A TWO-STAGE OPTIMIZATION APPROACH TO THE ASYNCHRONOUS MULTI-SENSOR REGISTRATION PROBLEM

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ABSTRACT

An important step in multi-sensor data fusion is sensor *registration*, namely, to estimate sensors' range and azimuth biases from their asynchronous measurements. Assuming the target moves in a straight line with an unknown constant velocity, we propose a two-stage nonlinear least square (LS) approach to this problem. More specifically, in stage I, each sensor first estimates its own range bias individually, and then in stage II, all sensors jointly estimate their azimuth biases. We show that both of the nonconvex LS problems can be solved to global optimality under mild conditions. Simulation results show that the root mean square error (RMSE) of the proposed approach is quite close to the Cramér-Rao lower bound (CRLB) when the level of the measurement noise is small.

***Index Terms*—** Asynchronous multi-sensor registration problem, nonconvex nonlinear LS, tightness of semidefinite program (SDP) relaxation

1. INTRODUCTION

In recent years, there is an increasing interest in integrating standalone sensors into multi-sensor systems for command, control, and communications [1]. Instead of developing expensive high-performance sensors, directly fusing data from existing multiple inexpensive sensors is a more cost-effective approach to improving the performance of tracking and surveillance system. However, the success of fusing multi-sensors' data requires an important process called *registration* (or *alignment*). Sensor registration process refers to align a set of data coming from different sensors into a common coordinate system and compensate each sensor's *biases* (or *offset errors*), i.e., range and azimuth biases. Since sensors' biases change slowly with time, they can be treated as constants during a relatively long period of time. The major problem involved in sensor registration process is to estimate these constant biases from data measured asynchronously by different sensors.

Various algorithms have been proposed for solving the sensor registration problem. These algorithms generally can be divided into two categories according to the sensors' work mode: synchronous mode and asynchronous mode. In the synchronous work mode, sensors simultaneously observe the target's position, with measurements from different sensors corresponding to the same (observation) time instants. Based on this assumption, many algorithms, such as maximum likelihood-based [2, 3, 4, 5] and LS type of algorithms [6, 7], have been proposed for solving the registration problem in the synchronous scenario.

However in practice, sensors usually are not synchronized in time due to different data rates and task requirements. When working in the asynchronous mode, sensors observe the target at different time instants, which makes sensor registration become difficult. To overcome this difficulty, researchers have resorted to exploit the *a priori* knowledge of the target's dynamic model. For instance, several Bayesian filtering based approaches were proposed in [8, 9, 10]. These methods utilize target dynamic model and sensor measurement model to recursively update target's states and estimate sensors' biases.

In this paper, we consider the asynchronous multi-sensor registration problem and propose a two-stage optimization approach by assuming the *a priori* knowledge that the target is moving (in a straight line) with a constant velocity. In sharp contrast to filtering approaches [8, 9, 10], the proposed approach estimates biases in two separate stages. More specifically, in stage I of the proposed approach, each sensor independently estimates its own range bias; while in stage II, all sensors jointly estimate their azimuth biases by solving a nonconvex Quadratically Constrained Quadratic Program (QCQP). We show that both of the nonconvex problems in stages I and II can be solved to global optimality under mild conditions. Simulation results show the effectiveness of our proposed two-stage optimization approach.

We adopt the following notation in this paper. Lower and upper case letters in bold are used for vectors and matrices, respectively. For a given matrix $\boldsymbol{H}$, we denote its transpose, Hermitian transpose, and inverse (if it is invertible) by $\boldsymbol{H}^T$, $\boldsymbol{H}^\dagger$, and $\boldsymbol{H}^{-1}$, respectively. We use $\boldsymbol{x}_n$ to denote the n-th component of the vector $\boldsymbol{x}$ and $\boldsymbol{x}_{n:m}$ (with $n < m$) to denote the vector formed by components of $\boldsymbol{x}$ from index n to index m.

