EFFICIENT SINGLE/MULTIPLE UNIMODULAR WAVEFORM DESIGN WITH LOW WEIGHTED CORRELATIONS

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ABSTRACT

A new method for designing single/multiple unimodular waveforms with good weighted correlation properties, which is based on minimizing the weighted integrated sidelobe levels of waveforms, is developed. The main contributions of the paper lie in formulating the objective as a quartic form where Hadamard product of matrices is involved, converting the nonconvex quartic optimization problem into a quadratic form and then solving it by means of majorization-minimization technique which seeks to find the solution iteratively. Corresponding algorithm enables good weighted correlations of the designed waveforms and shows fast convergence compared with existing methods.

Index Terms— Majorization-minimization, radar, waveform design, weighted correlations.

1. INTRODUCTION

Waveform design [1]–[4], which is a part of research areas such as radar signal processing [5]–[13], active sensing [14]–[16], communications [17], etc., has been a research field of significant interest for several decades. It plays an important role especially in radar signal processing since excellent waveforms can ensure good localization accuracy [5], high resolution [9], and superior delay-Doppler ambiguity of the potential target [18]. Besides, robust or adaptive waveform design can deal with heterogeneous clutter mitigation and active jammer suppression [3]. One of the most important factors that determine the quality of waveforms is the correlation property, i.e., the auto- and cross-correlations between different time lags of the designed waveforms. This property is of great importance for radar since perfect correlations indicate that the transmitted waveforms are uncorrelated to any of their time-delayed echoes, meaning that the target at the range bin of interest can be easily extracted after matched filtering, and the sidelobes from other range bins are unable to attenuate it. On the other hand, despite the rapid progress in developing modern hardware of amplifiers, waveforms with constant modulus are still preferable compared to other counterparts due to their constant energy at any time lag, which can reduce the cost of hardware.

There has been an extensive literature on waveform design for radar applications. The integrated sidelobe level (ISL), which serves as an expression for characterizing waveform correlation properties and evaluating accumulated sidelobes at all non-zero time lags, is typically used. To design a single waveform via ISL minimization, [11] has proposed to design unimodular waveform in the frequency domain using a cyclic procedure of iterative calculations. A substitute objective function that is minimized by cyclic algorithm has been introduced. The methods associated with ISL and weighted ISL (WISL) minimization therein have been named as CAN and WeCAN, respectively. These methods have been later extended to multiple-input multiple-output radar case [12]. The work of [19] has dealt with the same ISL minimization problem as CAN but has addressed it via majorization-minimization (MaMi) technique [20]–[22]. WISL minimization problem for waveform design has been considered in [23].

In this paper, we aim at designing single or multiple waveforms with good weighted correlation properties. The WISL metric is used as the designing criterion for obtaining the optimal set of unimodular waveforms. We derive the objective of the formulated WISL minimization based problem in a non-convex quartic form, specifically, as the sum of two quartic components where Hadamard product of matrices is highly involved. We convert this quartic optimization problem into a quadratic form, and solve it by means of MaMi technique where majorized objective functions are properly selected. The solution to the WISL minimization based design problem is achieved efficiently in a way of iterative calculations. Corresponding algorithm which enables good weighted correlations of the designed waveforms and shows fast convergence is proposed.

2. PROBLEM FORMULATION

Consider designing a set of M unimodular waveforms, denoted by the $P \times M$ matrix $\mathbf{Y} \triangleq [\mathbf{y}_1, \ldots, \mathbf{y}_M]$, whose mth column $\mathbf{y}_m \triangleq [y_m(1), \ldots, y_m(P)]^{\mathrm{T}}$ is the mth launched waveform of length P. Here, $(\cdot)^{\mathrm{T}}$ stands for the transpose operation, and the elements of $\mathbf{y}_m$ are denoted as $y_m(p) = e^{j\psi_m(p)}$, $p = 1, \ldots, P$ with $\psi_m(p)$ being an arbitrary phase value ranging between $-\pi$ and π. The main problem of waveform design lies in synthesizing $\mathbf{Y}$ which gives good weighted correlation properties.

