Design of Space-Time Block Coded Unique Word OFDM Systems

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***Abstract*—In this paper we develop space-time block codes for unique word - orthogonal frequency division multiplexing (UW-OFDM) systems to fully exploit the diversity gain when the channel state information is not available at the transmitter. To this end, we propose two novel space-time block codes (STBCs) for UW-OFDM systems, namely a frequency domain space-time block code and a time-reversal space-time code (TR-STC). The former one is an extension of the traditional space-frequency block codes (SFBCs) for CP-OFDM systems to UW-OFDM systems while the latter one makes use of the frame structure of the UW-OFDM symbols and has a low complexity decoder. Simulation results show that both of the proposed space-time block codes achieve a significant gain compared to the SFBC based CP-OFDM. Moreover, the frequency domain STBC yields a slightly better performance as compared to the TR-STC but has a higher computational complexity.**

***Index Terms*—Orthogonal Frequency Division Multiplexing (OFDM), Cyclic Prefix (CP), Unique Word (UW), Space-Time Block Codes (STBCs), Multiple-Input Multiple-Output (MIMO)**

I. Introduction

A new orthogonal frequency division multiplexing (OFDM) based signaling technique was introduced in [1] where the usual cyclic prefixes (CPs) in conventional OFDM are replaced by known deterministic sequences called unique word (UW). The main idea is to add redundancy in the frequency domain such that a zero tail is obtained at the output of the inverse fast Fourier transform (IFFT). Then a UW is added in the time domain for each OFDM symbol hence guaranteeing the cyclicity. The presence of redundant subcarriers leads to a better bit error rate (BER) performance in frequency selective environments, but at the cost of slightly increased complexity [2], [3]. Moreover, the UW can be designed to be used for particular receiver tasks such as channel estimation and synchronization.

The energy contribution of the redundant subcarriers plays a crucial role in the performance of UW-OFDM. The choice of the positions of the redundant subcarriers also has an enormous influence on the redundant energy. However even with the optimum positions of the redundant subcarriers, this so-called systematic approach suffers from a high energy contribution of the redundant subcarriers. Hence, in [4], the authors introduced a new non-systematic approach where the idea of the dedicated redundant subcarriers is abandoned, and the redundancy is distributed across all subcarriers. The results show that UW-OFDM based on the non-systematic approach not only outperforms systematic UW-OFDM but also conventional CP-OFDM [4].

Orthogonal space-time block codes (OSTBCs) are widely used in modern communication systems when no channel state information (CSI) is available at the transmitter. One of the main reasons for the popularity of OSTBCs is the availability of low complexity optimal detection algorithms such as maximum ratio combining (MRC) at the receiver. The application of these STBCs to CP-OFDM is straightforward, with the detection being performed on each subcarrier. However, this is different for UW-OFDM since the code generator matrix introduces correlation among the data symbols which need to be efficiently utilized at the receiver to get better performance. So far, UW-OFDM has been well investigated from various points of view for single-input single-output (SISO) systems where it was shown that a joint detection fully exploits this correlation. In [5], we have shown that a subcarrier-wise detection is also possible for MIMO UW-OFDM. We have shown that a similar performance to that of joint detection can be achieved if equalization combined with a minimum mean square error (MMSE) based code generator demodulation is performed. In this work, we propose two approaches to apply STBCs to UW-OFDM. In the first approach, we apply STBCs in the frequency domain which is an extension of space-frequency block codes for CP-OFDM . But for UW-OFDM, this approach has a much higher decoding complexity since detection based on MRC is not possible. Alternatively, in the second approach, we propose to apply STBCs in the time domain with a slight modification in the conventional UW-OFDM frame structure. We show that this approach results in a decoding procedure with a very low complexity. Moreover, a matched filter (MF) based demodulation can also be applied. Different types of STBCs have been investigated and recommended for CP-OFDM for different MIMO dimensions [6], [7]. In this paper, we have chosen Alamouti based OSTBCs as an example. Other higher-order STBCs such as orthogonal, quasi orthogonal, and Tarokh STBCs can also be applied to UW-OFDM, but the application and detection procedures will not change significantly.