2. ASYNCHRONOUS MULTI-SENSOR REGISTRATION PROBLEM

Consider a multi-sensor system consisting of $M > 1$ sensors located distributively on a 2-dimensional plane with known positions. There is a target moving in the surveillance space. Different sensors measure the relative range and azimuth between the target and sensors themselves in an asynchronous work mode. For ease of notation and presentation, measurements from different sensors are mapped onto a common time axis at the fusion center, indexed by k. Furthermore, we assume that, at time instant k, only one sensor observes the target and the corresponding sensor is denoted as $s_k \in \{1, 2, \ldots, M\}$.

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Please see Fig. 1 for an illustration of the asynchronous work mode with $M = 2$ sensors.

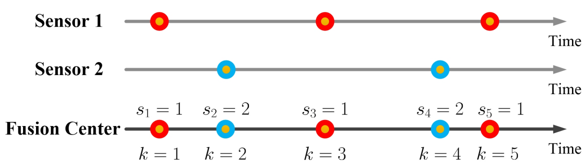

Fig. 1. An illustration of the asynchronous work mode ($M = 2$).

Let $\boldsymbol{\xi}_k = [x_k, y_k]^T$ denote the target's position in the common coordinate system at time instant k. Then, the measured range ρ_k and azimuth ϕ_k at sensor s_k are

$$\boldsymbol{z}_k \triangleq \begin{bmatrix} \rho_k \\ \phi_k \end{bmatrix} = h^{-1}(\boldsymbol{\xi}_k - \boldsymbol{p}_{s_k}) - \boldsymbol{\theta}_{s_k} + \boldsymbol{w}_k. \quad (1)$$

In the above, $h^{-1}(\cdot)$ is the inverse function of the 2-dimensional spherical-to-Cartesian transformation function $h(\cdot)$; $\boldsymbol{p}_m = [p_m^x, p_m^y]^T$ is the position of sensor m; $\boldsymbol{\theta}_m = [\Delta\rho_m, \Delta\phi_m]^T$ denotes the range and azimuth biases of sensor m; $\boldsymbol{w}_k = [w_k^\rho, w_k^\phi]^T$ is assumed to be an uncorrelated Gaussian random noise with zero mean [11], i.e.,

$$\boldsymbol{w}_k \sim \mathcal{N}\left(\boldsymbol{0}, \begin{bmatrix} \sigma_\rho^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}\right).$$

The asynchronous multi-sensor registration problem considered in this paper is to estimate sensors' biases $\{\boldsymbol{\theta}_m\}_{m=1}^M$ from the noisy measurements $\{\boldsymbol{z}_k\}_{k=1}^K$, where K is the total number of measurements. However, it is generally impossible to do so only based on measurements $\{\boldsymbol{z}_k\}_{k=1}^K$. This is because the number of unknown parameters is larger than that of the measurements, i.e., we have in total $2K + 2M$ unknown parameters $\{\boldsymbol{\xi}_k\}_{k=1}^K$ and $\{\boldsymbol{\theta}_m\}_{m=1}^M$ but in total only $2K$ measurements $\{\boldsymbol{z}_k\}_{k=1}^K$. Fortunately, the target usually moves in some known pattern such as with the constant velocity. By exploiting the a priori knowledge, we can still estimate $\{\boldsymbol{\theta}_m\}_{m=1}^M$ from measurements $\{\boldsymbol{z}_k\}_{k=1}^K$. More details on this will be shown in the next section.

3. PROBLEM FORMULATION AND PROPOSED TWO-STAGE APPROACH

In this section, we present our LS formulations of the asynchronous multi-sensor registration problem and propose an approach to solving the formulations. The proposed approach consists of two stages: in stage I, each sensor uses its local measurements to estimate its range bias independently; then in stage II, the fusion center combines all sensors' measurements and the estimated range biases information in stage I together to estimate all sensors' azimuth biases. We shall present stage I and stage II in Sections 3.1 and 3.2, respectively.

