EVENT-BASED CONSENSUS FOR A CLASS OF HETEROGENEOUS MULTI-AGENT SYSTEMS: AN LMI APPROACH

Amir Amini, Arash Mohammadi, and Amir Asif

Electrical and Computer Engineering, Concordia University, Montreal, QC, Canada

ABSTRACT

Based on the theory of linear matrix inequalities (LMI), this paper proposes an event-based distributed consensus algorithm for linear multi-agent/sensor networks that are heterogeneous. The proposed scheme is event-based in the sense that each agent transmits its information to its neighbouring nodes only under predefined circumstances. Ensuring the stability of the closed-loop system, the Lyapunov theorem is utilized to compute design parameters (heterogeneous controller gains and transmission threshold) used in the proposed consensus algorithm. Numerical simulations demonstrate a performance gain in the convergence time and a reduction in the number of data transmissions with the proposed approach.

Index Terms— Consensus algorithms, Event-based transmission, Distributed agent/sensor networks, Linear Matrix Inequalities (LMIs).

1. INTRODUCTION

As a fundamental cooperative behaviour in multi-agent/sensor networks, consensus has recently attracted significant attention. The consensus problem in which agents constantly transmit their information has been widely studied in several applications of practical importance [1–4]. In order to decrease the number of transmissions in the distributed scheme and extend the life of the nodes, incorporation of an efficient event-triggering mechanism is of great interest [5–10]. We note that most related works deal primarily with homogeneous agents/sensors, i.e., all agents have identical dynamics; an assumption which is often contradicted in practice [11]. Moreover, in most existing works, in order to achieve consensus, a common control gain is typically designed and shared among all agents [12, 13]. Such a design approach is not considered to be entirely distributed. In a fully distributed structure, each agent should be able to choose its own controller gain, according to its own dynamics and connectivity within the communication network. In addition, such strategies are not optimal for heterogeneous multi-agent/sensor systems. Ignoring heterogeneous control design often leads to unstable network performance, as discussed in Reference [14].

Among different strategies developed to deal with consensus, in this paper, we convert event-triggered consensus problem to an equivalent stability problem of a transformed version of the original system. The main reason for using this approach is to utilize the well-developed Lyapunov method, a powerful stability maintenance technique that also provides a variety of performance indices [15]. We are interested in synthesizing heterogeneous controller gains. The transformed system needs to conserve all system parameters. The transformations suggested by [13] and [16] are incapable of meeting these challenges.

We used linear matrix inequalities (LMI) to solve the transformed stability problem and design the consensus parameters, including the individual controller gains and a common transmission threshold used at each consensus iteration to determine if an agent communicates to its neighbours or not. As a powerful design method, LMI optimization guarantees system stability for desired control and communication specifications through convex optimization. For a multi-objective problem (such as a distributed event-based consensus network with heterogeneous agents as is being considered in this paper), an analytical solution to compute the design parameters is difficult (if not impossible) to derive. Analytical solutions proposed in the literature need strong assumptions on agent dynamics [21], control gain design [29], or/and network topology [22]. Formulating the problem within an LMI framework is a practically feasible solution to pursue [27]. Note that deriving optimization matrix inequalities in a linear structure is a non-trivial effort and some suggested consensus approaches result in bilinear matrix inequalities (BMI) that are even more difficult to solve [28, 30].

Motivated by the aforementioned discussions, in this paper, we investigate the problem of control design for event-based consensus in heterogeneous multi-agent/sensor systems. The proposed event-based consensus strategy offers these practical features: (i) Asynchronous triggering instants, i.e., each agent independently decides on its own triggering time; (ii) Guaranteed minimum number of data transmissions for a triggering function; (iii) Reliance of the control protocol on the most recently received data. Within the LMI framework, the $\mathcal{H}_\infty$ technique is used to reduce the impact of external disturbance on the closed-loop performance.

The remaining paper is organized as follows: Section 2 introduces the notation. In Section 3, we formulate the event-based consensus problem. Section 4 derives a theorem used for parameter estimation. A sample simulation example is included in Section 5. Finally, Section 6 concludes the paper.