The organization of the remaining paper is as follows. Section II reviews the basics of UW-OFDM systems. In Section III, STBCs for UW-OFDM are presented where we discuss two approaches to implement STBCs for UW-OFDM. We also propose appropriate detection schemes for these approaches. Section IV shows the simulation results and quantifies the system performance in terms of the BER. The performance is compared with CP-OFDM. The paper is summarized at the end in Section V.

Notation: We use lower-case bold face letters $(\boldsymbol{a}, \boldsymbol{b}, ...)$ to indicate vectors and upper-case bold face letters $(\boldsymbol{A}, \boldsymbol{B}, ...)$ to indicate matrices. The superscripts and subscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, and $\|\cdot\|_F$ represent complex conjugate, matrix transpose, complex conjugate transpose (Hermitian), and Frobenius norm, respectively. The $\mathrm{vec}\{\cdot\}$ operator stacks the columns of a matrix into a vector. The Kronecker product is represented by $\otimes$ and the $\mathrm{tr}\{\cdot\}$ operator defines the trace of a matrix.

II. UW-OFDM Basics

Let $\mathbf{x}_u \in \mathbb{C}^{N_u \times 1}$ be a unique word of length N_u which shall be generated at the tail of a time domain UW-OFDM symbol. In [1], [2], it is has been suggested to generate a UW-OFDM symbol $\mathbf{x} = \begin{bmatrix} \mathbf{x}_d^T & \mathbf{0} \end{bmatrix}^T$ with a zero-UW in a first step, and to add the desired UW in the time domain in a second step:

$$\boldsymbol{x}' = \begin{bmatrix} \boldsymbol{x}_d \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{x}_u \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_d \\ \boldsymbol{x}_u \end{bmatrix} \in \mathbb{C}^{N \times 1} \quad (1)$$

The UW-OFDM symbol $\boldsymbol{x}$ with a zero UW is obtained by introducing redundancy in the frequency domain. This is achieved by defining codewords with the help of an appropriate complex-valued code generator matrix $\boldsymbol{G} \in \mathbb{C}^{N_m \times N_d}$ with $N_m = N_d + N_r$ where N_r is the number of additionally required (redundant) subcarriers and $N_r = N_u$.

Let $\boldsymbol{d} \in \mathbb{C}^{N_d \times 1}$ represent the complex data symbols in the frequency domain for N_d data subcarriers. In the UW-OFDM symbol generation, the data symbols $\boldsymbol{d}$ are first fed to the code generator matrix $\boldsymbol{G}$. Then the resulting coded symbols are mapped to the corresponding data subcarriers by a mapping matrix $\boldsymbol{B} \in \{0,1\}^{N \times N_m}$, which consists of zero rows at the positions of the zero subcarriers and elsewhere unit row vectors. Thereafter, the resulting frequency domain signal is modulated on the subcarriers using the IFFT to obtain the time domain samples $\boldsymbol{x}$. In order to obtain a zero word UW in the time domain, a valid code generator matrix has to fulfill

$$\boldsymbol{F}_N^{-1}\boldsymbol{B}\boldsymbol{G} = \begin{bmatrix} * \\ \boldsymbol{0} \end{bmatrix}, \tag{2}$$

where $\boldsymbol{F}_N$ is the N-point DFT matrix with its elements $[\boldsymbol{F}_N]_{k,l} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}kl}$ for $k, l = 0, 1, 2, \ldots, N-1$. The choice of the $\boldsymbol{G}$ matrix plays an important role with respect to the performance of UW-OFDM, and different $\boldsymbol{G}$ matrices yield different BER-performances [4]. Various types of $\boldsymbol{G}$ matrices have been proposed for UW-OFDM in [3], [4] for SISO systems, same matrices can also be utilized for MIMO systems. It has also been shown in [4] that the $\boldsymbol{G}$ matrices based on the non-systematic approach show a better performance over flat and frequency selective channels. Therefore, in this work, we have used $\boldsymbol{G}$ matrices based on the non-systematic approach.