For ease of presentation, we assume that the target moves with an *unknown* constant velocity $\boldsymbol{v} = [v_x, v_y]^T \neq \boldsymbol{0}$. Let T_k denote the time difference between the time instants $k+1$ and k. Then, the target's positions at time instants $k+1$ and k satisfy

$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\xi}_k + T_k\boldsymbol{v}. \quad (2)$$

Notice that Eq. (1) can be equivalently rewritten as

$$\boldsymbol{\xi}_k = h(\boldsymbol{z}_k - \boldsymbol{w}_k + \boldsymbol{\theta}_{s_k}) + \boldsymbol{p}_{s_k}. \quad (3)$$

We can see from (2) and (3) that measurements from different sensors are connected with each other through (2). Our idea is to substitute (3) into (2) and establish the LS models. To do so, we still need to deal with the unknown noise $\boldsymbol{w}_k$ in (3). To circumvent this difficulty, we introduce the so-called *unbiased* spherical-to-Cartesian coordinate transformation for Gaussian noise [12]:

$$\bar{h}(\boldsymbol{z}_k) = \begin{bmatrix} \lambda^{-1}\rho_k\cos\phi_k \\ \lambda^{-1}\rho_k\sin\phi_k \end{bmatrix}, \quad (4)$$

where $\lambda = e^{-\sigma_\phi^2/2}$ is the noise compensation factor. The unbiased property refers to $\mathbb{E}_{\boldsymbol{w}_k}\{\bar{h}(\boldsymbol{z}_k)\} = h\left(\mathbb{E}_{\boldsymbol{w}_k}\{\boldsymbol{z}_k\}\right)$, which further implies

$$\boldsymbol{\xi}_k = \mathbb{E}_{\boldsymbol{w}_k}\{\bar{h}(\boldsymbol{z}_k + \boldsymbol{\theta}_{s_k})\} + \boldsymbol{p}_{s_k}. \quad (5)$$

We are now ready to formulate the asynchronous registration problem and the LS formulations will be presented in Sections 3.1 and 3.2.

3.1. Stage I: Individual estimation of range bias

In this subsection, with a slight abuse of notations, we still use $\{\boldsymbol{z}_k\}_{k=1}^K$ to denote the measurements of a single sensor and $\boldsymbol{\theta} \triangleq [\Delta\rho, \Delta\phi]^T$ to denote its unknown range and azimuth biases.

Combining (2) and (5), we can obtain the LS formulation for estimating $\boldsymbol{\theta}$ and $\boldsymbol{v}$:

$$\min_{\boldsymbol{\theta},\boldsymbol{v}} f(\boldsymbol{\theta},\boldsymbol{v}) \triangleq \sum_{k=1}^{K-1} \|\bar{h}(\boldsymbol{z}_{k+1} + \boldsymbol{\theta}) - \bar{h}(\boldsymbol{z}_k + \boldsymbol{\theta}) - T_k\boldsymbol{v}\|^2. \quad (6)$$

where $\|\cdot\|$ denotes the Euclidean norm. In light of (4), $f(\boldsymbol{\theta},\boldsymbol{v})$ is a (convex) quadratic function with respect to $\Delta\rho$ and $\boldsymbol{v}$. For any fixed $\Delta\phi$, problem (6) can be equivalently rewritten as the following convex quadratic program with respect to $\Delta\rho$ and $\boldsymbol{v}$:

$$\min_{\Delta\rho,\boldsymbol{v}} \quad \left\|\boldsymbol{H}_{\Delta\phi}\begin{bmatrix} \Delta\rho \\ \boldsymbol{v} \end{bmatrix} - \boldsymbol{y}_{\Delta\phi}\right\|^2, \quad (7)$$

where $\boldsymbol{H}_{\Delta\phi} \in \mathbb{R}^{2(K-1)\times 3}$ and $\boldsymbol{y}_{\Delta\phi} \in \mathbb{R}^{2(K-1)}$ depend on the value of $\Delta\phi$ and the sensor's measurements. Suppose $K \geq 3$ (such that $\boldsymbol{H}_{\Delta\phi}$ is of full column rank), then the optimal solution for problem (7) is

$$\begin{bmatrix} \Delta\rho^* \\ \boldsymbol{v}^* \end{bmatrix} = (\boldsymbol{H}_{\Delta\phi}^T\boldsymbol{H}_{\Delta\phi})^{-1}\boldsymbol{H}_{\Delta\phi}^T\boldsymbol{y}_{\Delta\phi}. \quad (8)$$

The following Theorem 1 shows, somewhat surprisingly, that $\Delta\rho^*$ in (8) does not depend on the choice of $\Delta\phi$ and hence is optimal to problem (6).

