14] that requires near triple the computing time in MATLAB. Fig. 3 illustrates the velocity estimation performance. It turns out the angle measurements do not have much impact on the velocity estimate. This is expected because the angles depend only on the target and receiver positions, and the velocity information mostly comes from Doppler shifts.

6. CONCLUSION

A closed-form solution was developed for multistatic sonar localization of a moving target using time delay, Doppler shift and arrival angle measurements. This highly nonlinear estimation problem is solved by introducing nuisance variables and using successive processing stages to refine the location estimate. We have shown analytically and verified by simulations that the proposed solution is able to reach the CRB accuracy, when the measurement noise is small compared to the true measurements and the target is not near to any receiver. The angle measurements can significantly improve the positioning accuracy but do not affect the velocity estimation much.

7. REFERENCES

[1] H. Cox, "Fundamentals of bistatic active sonar," in *Proc. NATO Advanced Study Inst. Underwater Acoustic Data Process.*, Y. T. Chan, Ed. Norwood, MA: Kluwer, 1989.

[2] D. Orlando and F. Ehlers, "Advances in multistatic sonar," in *Sonar System*, N. Z. Kolev, Ed. InTech, Sep. 2011.

[3] L. Bjørnø, "Developments in sonar technologies and their applications," *IEEE Int. Underwater Technology (UT) Symp.*, pp. 1-8, 2013.

[4] F. Ehlers, "Cooperative vehicle target tracking," *Springer Handbook of Ocean Engineering*, Springer, pp. 499-516, 2016.

[5] J. Zhang, T. Jin, Y. He, L. Qiu, and Z. Zhou, "Range-Doppler-based centralised framework for human target tracking in multistatic radar," to appear in *IET Radar Sonar Navig.*, 2016.

[6] S. He, "Asynchronous time difference of arrival positioning system and implementation," Ph.D. dissertation, Dept. Elect. Eng., Univ. of Victoria, BC, Canada, 2016.

[7] S. Simakov, "Localization in airborne multistatic sonars," *IEEE J. Ocean Eng.*, vol. 33, no. 3, pp. 278-288, Jul. 2008.

[8] L. Rui and K. C. Ho, "Efficient closed-form estimators for multistatic sonar localization," *IEEE Aerosp. Electron. Syst.*, vol. 51, no. 1, pp. 600-614, Jan. 2015.

[9] L. Yang, L. Yang, and K. C. Ho, "Moving target localization in multistatic sonar by differential delays and Doppler shifts," *IEEE Signal Process. Lett.*, vol. 23, no. 9, pp. 1160-1164, Sep. 2016.

[10] N. H. Nguyen, K. Doǧançay, and L. M. Davis, "Adaptive waveform selection and target tracking by wideband multistatic radar/sonar systems," in *Proc. 22nd European Signal Process. Conf. (EUSIPCO)*, Sep. 2014, pp. 1910-1914.

[11] B. K. Chalise, Y. D. Zhang, M. G. Amin, and B. Himed, "Target localization in a multistatic passive radar system through convex optimization," *Signal Process.*, vol. 102, pp. 207-215, Sep. 2014.

[12] S. Kim, B. Ku, W. Hong, and H. Ko, "Performance comparison of target localization for active sonar systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 4, pp. 1371-1380, Oct. 2008.

[13] Y. Wang and K. C. Ho, "An asymptotically efficient estimator in closed-form for 3-D AOA localization using a sensor network," *IEEE Trans. Wireless Commun.*, vol. 14, no. 12, pp. 6524-6535, Dec. 2015.

[14] P. Falcone, F. Colone, A. Macera, and P. Lombardo, "Two-dimensional location of moving target within local areas using WiFi-based multistatic passive radar," *IET Radar Sonar Navig.*, vol. 8, no. 2, pp. 123-131, 2014.

[15] T. Lang and G. Hayes, "Exploitation of bistatic Doppler measurements in multistatic tracking," in *Proc. 10th Int. Conf. Inf. Fusion (FUSION)*, Jul. 2007, pp. 403-410.

[16] E. Hanusa, D. Krout, and M. R. Gupta, "Estimation of position from multistatic Doppler measurements," in *Proc. 13th Int. Conf. Inf. Fusion (FUSION)*, Jul. 2010, pp.1-7.

[17] B. Shtarkalev and B. Mulgrew, "Multistatic moving target detection in unknown coloured Gaussian interference," *Signal Process.*, vol. 115, pp. 130-143, 2015.

[18] K. C. Ho, "Bias reduction for an explicit solution of source localization using TDOA," *IEEE Trans. Signal Process.*, vol. 60, pp. 2101-2114, May 2012.

[19] S. M. Kay, *Fundamentals of Statistical Signal Process., Estimation Theory.* Englewood Cliffs, NJ: Prentice-Hall, 1993.