The constraints in Eq. (2) has to be fulfilled for any code matrix to obtain a valid UW-OFDM symbol. Let $\boldsymbol{U} = \left[\boldsymbol{F}_N^{-1}\boldsymbol{B}\right]_{\text{last } N_u \text{ rows}}$ be the $N_r \times N_m$ matrix containing the N_r lowermost rows of $\boldsymbol{F}_N^{-1}\boldsymbol{B}$. Then the constraint in (2) can be reformulated as

$$\boldsymbol{U}\boldsymbol{G} = \boldsymbol{0}, \tag{3}$$

which implies that the columns of $\boldsymbol{G}$ must be in the null space of the matrix $\boldsymbol{U}$. One solution can be a matrix that contains an orthonormal basis of the null space. This can be achieved by computing the singular value decomposition of $\boldsymbol{U}$. In this work, we denote the code matrix as $\boldsymbol{G}'$ which is computed in this way.

In [4], it is suggested to derive optimum code generator matrices by minimizing the trace of the error covariance matrices of the best linear unbiased estimator (BLUE) and the LMMSE estimator in the AWGN channel for a fixed signal-to-noise ratio. Furthermore, it is also shown in [4] that the solution of the optimization problem is not unique, but all solutions fulfill $\boldsymbol{G}^H\boldsymbol{G} = s^2\boldsymbol{I}_{N_d}$ with s denoting all the identical singular values and $\boldsymbol{I}_{N_d}$ is the identity matrix. The code generator matrix, in this approach, distributes the redundancy over all the subcarriers. Furthermore, it is also shown in [4] that the optimization problems can be converted to unconstrained problems which can numerically be solved, e.g., using the steepest descent method. Different initializations to solve these problems result in different code generator matrices for the same problem. In this work, we use a generator matrix $\boldsymbol{G}''$ which has been obtained by a random initialization and a minimization of the trace of the error covariance matrices of the LMMSE estimator in the AWGN channel for a fixed signal-to-noise ratio.

III. Space Time Block Codes

In this section, we investigate space-time block codes for UW-OFDM and propose new transceiver architectures for UW-OFDM with STBCs. We discuss two approaches for realizing OSTBCs in UW-OFDM. For the sake of simplicity, we consider Alamouti based STBCs, but the same approaches can be applied to higher MIMO dimensions. The conventional Alamouti STBCs were designed for flat fading channels and the encoding rule was applied to two consecutive symbols instead of applying it to blocks of data. Later on, in [8], Alamouti-based space-frequency coding was proposed for CP-OFDM. Here we propose two approaches to apply Alamouti-based STBCs to UW-OFDM systems.

A. Space-time encoder in the frequency domain

We consider a system with $M_T = 2$ antennas at the transmitter and M_R receive antennas. In the first approach, we apply the space-time encoding in the frequency domain. The modulated data symbols $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ are processed by the space-time encoder to produce signals $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ for two transmit antennas in two successive time frames as shown in Table I. The two data vectors at the output of the space-time encoder are independently multiplied with the code generator matrix. Then after applying the IFFT and adding the UW, they are transmitted by the two antennas.

	Antenna 1	Antenna 2
Time frame 1	$\boldsymbol{s}_{1,1} = \boldsymbol{d}_1$	$\boldsymbol{s}_{1,2} = \boldsymbol{d}_2$
Time frame 2	$\boldsymbol{s}_{2,1} = -\boldsymbol{d}_2^*$	$\boldsymbol{s}_{2,2} = \boldsymbol{d}_1^*$

TABLE I: STBC encoder for $M_T = 2$ with $\boldsymbol{d}_1,\ \boldsymbol{d}_2 \in \mathbb{C}^{N_d \times 1}$

The received signal at the jth receive antenna for the two time frames is given by

$$\boldsymbol{y}_{t,j} = \sum_{i=1}^{M_T} \boldsymbol{H}_{j,i}\boldsymbol{F}_N^{-1}(\boldsymbol{B}\boldsymbol{G}\boldsymbol{s}_{t,i} + \tilde{\boldsymbol{x}}_u) + \boldsymbol{v}, \tag{4}$$

where $\boldsymbol{H}_{j,i}$ is the channel convolutional matrix containing the channel impulse response from the ith transmit antenna to the jth receive antenna, $\tilde{\boldsymbol{x}}_u \in \mathbb{C}^{N \times 1}$ is the corresponding UW in the frequency domain, $\boldsymbol{v}$ is a zero-mean Gaussian noise vector and the subscript $(.)_{t,j}$ represents the time frames and the transmit antennas, respectively. After applying the FFT at the receiver and removing the UW, the received signal at the jth receive antenna in the frequency domain during the tth time frame is given by