Theorem 1. *Given any $\Delta\phi$ in problem* (7)*, problems* (6) *and* (7) *have the same optimal $\Delta\rho^*$ given by* (8) *and the same optimal objective value. However, the optimal $\boldsymbol{v}^*$ in* (8) *depends on the choice of $\Delta\phi$.*

For space reason, we do not give a rigorous proof of Theorem 1 here. Instead, we give an intuitive explanation by using Fig. 2. Without loss of generality, we set $\lambda = 1$ in Fig. 2. Given the original measurements (green points), problem (6) aims at finding an azimuth bias $\Delta\phi$, a range bias $\Delta\rho$, and a velocity vector $\boldsymbol{v}$ to minimize the matching errors (corresponding to the square sum of the length of those black lines in Fig. 2). As shown in Fig. 2, when we rotate green points to blue points by $\Delta\phi$ or to blue circles by $\Delta\phi'$, the relative positions of the obtained points (circles) do no change and neither

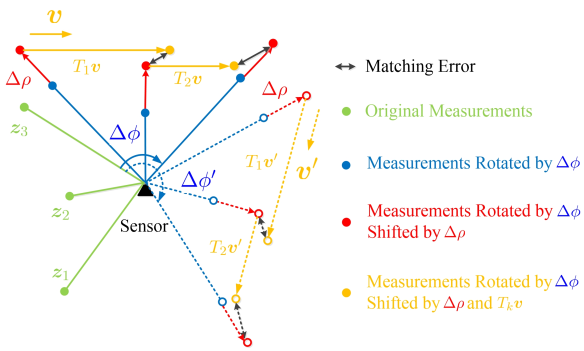

Fig. 2. A geometrical explanation of Theorem 1.

do the optimal $\Delta\rho$ and the optimal value of problem (6). However, the optimal velocity of problem (6) indeed changes, i.e., it changes from $\boldsymbol{v}$ to $\boldsymbol{v}'$ when the rotation changes from $\Delta\phi$ to $\Delta\phi'$.

In summary, Theorem 1 tells us that each sensor can estimate its range bias $\Delta\rho$ independently by solving problem (6) (or problem (7)). However, each sensor can not estimate its azimuth bias $\Delta\phi$ and target's velocity $\boldsymbol{v}$ independently by solving problem (6). Because there is an ambiguity of $\Delta\phi$ and $\boldsymbol{v}$ in problem (6), i.e., problem (6) has multiple optimal pairs $(\Delta\phi, \boldsymbol{v})$. In view of this, we propose to estimate the azimuth biases of all sensors together with the target's velocity by combining the measurements from all sensors.

3.2. Stage II: Joint estimation of azimuth biases

In stage II, we assume that all sensors' range biases have already been estimated in stage I, denoted by $\Delta\rho_m^*, m = 1, \ldots, M$. Now, we estimate all sensors' azimuth biases and target's velocity jointly by combining all sensors' measurements.

We continue with the LS model in stage I to estimate all sensors' azimuth biases $\boldsymbol{\Delta\phi} = [\Delta\phi_1, \ldots \Delta\phi_M]^T$ and target's velocity $\boldsymbol{v}$, while assuming $\Delta\rho_m$ has been fixed $(m = 1, 2, \cdots, M)$:

$$\min_{\boldsymbol{\Delta\phi}, \boldsymbol{v}} \sum_{k=1}^{K-1} \left\| g_{k+1}\left(\Delta\phi_{s_{k+1}}\right) - g_k\left(\Delta\phi_{s_k}\right) - T_k \boldsymbol{v} \right\|^2, \tag{9}$$

where

$$g_k(\Delta\phi_{s_k}) \triangleq \bar{h}\left(\boldsymbol{z}_k + \begin{bmatrix} \Delta\rho_{s_k}^* \\ \Delta\phi_{s_k} \end{bmatrix}\right) + \boldsymbol{p}_{s_k}. \tag{10}$$

The difference between problems (9) and (6) lies in that the measurements from all sensors are used in (9) while the measurements from a single sensor are used in (6).