2. NOTATION AND PROBLEM STATEMENT

Throughout the paper, we use the following notation for a $m\times n$ dimensional matrix A=$\{a_{ij}\}$. $|A|$: Matrix with component-wise absolute values of A; $\|A\|$: Frobenius norm of A; $A^\dagger$: Pseudo inverse of A; $(A > 0)$: A is symmetric positive definite.

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I_n denotes an Identity matrix of order n; $\boldsymbol{J}_{m\times n}$: $(m \times n)$ matrix with all entries equal to one; $\mathbf{1}_n$: column vector of order n with unit entries. Let $\otimes$ and $\circ$, respectively, denote Kronecker product and Hadamard product; $a_{(i,\bullet)}$: i-th row of matrix A, i.e., $[a_{i1}, \dots, a_{in}]$. Moreover, for two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, $\boldsymbol{u} \leq \boldsymbol{v}$ refers to their component-wise inequality, i.e., $u_i \leq v_i$. Notation $*$ denotes the transpose of the corresponding block in the upper triangle of a symmetric matrix.

Graph Theory: $\mathcal{A} = \{a_{ij}\} \in \mathbb{R}^{N\times N}$: Weighted adjacency matrix for graph $\mathcal{G}$; L: Laplacian matrix; $\mathcal{N}_i$: neighbouring set for agent i. For details on algebraic graph theory, refer to [19].

We consider an agent network comprising of N agents. The general linear dynamics of agent i, for $(1 \leq i \leq N)$, is

$$\dot{\boldsymbol{x}}_i(t) = A\boldsymbol{x}_i(t) + B_i\boldsymbol{u}_i(t) + D_i\boldsymbol{\omega}_i(t), \tag{1}$$

where $\boldsymbol{x}_i(t) \in \mathbb{R}^n$ is the state vector in time instant t, $\boldsymbol{u}_i(t) \in \mathbb{R}^m$ is the control input vector and $\boldsymbol{\omega}_i(t)$ defines the external disturbance for agent i. Despite the common practice [10,19] that assumes similar state matrices (A, B_i, and D_i) across the network, we only consider A to be identical among agents. We note that pairs (A, B_i) are controllable.

Definition 1. *Under a proposed distributed protocol $\boldsymbol{u}_i(t)$, the consensus problem is solved if and only if* $\lim_{t\to\infty} \| \boldsymbol{x}_i(t) - \boldsymbol{x}_j(t) \| = 0$*, $(1 \leq i, j \leq N)$, for any initial states* [26].

The agents share their information with the neighbours to asymptotically reach consensus. However, in order to decrease the number of transmissions, agent i transmits its states to the neighbouring nodes only under certain condition. Denoting $t_0^i, t_1^i, \dots$ as the triggering time sequence of agent i, we define the most recently broadcasted state of agent i as $\hat{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t_k^i)$ for any interval between two consecutive triggering instants, i.e., $t \in [t_k^i \quad t_{k+1}^i)$. In order to achieve consensus the following control law is proposed for agent i

$$\boldsymbol{u}_i(t) = K_i \sum_{j\in\mathcal{N}_i} (\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t)), \tag{2}$$

where $K_i \in \mathbb{R}^{m\times n}$ is the control gain to be computed.

3. PROBLEM FORMULATION

We proceed to formulate the problem in this section. Let $\boldsymbol{e}_i(t) = \hat{\boldsymbol{x}}_i(t) - \boldsymbol{x}_i(t)$ denote the error between the most recently transmitted state, $\hat{\boldsymbol{x}}_i(t)$, and its instantaneous value, $\boldsymbol{x}_i(t)$, for agent i. For the purpose of analysis in a collective manner, we define $\boldsymbol{x}(t){=}[\boldsymbol{x}_1^T(t), \dots, \boldsymbol{x}_N^T(t)]^T$ as the stacked state vector and $\hat{\boldsymbol{x}}(t){=}[\hat{\boldsymbol{x}}_1^T(t), \dots, \hat{\boldsymbol{x}}_N^T(t)]^T$ as the stacked vector for last transmitted states, which together define the stacked error vector $\boldsymbol{e}(t){=}\hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$. We also define $\boldsymbol{\omega}(t){=}[\boldsymbol{\omega}_1^T(t), \dots, \boldsymbol{\omega}_N^T(t)]^T$ as the stacked external disturbance vector. Based on the predefined parameters, the following augmented closed-loop system is derived