$$\tilde{\boldsymbol{y}}_{t,j} = \sum_{i=1}^{M_T} \tilde{\boldsymbol{H}}_{j,i}\boldsymbol{G}\boldsymbol{s}_{t,i} + \tilde{\boldsymbol{v}}, \tag{5}$$

where the diagonal matrix $\tilde{\boldsymbol{H}}_{j,i} = \boldsymbol{B}^T\boldsymbol{F}_N\boldsymbol{H}_{j,i}\boldsymbol{F}_N^{-1}\boldsymbol{B} \in \mathbb{C}^{N_m \times N_m}$ contains the channel frequency response on its main diagonal and $\tilde{\boldsymbol{v}} = \boldsymbol{B}^T\boldsymbol{F}_N\boldsymbol{v}$ is the zero-mean Gaussian noise vector in the frequency domain. After taking the conjugate of the received signal in the second time frame, the combined signal for the two time frames in the frequency domain can be written as

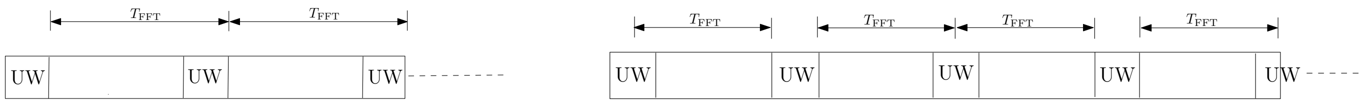
$$\begin{bmatrix} \tilde{\boldsymbol{y}}_{1,1} \\ \vdots \\ \tilde{\boldsymbol{y}}_{1,M_R} \\ \tilde{\boldsymbol{y}}_{2,1}^* \\ \vdots \\ \tilde{\boldsymbol{y}}_{2,M_R}^* \end{bmatrix} = \boldsymbol{H}_{\text{eff}}\boldsymbol{G}_{\text{eff}} \begin{bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{v}}_{1,1} \\ \vdots \\ \tilde{\boldsymbol{v}}_{1,M_R} \\ \tilde{\boldsymbol{v}}_{2,1}^* \\ \vdots \\ \tilde{\boldsymbol{v}}_{2,M_R}^* \end{bmatrix}, \tag{6}$$

where

$$\boldsymbol{G}_{\text{eff}} = \begin{bmatrix} \boldsymbol{I}_{M_T} \otimes \boldsymbol{G} \\ \boldsymbol{I}_{M_T} \otimes \boldsymbol{G}^* \end{bmatrix} \in \mathbb{C}^{2M_TN_m \times M_TN_m} \tag{7}$$

$$\boldsymbol{H}_{\text{eff}} = \begin{bmatrix} \tilde{\boldsymbol{H}}_{1,1} & \tilde{\boldsymbol{H}}_{1,2} & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\boldsymbol{H}}_{M_R,1} & \tilde{\boldsymbol{H}}_{M_R,2} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \tilde{\boldsymbol{H}}_{1,2}^* & -\tilde{\boldsymbol{H}}_{1,1}^* \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \tilde{\boldsymbol{H}}_{M_R,1}^* & -\tilde{\boldsymbol{H}}_{M_R,2}^* \end{bmatrix} \in \mathbb{C}^{2M_RN_m \times 4N_m}$$

The channel transfer functions (CTFs) $\tilde{\boldsymbol{H}}_{j,i} \in \mathbb{C}^{N_m \times N_m}$ are still the diagonal matrices with the channel from the ith transmit antenna to the jth receive antenna on the main diagonal. To apply the subcarrier-wise operation, an equivalent representation of Eq. (6) can be written as



(a) Conventional UW-OFDM frame structure (also with STBC in the frequency domain)