To solve problem (9) efficiently, we first give its equivalent complex QCQP reformulation as follows:

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{C}^M, v \in \mathbb{C}} \quad & \|\boldsymbol{H}\boldsymbol{x} - \boldsymbol{t}v + \boldsymbol{c}\|^2 \\ \text{s.t.} \quad & |\boldsymbol{x}_m|^2 = 1, \quad \forall\, m, \end{aligned} \tag{11}$$

where $\boldsymbol{x}_m$ denotes the unknown azimuth bias of sensor m in the sense that $\Delta\phi_m = \angle\boldsymbol{x}_m$ ($\angle\boldsymbol{x}_m$ represents the phase of $\boldsymbol{x}_m$), and complex scalar $v = v_x + jv_y$ represents the unknown constant velocity. In (11), matrix $\boldsymbol{H} \in \mathbb{C}^{(K-1)\times M}$ are determined by sensors' measurements $\{\boldsymbol{z}_k\}_{k=1}^K$, observation order $\{s_k\}_{k=1}^K$ and those fixed $\{\Delta\rho_m^*\}_{m=1}^M$; $\boldsymbol{t} \in \mathbb{R}^{K-1}$ relates to time differences $\{T_k\}_{k=1}^{K-1}$; $\boldsymbol{c} \in \mathbb{C}^{K-1}$ is determined by sensors' positions $\{\boldsymbol{p}_m\}_{m=1}^M$. Detailed forms of the above parameters are omitted due to space limitation.

Problem (11) is an unconstrained quadratic program with respect to v. Its closed-form solution is given by

$$v = (\boldsymbol{t}^\dagger \boldsymbol{t})^{-1} \boldsymbol{t}^\dagger (\boldsymbol{H}\boldsymbol{x} + \boldsymbol{c}). \tag{12}$$

Plugging (12) into (11), we get

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{C}^M} \quad & \|\boldsymbol{P}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{P}\boldsymbol{c}\|^2 \\ \text{s.t.} \quad & |\boldsymbol{x}_m|^2 = 1, \quad \forall\, m, \end{aligned} \tag{13}$$

where $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{t}\boldsymbol{t}^\dagger / \|\boldsymbol{t}\|^2$.

Problem (13) is a non-convex QCQP, and such class of problems is known to be NP-hard in general [13]. One efficient convex relaxation technique for solving such class of problems, SDP relaxation, has shown its effectiveness in signal processing and communication communities [14]. We also apply the SDP relaxation technique to solve problem (13). To do so, we reformulate problem (13) in a homogeneous form as follows:

$$\begin{aligned} \min_{\boldsymbol{x} \in \mathbb{C}^{M+1}} \quad & \boldsymbol{x}^\dagger \boldsymbol{C} \boldsymbol{x} \\ \text{s.t.} \quad & |\boldsymbol{x}_m|^2 = 1, \quad \forall\, m, \end{aligned} \tag{14}$$

where

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{H}^\dagger \boldsymbol{P} \boldsymbol{H} & \boldsymbol{H}^\dagger \boldsymbol{P} \boldsymbol{c} \\ \boldsymbol{c}^\dagger \boldsymbol{P} \boldsymbol{H} & 0 \end{bmatrix}.$$

It is simple to show that problems (13) and (14) are equivalent in the sense that $\boldsymbol{x}^* \in \mathbb{C}^{M+1}$ is the optimal solution for problem (14) if and only if $\boldsymbol{x}_{1:M}^* / \boldsymbol{x}_{M+1}^* \in \mathbb{C}^M$ is the optimal solution for problem (13).