$$\dot{\boldsymbol{x}}(t) = \left(A_{\langle N\rangle} + BKL_{\langle n\rangle}\right)\boldsymbol{x}(t) + BKL_{\langle n\rangle}\boldsymbol{e}(t) + D\boldsymbol{\omega}(t), \tag{3}$$

where $L_{\langle n\rangle} = L \otimes I_n$, $A_{\langle N\rangle} = I_N \otimes A$, $B = \mathrm{diag}(B_1, \dots, B_N)$, $D = \mathrm{diag}(D_1, \dots, D_N)$, and $K = \mathrm{diag}(K_1, \dots, K_N)$. With the aim to minimize the impact of noise on the closed-loop performance the $\mathcal{H}_\infty$ control problem [16] is defined next.

Definition 2. *Under zero-initial states, the $\mathcal{H}_\infty$ disturbance rejection performance for system variable, $\boldsymbol{x}(t)$, accompanied by a given positive definite weighting matrix R, is achieved if* $J = \int_0^\infty \left(\boldsymbol{x}^T(t)R\boldsymbol{x}(t) - \rho^2\boldsymbol{\omega}^T(t)\boldsymbol{\omega}(t)dt\right) < 0$ *is fulfilled for a given positive-valued scalar ρ.*

System Transformation: We note that the control design problem for system (3) is troublesome, since $L_{\langle n\rangle}$ contains a zero eigenvalue [16]. A common solution is to convert the consensus problem to stability problem by applying proper state transformation, e.g., $\bar{\boldsymbol{x}}(t) = T\boldsymbol{x}(t)$ [23]. However, the stability of the transformed system, (i.e., $\lim_{t\to\infty} \bar{\boldsymbol{x}}(t) = 0$), must be equivalent to the consensus problem for (3). Therefore, let $\hat{L} \in \mathbb{R}^{(N-1)\times N}$ denote reduced Laplacian matrix which is obtained by removing an arbitrary row of the L. Then, the following transformation is proposed

$$\boldsymbol{x}_{(\mathrm{r})}(t) = \hat{L}_{\langle n\rangle}\boldsymbol{x}(t), \tag{4}$$

where $\hat{L}_{\langle n\rangle} = \hat{L} \otimes I_n$. According to Lemma 1, the consensus problem for system (3) is equivalent to the stability problem for the transformed system expressed as $\boldsymbol{x}_{(\mathrm{r})}(t)$.

Lemma 1. $\boldsymbol{x}_{(\mathrm{r})}(t) = 0$ *if and only if* $\boldsymbol{x}_1(t) = \cdots = \boldsymbol{x}_N(t)$*. In other words, the consensus is satisfied if* $\boldsymbol{x}_{(\mathrm{r})}(t) = 0$.

The proof of Lemma 1 is omitted to save on space. Using Transformation (4), we convert System (3) as follows

$$\dot{\boldsymbol{x}}_{(\mathrm{r})}(t) = \mathcal{A}_x\boldsymbol{x}_{(\mathrm{r})}(t) + \mathcal{A}_e\boldsymbol{e}_{(\mathrm{r})}(t) + \hat{L}_{\langle n\rangle}D\boldsymbol{\omega}(t), \tag{5}$$