(b) UW-OFDM frame structure with TR-STC

Fig. 1: Different frame structures for UW-OFDM

$$\begin{bmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \\ \vdots \\ \boldsymbol{z}_{N_m} \end{bmatrix} = \boldsymbol{H}'_{\text{eff}}\boldsymbol{G}'_{\text{eff}} \begin{bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \end{bmatrix} + \begin{bmatrix} \hat{\boldsymbol{v}}_1 \\ \hat{\boldsymbol{v}}_2 \\ \vdots \\ \hat{\boldsymbol{v}}_{N_m} \end{bmatrix}, \quad (8)$$

where $\boldsymbol{z}_l = [\hat{y}_{1,1} \cdots \hat{y}_{1,M_R} \quad \hat{y}^*_{2,1} \cdots \hat{y}^*_{2,M_R}]^T_l$ is the combined received signal for the two time frames at the lth subcarrier and

$$\boldsymbol{H}'_{\text{eff}}\boldsymbol{G}'_{\text{eff}} = \text{blkdiag}\left[\boldsymbol{H}'_1, \boldsymbol{H}'_2, \cdots, \boldsymbol{H}'_{N_m}\right] \begin{bmatrix} \boldsymbol{I}_{M_T} \otimes [\boldsymbol{G}]_1 \\ \boldsymbol{I}_{M_T} \otimes [\boldsymbol{G}]^*_1 \\ \vdots \\ \boldsymbol{I}_{M_T} \otimes [\boldsymbol{G}]_{N_m} \\ \boldsymbol{I}_{M_T} \otimes [\boldsymbol{G}]^*_{N_m} \end{bmatrix}.$$

Here $[\boldsymbol{G}]_l$ is the lth row of the $\boldsymbol{G}$ matrix, and $\boldsymbol{H'}_{\text{eff}}$ is a block diagonal matrix containing the N_m subcarrier wise CTF which is given as

$$\boldsymbol{H}'_l = \begin{bmatrix} h_{l,11} & h_{l,12} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ h_{l,M_R1} & h_{l,M_R2} & 0 & 0 \\ 0 & 0 & h^*_{l,12} & -h^*_{l,11} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & h^*_{l,M_R2} & -h^*_{l,M_R1} \end{bmatrix} \forall\, l = 1, \ldots, N_m$$

Clearly, $\boldsymbol{H'}_l$ is a non-orthogonal matrix and hence the MRC scheme cannot be used. We can apply a two step detection procedure as proposed in [5], where in the first stage a ZF or MMSE based equalization is performed and in the second step a MMSE based demodulation is performed. Alternatively, joint detection can be applied. In this case, joint detection is the best choice due to its lower complexity. Thus, the estimated received signal is obtained by using the BLUE or the LMMSE estimator, which are given by

$$\boldsymbol{E}_{\text{BLUE}} = \left(\boldsymbol{G}^H_{\text{eff}}\boldsymbol{H}^H_{\text{eff}}\boldsymbol{H}_{\text{eff}}\boldsymbol{G}_{\text{eff}}\right)^{-1}\boldsymbol{G}^H_{\text{eff}}\boldsymbol{H}^H_{\text{eff}} \quad (9)$$

$$\boldsymbol{E}_{\text{LMMSE}} = \left(\boldsymbol{G}^H_{\text{eff}}\boldsymbol{H}^H_{\text{eff}}\boldsymbol{H}_{\text{eff}}\boldsymbol{G}_{\text{eff}} + \frac{\sigma^2_n}{\sigma^2_d}\boldsymbol{I}\right)^{-1}\boldsymbol{G}^H_{\text{eff}}\boldsymbol{H}^H_{\text{eff}} \quad (10)$$

Note that the $\boldsymbol{G}'_{\text{eff}}$ and $\boldsymbol{H}'_{\text{eff}}$ are the equivalent transformations of $\boldsymbol{G}_{\text{eff}}$ and $\boldsymbol{H}_{\text{eff}}$ and can also be applied to Eq. (9) and Eq. (10).