The SDP relaxation of problem (14) is

$$\begin{aligned} \min_{\boldsymbol{X} \in \mathbb{H}^{M+1}} \quad & \mathrm{Tr}(\boldsymbol{C}\boldsymbol{X}) \\ \text{s.t.} \quad & \mathrm{Diag}(\boldsymbol{X}) = \boldsymbol{1}, \\ & \boldsymbol{X} \succeq \boldsymbol{0}, \end{aligned} \tag{15}$$

where $\mathbb{H}^{M+1}$ denotes the set of $(M+1) \times (M+1)$ Hermitian matrices, $\mathrm{Tr}(\cdot)$ is the trace operation, and $\mathrm{Diag}(\boldsymbol{X})$ represents the vector formed by all diagonal elements of matrix $\boldsymbol{X}$. Problem (15) can be efficiently solved by the interior-point algorithm [15]. If the optimal solution $\boldsymbol{X}^*$ for problem (15) is of rank one, i.e., $\boldsymbol{X}^* = \boldsymbol{x}^*(\boldsymbol{x}^*)^\dagger$, then the optimal solution for problem (9) is obtained as follows:

$$\Delta\phi_m^* = \angle \frac{\boldsymbol{x}_m^*}{\boldsymbol{x}_{M+1}^*}, \quad m = 1, \cdots, M, \tag{16}$$

and

$$v^* = (\boldsymbol{t}^\dagger \boldsymbol{t})^{-1} \boldsymbol{t}^\dagger \left(\boldsymbol{H} \frac{\boldsymbol{x}_{1:M}^*}{\boldsymbol{x}_{M+1}^*} + \boldsymbol{c}\right). \tag{17}$$

The following result shows that problem (15) has a rank-one solution under mild conditions.

Theorem 2. *If the level of the measurement noise is sufficiently small, problem* (15) *always has a unique solution that is rank one. In other words, the azimuth biases can be estimated by solving a convex SDP.*

The proposed two-stage optimization approach for the asynchronous multi-sensor registration problem is summarized as Algorithm 1.

Based on Theorems 1 and 2, we have the following corollary for Algorithm 1.

Algorithm 1 The Proposed Two-Stage Approach

Input: Measurements $\{z_k\}_{k=1}^{K}$ collected by all sensors

Stage I: Each sensor estimates its own range bias by (8), and obtain $\Delta\rho_m^*, m = 1, 2, \ldots, M$.

Stage II: Solve SDP (15) to obtain $\boldsymbol{X}^* = \boldsymbol{x}^*(\boldsymbol{x}^*)^\dagger$ and extract $\{\Delta\phi_m^*\}_{m=1}^{M}$ and $\boldsymbol{v}^*$ by (16) and (17), respectively.

Output: Estimated biases $\{\boldsymbol{\theta}_m^*\}_{m=1}^{M}$ and velocity $\boldsymbol{v}^*$

Corollary 1. *In the absence of the measurement noise, Algorithm 1 exactly recovers the true* $\{\boldsymbol{\theta}_m\}_{m=1}^{M}$ *and* $\boldsymbol{v}$.

Now we make some remarks on applying the proposed algorithm to solve a special class of the problem without measurement noise. In this case, solving the sensor registration problem is equivalent to solving a set of nonlinear equations (1) and (2). Corollary 1 shows that our proposed Algorithm 1 is able to exactly solve these equations. This exact recovery property makes our proposed algorithm sharply different from the existing ones [1, 2, 3, 4, 5, 7, 8], since they use the first-order approximation to handle nonlinearity in the registration problem while our proposed algorithm circumvents the nonlinearity difficulty by exploiting the special structure of the problem, like the hidden convexity in problems (6) and (9).

4. NUMERICAL SIMULATION

In this section, we present some simulation results to evaluate the effectiveness of the proposed approach (Algorithm 1). Consider a scenario with 3 sensors and a target moving with velocity $\boldsymbol{v} = [180, 0]^T$ m/s starting from position $[-9, 0]^T$ km. Each sensor observes the target every 10 seconds with different initial time. The observation lasts 97 seconds in total and each sensor has 10 measurements. Other details of the simulation setup are listed in Table 1. In our numerical simulation, SDP (15) is solved by CVX [16] and it is observed that the obtained solution is always rank one.