where $\boldsymbol{e}_{(\mathrm{r})}(t){=}\hat{L}_{\langle n\rangle}\boldsymbol{e}(t)$, $\mathcal{A}_x{=}A_{\langle N-1\rangle} + \hat{L}_{\langle n\rangle}BK\mathcal{L}$, $\mathcal{A}_e{=}\hat{L}_{\langle n\rangle}BK\mathcal{L}$, with $\mathcal{L}{=}L_{\langle n\rangle}\hat{L}_{\langle n\rangle}^\dagger$ and $A_{\langle N-1\rangle}{=}I_{N-1} \otimes A$. Note that, the equalities $\hat{L}_{\langle n\rangle}A_{\langle N\rangle}{=}A_{\langle N-1\rangle}\hat{L}_{\langle n\rangle}$, and, $L{=}L\hat{L}^\dagger\hat{L}$, are also used to derive (5). Moreover, $\boldsymbol{e}_{(\mathrm{r})}(t){=}\hat{\boldsymbol{x}}_{(\mathrm{r})}(t) - \boldsymbol{x}_{(\mathrm{r})}(t)$ holds in the transformed domain. Before proceeding to the next section, we note that for the convenience of discussion and without loss of generality, we remove the N-th row of L to obtain $\hat{L}$.

Event-Triggering Scheme: In order to propose and formulate the event-triggering condition, we first define disagreement vector for agent i as $\hat{\mathbb{X}}_i(t) = l_{(i,\bullet)}^{\langle n\rangle}\hat{\boldsymbol{x}}(t)$, with $l_{(i,\bullet)}^{\langle n\rangle} = l_{(i,\bullet)} \otimes I_n$. Similarly, let $\hat{\mathbb{X}}(t) = [\hat{\mathbb{X}}_1(t), \dots, \hat{\mathbb{X}}_N(t)]^T$ denote the stacked disagreement vector. With the latter definition, $\hat{\boldsymbol{x}}_{(\mathrm{r})}(t)$ is the independent rows of $\hat{\mathbb{X}}(t)$. The triggering function for agent i is given by the following inequality

$$\|\boldsymbol{e}_i(t)\| \leq \phi\|\hat{\mathbb{X}}_i(t)\|, \tag{6}$$

where ϕ is the transmission threshold to be maximized to provide minimum number of transmission. While $\|\boldsymbol{e}_i(t)\|$ is lower than $\phi\|\hat{\mathbb{X}}_i(t)\|$, the state vector is not transmitted. However, when the two sides of (6) become equal, the new information, $\boldsymbol{x}_i(t)$, is transmitted to the neighbours. The set of inequities in (6) are integrated in the following component-wise inequality by defining the normed-vectors $\boldsymbol{e}^{[\mathrm{Nr}]} = [\|\boldsymbol{e}_1(t)\|, \dots, \|\boldsymbol{e}_N(t)\|]^T$, and $\hat{\mathbb{X}}^{[\mathrm{Nr}]} = [\|\hat{\mathbb{X}}_1(t)\|, \dots, \|\hat{\mathbb{X}}_N(t)\|]^T$.

$$\boldsymbol{e}^{[\mathrm{Nr}]} \leq \phi\hat{\mathbb{X}}^{[\mathrm{Nr}]}. \tag{7}$$

In order to derive maximum stable threshold ϕ, the event-triggering condition (7) should be expressed by $\boldsymbol{x}_{(\mathrm{r})}(t)$ and $\boldsymbol{e}_{(\mathrm{r})}(t)$. Therefore, the following two Lemmas are given.

Lemma 2. *Event-triggering condition* (7) *is equivalent to*

$$\hat{L}\boldsymbol{e}^{[\mathrm{Nr}]} \leq \phi|\hat{L}|\hat{\mathbb{X}}^{[\mathrm{Nr}]}. \tag{8}$$