B. Space-time encoder in the time domain

The application of a conventional Alamouti based space-time encoder for UW-OFDM in the frequency domain results in a non-orthogonal equivalent channel matrix which is a disadvantage because it increases the decoding complexity. An alternative strategy is the implementation of a space-time encoder in the time domain just after the IFFT operation. Such an encoding strategy is also referred to as time-reversal space-time coding (TR-STC). An Alamouti based TR-STC scheme was proposed for single-carrier block transmission with frequency domain equalization in [9]. This TR-STC scheme can also be implemented for the UW-OFDM with a slight modification of the UW-OFDM frame structure. In this approach, the time domain samples $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ at the output of the IFFT are first encoded by the space-time encoder according to Table II for $n = 0, 1, ..., N-1$. Then a UW is added in the time domain signal. To correctly apply this TR-STC scheme, we propose to generate a zero UW at the start of the time domain signal instead of generating it at the end. This can be accomplished by designing a $\boldsymbol{G}$ matrix which fulfills $\boldsymbol{F}^{-1}_N\boldsymbol{B}\boldsymbol{G} = [\boldsymbol{0} \quad *]^T$.

Moreover, we propose to generate $N_u/2 + 1$ zeros instead of N_u zeros, thus requiring only $N_r/2 + 1$ redundant subcarriers instead of N_r. Additionally, a UW of length N_u needs to be inserted in between two encoded symbols to ensure cyclicity, as shown in Fig. 1b. Therefore, the extra addition of a UW is roughly compensated by the reduction of the redundancy in the frequency domain. Note that the length of the UW in this scheme is still the same as in the conventional UW-OFDM or when STBCs are applied in the frequency domain as shown in Fig. 1.

	Antenna 1	Antenna 2
Time frame 1	$x_{1,1}[n] = x_1[n]$	$x_{1,2}[n] = x_2[n]$
Time frame 2	$x_{2,1}[n] = -x^*_2[(-n)_N]$	$x_{2,2}[n] = x^*_1[(-n)_N]$

TABLE II: TR-STC for UW-OFDM

The combined signal in the frequency domain after removing the UW can be written as

$$\begin{bmatrix} \hat{\boldsymbol{z}}_1 \\ \hat{\boldsymbol{z}}_2 \\ \vdots \\ \hat{\boldsymbol{z}}_{N_m} \end{bmatrix} = \boldsymbol{H}''_{\text{eff}}\boldsymbol{G}''_{\text{eff}} \begin{bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \end{bmatrix} + \begin{bmatrix} \hat{\boldsymbol{v}}_1 \\ \hat{\boldsymbol{v}}_2 \\ \vdots \\ \hat{\boldsymbol{v}}_{N_m} \end{bmatrix}, \quad (11)$$

where $\hat{\boldsymbol{z}}_l = [\hat{y}_{1,1} \cdots \hat{y}_{1,M_R} \quad \hat{y}^*_{2,1} \cdots \hat{y}^*_{2,M_R}]^T_l$ is the equivalent received signal for two time frames, $\boldsymbol{G}''_{\text{eff}} = \boldsymbol{I}_{M_T} \otimes \boldsymbol{G} \in \mathbb{C}^{M_T N_m \times M_T N_m}$, and $\boldsymbol{H}''_{\text{eff}} \in \mathbb{C}^{2M_R N_m \times M_T N_m}$ is a block diagonal matrix with the N_m subcarrier wise combined CTFs at its main diagonal. The combined frequency response at the lth subcarrier is given as