Table 1. Simulation setup.

	Position	Initial Time	$\Delta\rho$	$\Delta\phi$
Sensor 1	$[-5, -5]^T$ km	0 s	1500 m	$-2°$
Sensor 2	$[5, -5]^T$ km	3.5 s	-800 m	$2°$
Sensor 3	$[0, 5]^T$ km	7 s	-1000 m	$3°$

We first give the simulation results in the absence of measurement noise. In this case, sensors' biases are exactly estimated by the proposed approach (with the numerical error being less than 10^{-9}). As shown in Fig. 3, those asynchronous measurements are aligned to the ground truth positions after registration process. This validates the exact recovery property (in Corollary 1) of the proposed approach in the absence of the measurement noise.

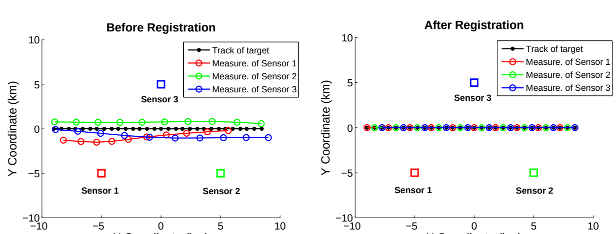

Fig. 3. Asynchronous measurements before and after registration.

In the next, we evaluate the performance of the proposed approach under different levels of the measurement noise. The RMSE is adopted as our measure and we use the well known CRLB [17] as our benchmark. These results are obtained by averaging over 100 Monte Carlo runs. Figs. 4 and 5 plot RMSEs of the proposed approach with different σ_ρ and σ_ϕ. From Figs. 4 and 5, we can observe that RMSEs of our proposed approach are quite close to the CRLBs when the level of the measurement noise is small, which demonstrates the effectiveness of the proposed approach.

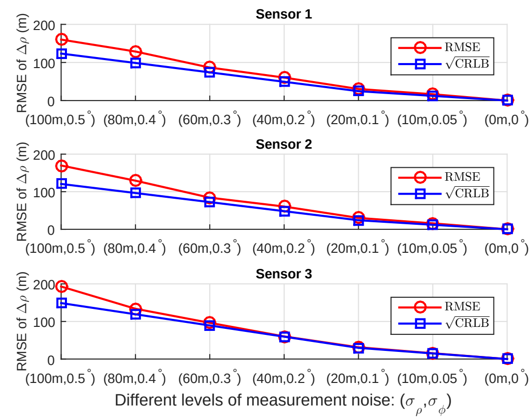

Fig. 4. RMSEs of sensors' range biases with different σ_ρ and σ_ϕ.

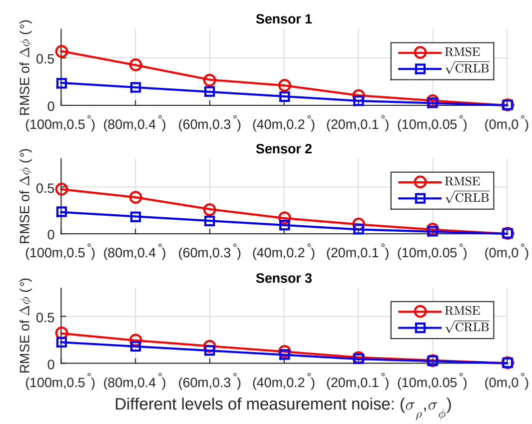

Fig. 5. RMSEs of sensors' azimuth biases with different σ_ρ and σ_ϕ.

5. CONCLUSION

In this paper, we proposed a two-stage (optimization) approach to solving the asynchronous multi-sensor registration problem. We gave two nonlinear LS formulations of the problem (in two stages) by exploiting the a priori knowledge of the target's dynamic model and showed that both of the LS problems can be solved efficiently and globally under mild conditions. Numerical results demonstrate the effectiveness of our proposed approach.

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