Lemma 3. *Let* $\boldsymbol{\psi}_e = [\|l^{\langle n\rangle}_{(1,\bullet)}\boldsymbol{e}(t)\|, \ldots, \|l^{\langle n\rangle}_{(N-1,\bullet)}\boldsymbol{e}(t)\|]^T$ *and* $\boldsymbol{\psi}_{\hat{x}} = [\|\phi m^{\langle n\rangle}_{(1,\bullet)}\hat{\boldsymbol{x}}_{(r)}(t)\|, \ldots, \|\phi m^{\langle n\rangle}_{(N-1,\bullet)}\hat{\boldsymbol{x}}_{(r)}(t)\|]^T$, *where* $m^{\langle n\rangle}_{(i,\bullet)} = m_{(i,\bullet)}\otimes I_n$ *and triggering matrix* $M = \{m_{ij}\} \in \mathbb{R}^{(N-1)\times(N-1)} = \{l_{ij} + \alpha_j l_{iN}\}$ *with* $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_{N-1}] = l_{(N,\bullet)}\hat{L}^{\dagger}$. *If a certain* ϕ *satisfies the following entry-wise inequality*

$$\boldsymbol{\psi}_e \leq \boldsymbol{\psi}_{\hat{x}} \tag{9}$$

it also satisfies the inequality given in (8).

Lemmas 2 and 3 are proved using the reverse triangle inequality and sub-additivity property associated with the Euclidean norm. Due to space limitation, the proofs are not included here. Based on Lemma 3, any ϕ satisfying (9) also satisfies (8). Moreover, Inequality (8) is equivalent to (7) according to Lemma 2. Inequality (9) can be expressed as a global quadratic constraint in the form of $\boldsymbol{e}^T_{(\mathrm{r})}\boldsymbol{e}_{(\mathrm{r})} \leq \hat{\boldsymbol{x}}^T_{(\mathrm{r})}M^T_{\langle n\rangle}\Phi^2 M_{\langle n\rangle}\hat{\boldsymbol{x}}_{(\mathrm{r})}$, where $\Phi = \phi I_{(N-1)n}$, and $M_{\langle n\rangle} = M \otimes I_n$. The quadratic form of (9) is useful for maximizing ϕ, when $\hat{\boldsymbol{x}}_{(\mathrm{r})}$ is replaced by $\hat{\boldsymbol{x}}_{(\mathrm{r})} = \boldsymbol{e}_{(\mathrm{r})} + \boldsymbol{x}_{(\mathrm{r})}$. The following inequality will be used in Section 4 to determine the maximum possible threshold ϕ.

$$\boldsymbol{e}^T_{(\mathrm{r})}\boldsymbol{e}_{(\mathrm{r})} \leq \left(\boldsymbol{e}_{(\mathrm{r})} + \boldsymbol{x}_{(\mathrm{r})}\right)^T M^T_{\langle n\rangle}\Phi^2 M_{\langle n\rangle}\left(\boldsymbol{e}_{(\mathrm{r})} + \boldsymbol{x}_{(\mathrm{r})}\right). \tag{10}$$

4. PARAMETER ESTIMATION

Theorem 1 is derived using the Lyapunov framework. It computes the heterogeneous control gains K_i's and maximum transmission threshold ϕ. It guarantees stability of the reduced system (5) and ensures consensus in system (3).

Theorem 1. *Given scalar* $\rho > 0$ *and positive definite weighting matrices* $R_i \in \mathbb{R}^{n\times n}$ $(1 \leq i \leq N-1)$, *the following optimization problem computes the control gains,* K_i, $(1 \leq i \leq N)$ *and the transmission threshold,* ϕ, *for state model* (3)

$$\min_{\Theta_i,\gamma,\tau,\mathcal{P}} \quad \gamma \tag{11}$$

$$\text{subject to:} \quad \begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix} < 0, \quad \mathcal{P} > 0, \quad \tau > 0, \quad \gamma > 0,$$

$$\textit{with} \quad \Pi_1 = \begin{bmatrix} \pi_{11} & \Xi\mathcal{L} \\ * & -\tau I \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} P\hat{L}_{\langle n\rangle}D & \tau M^T_{\langle n\rangle} \\ 0 & \tau M^T_{\langle n\rangle} \end{bmatrix},$$