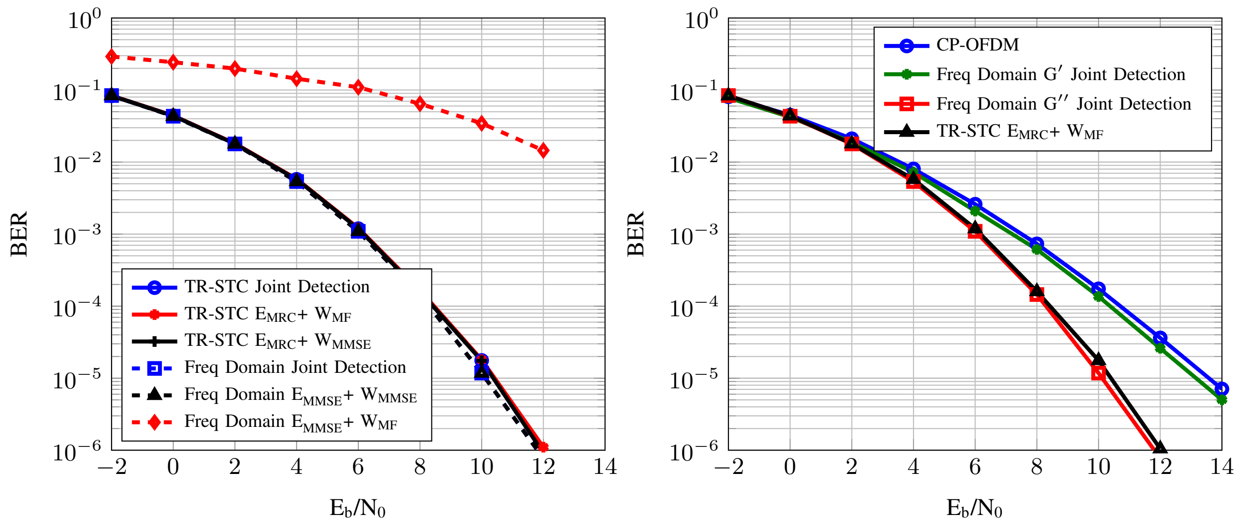
$$\boldsymbol{H}''_l = \begin{bmatrix} h_{l,11} & h_{l,12} \\ \vdots & \vdots \\ h_{l,M_R1} & h_{l,M_R2} \\ h^*_{l,12} & -h^*_{l,11} \\ \vdots & \vdots \\ h^*_{l,M_R2} & -h^*_{l,M_R1} \end{bmatrix} \forall\, l = 1, \ldots, N_m \quad (12)$$

A better estimate of the data symbols can be achieved if the redundancy introduced during the UW-OFDM symbol generation is well exploited at the receiver. In [5], we have shown that a low complexity two-step linear detection scheme for MIMO systems yields a similar performance as of a joint detection (Eq. (9) and Eq. (10)) when a MMSE based code generator demodulator matrix is used. In this case, the proposed two step approach results in a low complex solution. In the first step, we remove the fading impact of the channel on the lth subcarrier by multiplying the combined received signal after two time frames on the lth subcarrier with an equalizer,

$$\hat{\boldsymbol{x}}_l = \boldsymbol{E}_l\hat{\boldsymbol{z}}_l \in \mathbb{C}^{M_T \times 1}, \quad (13)$$

where $\boldsymbol{E}_l \in \mathbb{C}^{M_T \times M_R}$ are the equalizer weights. Since $\boldsymbol{H}''_l$ is an orthogonal matrix, the MRC method can be used. Hence $\boldsymbol{E}_l = [\boldsymbol{H}''_l]^H / \parallel \hat{\boldsymbol{H}}''_l \parallel^2_F$, where

$$\hat{\boldsymbol{H}}''_l = \begin{bmatrix} h_{l,11} & h_{l,12} \\ \vdots & \vdots \\ h_{l,M_R1} & h_{l,M_R2} \end{bmatrix}$$



(a) Different detection schemes for both approaches using $\boldsymbol{G}''$

(b) Performance comparison of UW-OFDM with STBCs

Fig. 2: Performance comparison of 2×2 MIMO with Alamouti based STBCs

In the second step, all equalized symbols are jointly multiplied by a code generator demodulator matrix to get the estimated data symbols

$$\hat{\boldsymbol{D}} = \boldsymbol{W}\hat{\boldsymbol{X}} \in \mathbb{C}^{N_d \times M_T}, \quad (14)$$

where $\hat{\boldsymbol{X}} = [\hat{\boldsymbol{x}}_1, ..., \hat{\boldsymbol{x}}_{N_m}]^T \in \mathbb{C}^{N_m \times M_T}$, and $\boldsymbol{W}$ is the code generator demodulator matrix which can be a MF based demodulator $\boldsymbol{W}_{MF} = \boldsymbol{G}^H$ or a MMSE based demodulator

$$\boldsymbol{W}_{\text{MMSE}} = \boldsymbol{G}^H(\boldsymbol{G}\boldsymbol{G}^H + \sigma_n^2/\sigma_d^2 \boldsymbol{Z})^{-1}, \quad (15)$$

where $\boldsymbol{Z} \in \mathbb{R}^{N_m \times N_m}$ is a diagonal matrix with $\text{tr}(\boldsymbol{E}_l \boldsymbol{E}_l^H)$ $\forall$ $l = 1, \ldots, N_m$ on its main diagonal. Note that $\boldsymbol{E}_l$ $\forall$ $l = 1, \ldots, N_m$ are already calculated in the first step. We can also apply a joint detection by using $\boldsymbol{H}''_{\text{eff}}$ and $\boldsymbol{G}''_{\text{eff}}$ in Eq. (9) or Eq. (10) but this approach is computationally more expensive.

	CP-OFDM	UW-OFDM
Modulation Scheme	QPSK	
Bandwidth	3 MHz	
Data Carriers (N_d)	180	162, 171
Redundant Carriers (N_r)		18, 9
FFT length (N)	256	
CP/UW duration	4.69 μs	
Channel	Extended Vehicular A (EVA)	
MIMO Configuration $M_R \times M_T$	2×2	

TABLE III: Simulation parameters

IV. Simulation Results

In this section, we evaluate the performance of the proposed space-time block coded UW-OFDM system.To this end, we use the LTE parameters in Table III. We assume that the whole bandwidth is allocated to a singe user and no channel coding is applied. Moreover, the 3GPP channel model EVA is used for the simulations. All the simulation results are averaged over 10,000 channel realizations.

First we compare the performance of the proposed frequency domain STBC approach, denoted as "freq domain", and the time domain STBC approach, denoted as "TR-STC" with different detection schemes in Fig. 2a. The frequency domain STBC yields a slightly better performance over the TR-STC (approximately 0.2 dB at high E_b/N_0) when a joint detection based on the BLUE or MMSE criterion is applied. The two-step detection scheme also yields similar results as those of the joint detection for both schemes, when a subcarrier-wise equalization is performed in the first step and a MMSE based demodulator is used in the second step. Note that we have applied MRC based detection for UW-OFDM with TR-STC while MMSE based detection for UW-OFDM with STBCs in the frequency domain. However, the results are different when a MF demodulator is used in the second step. Furthermore, it is noteworthy that an MRC based detection combined with a MF based demodulation shows the same performance as the joint detection for UW-OFDM with TR-STC. This is due to that fact that the received combined CTF on each subcarrier is orthogonal. This can be verified analytically from Eq. (9) which reduces to $\boldsymbol{E}_{\text{BLUE}} = \boldsymbol{G}_{\text{eff}}^H \boldsymbol{H}_{\text{eff}}^H$ when $\boldsymbol{H}_{\text{eff}}^H \boldsymbol{H}_{\text{eff}} = \boldsymbol{I}$. However, when the channel matrix is non-orthogonal e.g., when STBCs are applied in the frequency domain, a MF based demodulation is not feasible.

In Fig. 2b, we compare the performance of CP-OFDM with UW-OFDM using two code generator matrices $(\boldsymbol{G}', \boldsymbol{G}'')$. The simulation results show that the design of the code generator matrix plays an important role on the performance of UW-OFDM, where UW-OFDM with $\boldsymbol{G}''$ outperforms UW-OFDM with $\boldsymbol{G}'$. Note that $\boldsymbol{G}''$ is optimized using a MMSE criterion in an AWGN channel while $\boldsymbol{G}'$ is using the SVD of $\boldsymbol{U}$. Moreover, UW-OFDM with both code generator matrices outperforms CP-OFDM. But the obtained gain is significant for UW-OFDM with $\boldsymbol{G}''$. Clearly UW-OFDM combined with TR-STC is a better and low complex choice for applying STBCs, since the effective channel matrix is orthogonal and therefore a subcarrier-wise detection based on MRC can be applied. Moreover, we can also use a MF based demodulation in this case.

V. Conclusion

In this work, we have proposed two approaches to apply OSTBCs to UW-OFDM. We have also proposed different detection procedures for these approaches. We have shown that just like CP-OFDM, a low complex MRC based detection can also be applied to UW-OFDM when time reversal based space-time coding is used. Moreover, we can also apply a MF based demodulation in the second step if the combined channel matrix is orthogonal. This combination offers a slightly higher computational complexity as CP-OFDM where the additional complexity lies only in the multiplication by $\boldsymbol{G}^H$. But the performance of UW-OFDM with STBCs is far better than for CP-OFDM.

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