$$\textit{and} \quad \Pi_3 = diag\left(-\rho^2 I, -\gamma I\right).$$

The undefined parameters in (11) *are given by:* $\pi_{11} = A^T_{\langle N-1\rangle}P + PA_{\langle N-1\rangle} + R + \Xi\mathcal{L} + \mathcal{L}^T\Xi^T$, $\Xi = \left(\hat{L} \otimes \boldsymbol{J}_{n\times n}\right) \circ (\boldsymbol{1}_{N-1} \otimes [\Theta_1, \ldots, \Theta_N])$, $P = I_{N-1} \otimes \mathcal{P}$, $R = diag(R_1, ..., R_{N-1})$, *where* $\mathcal{P} \in \mathbb{R}^{n\times n}$, $\Theta_i \in \mathbb{R}^{n\times n}$ $(1 \leq i \leq N)$ *and scalars* $\{\tau, \gamma\}$ *are the optimization parameters. Expressed in terms of these optimization parameters, terms* K_i*'s and* ϕ *are computed as*

$$\phi = \sqrt{\tau\gamma^{-1}}, \quad K_i = B_i^{\dagger}\mathcal{P}^{-1}\Theta_i, \quad i = 1, ..., N. \tag{12}$$

Proof. We start the proof with the following inequality which ensures stability conditions for the reduced system (5) and the $\mathcal{H}_\infty$ performance at the same time [20]

$$\dot{V}(\boldsymbol{x}_{(\mathrm{r})}) + \boldsymbol{x}^T_{(\mathrm{r})}R\boldsymbol{x}_{(\mathrm{r})} - \rho^2\boldsymbol{\omega}^T\boldsymbol{\omega} < 0, \tag{13}$$

where $V(\boldsymbol{x}_{(\mathrm{r})}) = \boldsymbol{x}^T_{(\mathrm{r})}P\boldsymbol{x}_{(\mathrm{r})}$ is the Lyapunov candidate function. Now considering $\Upsilon = [\boldsymbol{x}^T_{(\mathrm{r})}, \boldsymbol{e}^T_{(\mathrm{r})}, \boldsymbol{\omega}^T]^T$, we expand (13)

$$\Upsilon^T \begin{bmatrix} \mathcal{A}_x^T P + P\mathcal{A}_x + R & P\mathcal{A}_e & P\hat{L}_{\langle n\rangle}D \\ * & 0 & 0 \\ * & * & -\rho^2 I \end{bmatrix} \Upsilon < 0. \tag{14}$$

Now the event-triggering condition (10), must be placed into (14). To this end, we apply S-procedure [17]. The slack variable τ appears in the resulting integrated inequality as

$$\bar{\Pi} = \begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & P\hat{L}_{\langle n\rangle}D \\ * & \bar{\Pi}_{22} & 0 \\ * & * & -\rho^2 I \end{bmatrix} < 0, \tag{15}$$

with $\bar{\Pi}_{11} = \mathcal{A}_x^T P + P\mathcal{A}_x + R + \tau M^T_{\langle n\rangle}\Phi^2 M_{\langle n\rangle}$, $\bar{\Pi}_{12} = P\mathcal{A}_e + \tau M^T_{\langle n\rangle}\Phi^2 M_{\langle n\rangle}$ and $\bar{\Pi}_{22} = -\tau I + \tau M^T_{\langle n\rangle}\Phi^2 M_{\langle n\rangle}$. Although feasible solutions can be found for (15), however, maximization over ϕ is still a non-convex problem. To tackle this problem, we first apply Schur complement [17] on (15)

$$\begin{bmatrix} \mathcal{A}_x^T P + P\mathcal{A}_x + R & P\mathcal{A}_e & P\hat{L}_{\langle n\rangle}D & \tau M^T_{\langle n\rangle}\Phi \\ * & -\tau I & 0 & \tau M^T_{\langle n\rangle}\Phi \\ * & * & -\rho^2 I & 0 \\ * & * & * & -\tau I \end{bmatrix} < 0. \tag{16}$$

Algorithm 1: Proposed Event-based Consensus

Input: $\mathcal{A} = \{a_{ij}\}$, Agents' dynamics given in (1).
Output: Asymptotic Event-triggered State Consensus

Parameter Estimation: (E1 – E5)

I. *Initialization*

E1. *Transformation Matrix*: Remove N^{th} row of L in order to determine the reduced Laplacian matrix, $\hat{L}$.
E2. *System Transformation:* Determine reduced system (5).
E3. *Triggering Matrix*: Using Lemma 3, determine $M_{\langle n\rangle}$.

II. *Design*

E4. *Solving the LMI's*: Using convex optimization solvers, solve the LMIs (11) for a given $\mathcal{H}_\infty$ parameters, $\{R, \rho\}$.
E5. *Feasibility Verification*: If a solution exists for (11), obtain ϕ, and K_i's from (12). Otherwise, change parameters $\{R, \rho\}$, and repeat step E4.

Event-triggered Consensus: (C1 – C3)

C1. *Initialization*: Initialize by allowing all agents to transmit their initial states $x_i(0)$ to their neighbours.
C2. *Execution*: Using K_i's derived in Step E4, the states of agent i in (1) is excited by (2). Condition (6) is responsible to determine the next state transmission to neighbours for agent i as the states evolves to reach consensus.
C3. *Consensus Achievement*: Agent i repeats Step C2 until convergence is achieved for the disagreement state vector, i.e., $\|\hat{\mathbb{X}}_i(t)\| < \delta_i$ where δ_i is the stopping criterion.

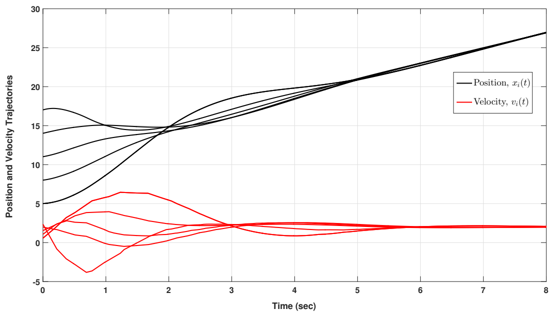

Fig. 1: Evolution of state consensus using Theorem 1.

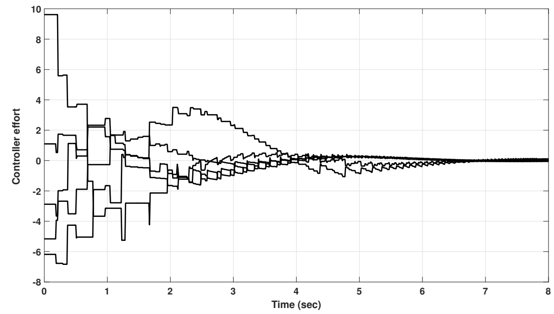

Fig. 2: Controller effort $u_i(t)$ for the proposed approach.

Then, pre and post-multiplying of (16) by positive definite matrix $\Omega = \mathrm{diag}(I, I, I, \Phi^{-1})$ leads to

$$\begin{bmatrix} \mathcal{A}_x^T P + P\mathcal{A}_x + R & P\mathcal{A}_e & P\hat{L}_{\langle n \rangle} D & \tau M_{\langle n \rangle}^T \\ * & -\tau I & 0 & \tau M_{\langle n \rangle}^T \\ * & * & -\rho^2 I & 0 \\ * & * & * & -\tau \Phi^{-2} \end{bmatrix} < 0. \quad (17)$$

The matrix inequality (17) is not linear due to numerous products of variables. To prevent difficult computations over BMIs, proper change of variables is necessary to derive LMIs condition. Therefore, we first expand $P\mathcal{A}_e$:

$$P\mathcal{A}_e = \left(\hat{L} \otimes \boldsymbol{J}_{n\times n}\right) \circ \left(\mathbf{1}_{N-1} \otimes [\mathcal{P}B_1K_1, \ldots, \mathcal{P}B_NK_N]\right)\mathcal{L} \quad (18)$$

Now, by changing variables as $\Theta_i = \mathcal{P}B_iK_i$, the term Ξ in (11) is obtained. The same change of variables can be applied for expanded $P\mathcal{A}_x$. Moreover, we define the alternative variable γ to represent $\tau\phi^{-2}$, i.e., $\gamma I_{n(N-1)} = \tau\Phi^{-2}$. As a result, maximization over ϕ is now equivalent to minimizing γ. Once the LMIs are solved, optimization variables $\{\tau, P, \Theta_i, \gamma\}$ are obtained and ϕ, and K_i's can be derived from (12). □

Algorithm 1 summarizes the proposed consensus approach.

5. EXPERIMENTAL RESULTS

We consider the 2^{nd} order, 5-agent system [14, 18] given by

$$\begin{aligned} \dot{r}_i(t) &= v_i(t), \\ m_i\dot{v}_i(t) &= u_i(t) + \omega_i(t), \quad (1 \le i \le 5), \end{aligned} \quad (19)$$

where $r_i(t)$, $v_i(t)$, m_i, and $\omega_i(t)$ denote the position, velocity, inertia, and external disturbance for agent i, respectively. Unlike most earlier work where the inertias in second-order systems are not taken into account by simply assuming $m_i = 1$, here we consider heterogeneous inertias as fully discussed in [25]. Therefore, our heterogeneous approach is relevant since the underlying dynamics justify the need for a heterogeneous control design. Considering $\boldsymbol{x}_i(t) = [r_i(t), v_i(t)]^T$, we derive the state matrices for (19) based on (1).

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 + 0.1i \end{bmatrix}, \quad D_i = \begin{bmatrix} 0 \\ 1 + 0.1i \end{bmatrix}. \quad (20)$$

The directed communication topology of the multi-agent system (20) is given by $L = [2, -1, 0, -1, 0; 0, 2, -1, 0, -1; 0, -1, 3, -1, -1; 0, -1, 0, 2, -1; -1, 0, 0, -1, 2]$. To solve the consensus problem using Theorem 1, we initialize $R = 0.04I_8$, and $\rho = 0.03$. The initial state values are picked at random as $x_1(0)=[1, 2]^T$, $x_2(0)=[2, 5]^T$, $x_3(0)=[4, 8]^T$, $x_4(0)=[5, 6]^T$ and $x_5(0)=[7, 7]^T$. Using the YALMIP parser and SDPT3 solver in MATLAB [24], we solve (11) for the aforementioned given parameters. The computed values for the gains are $K_1=[0.50, 0.38]$, $K_2=[0.41, 0.47]$, $K_3=[0.49, 0.63]$, $K_4=[0.43, 0.46]$, and $K_5=[0.74, 0.87]$. The transmission threshold ϕ is calculated from (12) as $\phi=0.212$. The state trajectories of the five agents are shown in Fig. 1 for the stopping criterion $\delta_i=0.01$. Fig. 2 plots the controllers' force as defined in (2) based on the values of K_i's and ϕ. We compare the results from Theorem 1 with the event-based consensus approach proposed in [29]. In [29], the stable K needs to satisfy $ABK=A$. Therefore, position measurements are ignored in second-order agents. Table 1 provides a comparison between our proposed approach given in Theorem 1 with its counterpart [29] under identical initial conditions.

Table 1: Performance comparison for the two approaches.

Approach	# transmissions per agent					Consensus time (sec)
	1	2	3	4	5	
Theorem 1	39	128	119	67	123	7.32
Method [29]	74	106	140	88	104	8.95

As shown in Table 1, the proposed approach uses a lower total number of transmissions (476 versus 512, or about a 7 to 8% improvement). In terms of the CPU time, consensus is achieved faster with the proposed approach. In larger networks, we observed a higher performance gain.

6. SUMMARY AND FUTURE WORK

The paper addresses the problem of achieving distributed event-based consensus in heterogeneous, multi-agent/sensor networks. The Lyapunov stability theorem is used to compute the heterogeneous control gains and the transmission threshold based on the LMI optimization framework. The effectiveness of the proposed algorithm is examined through simulations for heterogeneous, 2^{nd} order multi-agent systems. In future, we are interested in applying the proposed event-based consensus algorithm to distributed state estimation networks. In such an application, the proposed event-based consensus algorithm will be extended to achieve consensus for the localized state estimates and their corresponding statistics.

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