The WISL of the waveform matrix $\mathbf{Y}$ can be expressed as

$$\zeta=\sum_{m=1}^{M}\sum_{\substack{p=-P+1\\p\neq 0}}^{P-1}\gamma_p^2|r_{mm}(p)|^2+\sum_{m=1}^{M}\sum_{\substack{m'=1\\m'\neq m}}^{M}\sum_{p=-P+1}^{P-1}\gamma_p^2|r_{mm'}(p)|^2 \tag{1}$$

where $r_{mm'}(p)\triangleq\sum_{k=p+1}^{P}y_m(k)y_{m'}^*(k-p)$ stands for the cross-correlation level of the mth and m'th waveforms at the pth time lag, $\{\gamma_p\}_{p=-P+1}^{P-1}$ are real-valued symmetric weights used to control the sidelobe levels corresponding to different time lags, i.e., $\gamma_p=\gamma_{-p},\ p\in\{1,\ldots,P-1\}$, and $|\cdot|$ and $(\cdot)^*$ are modulus and conjugation operators, respectively. Zero-valued element of γ_p means that the sidelobe level associated with the pth time lag is not considered. Therefore, the problem of unimodular waveform design associated with WISL minimization can be expressed as

$$\min_{\mathbf{y}}\ \zeta\quad \text{s.t. } |y_m(p)|=1,\ m=1,\ldots,M;\ p=1,\ldots,P \tag{2}$$

where the constraint ensures the modularity of waveforms.

3. UNIMODULAR WAVEFORM DESIGN

After transforming (1) into frequency domain and performing some derivations, then the WISL ζ can be expressed as [12]

$$\zeta=\frac{1}{2P}\sum_{p=1}^{2P}\left\|\mathbf{Y}^{\mathrm{H}}\big((\mathbf{a}_p\mathbf{a}_p^{\mathrm{H}})\odot\mathbf{\Gamma}\big)\mathbf{Y}-\gamma_0P\mathbf{I}_M\right\|^2 \tag{3}$$

where $\mathbf{a}_p\triangleq\left[1,e^{j\omega_p},\ldots,e^{j(P-1)\omega_p}\right]^{\mathrm{T}},\ p=1,\ldots,2P$ with $\omega_p\triangleq\frac{2\pi}{2P}p$, $\mathbf{\Gamma}$ is a $P\times P$ Toeplitz matrix constructed by the weights $\{\gamma_p\}_{p=0}^{P-1}$, $\odot$ and $(\cdot)^{\mathrm{H}}$ are Hadamard product and Hermitian operators, respectively, and $\mathbf{I}_M$ is an M-dimension identity matrix.

In order to solve (2) efficiently, we start by simplifying (3) and select to rewrite it into proper quadratic form. Expanding the square of norm in (3) yields the expression, i..e,

$$\zeta=\frac{1}{2P}\sum_{p=1}^{2P}\Big(\left\|\mathbf{Y}^{\mathrm{H}}\big((\mathbf{a}_p\mathbf{a}_p^{\mathrm{H}})\odot\mathbf{\Gamma}\big)\mathbf{Y}\right\|^2+\gamma_0^2MP^2 - 2\gamma_0P\mathrm{tr}\big\{\mathbf{Y}^{\mathrm{H}}\big((\mathbf{a}_p\mathbf{a}_p^{\mathrm{H}})\odot\mathbf{\Gamma}\big)\mathbf{Y}\big\}\Big). \tag{4}$$

Note that

$$\sum_{p=1}^{2P}\mathrm{tr}\big\{\mathbf{Y}^{\mathrm{H}}\big((\mathbf{a}_p\mathbf{a}_p^{\mathrm{H}})\odot\mathbf{\Gamma}\big)\mathbf{Y}\big\}=\mathrm{tr}\left\{\mathbf{Y}^{\mathrm{H}}\left(\sum_{p=1}^{2P}(\mathbf{a}_p\mathbf{a}_p^{\mathrm{H}})\odot\mathbf{\Gamma}\right)\mathbf{Y}\right\}$$
$$=2P\mathrm{tr}\big\{\mathbf{Y}^{\mathrm{H}}(\mathbf{I}_P\odot\mathbf{\Gamma})\mathbf{Y}\big\}=2\gamma_0P\|\mathbf{Y}\|^2=2\gamma_0MP^2 \tag{5}$$

where the properties $\sum_{p=1}^{2P}\mathbf{a}_p\mathbf{a}_p^{\mathrm{H}}=2P\mathbf{I}_P$ and $\mathrm{tr}\big\{\mathbf{Y}^{\mathrm{H}}\mathbf{Y}\big\}=\|\mathbf{Y}\|^2$ have been used in the derivation. Therefore, we only need to consider the first component of the sum on the right hand side of (4).

Let $\mathbf{\Gamma}=\sum_{k=1}^{K}\lambda_k\mathbf{q}_k\mathbf{q}_k^{\mathrm{H}}=\sum_{k=1}^{K}\mathbf{u}_k\mathbf{v}_k^{\mathrm{H}}$ be the general eigenvalue decomposition of the weight matrix $\mathbf{\Gamma}$ which can be non-positive semi-definite, where λ_k and $\mathbf{q}_k,\ k\in\{1,\ldots,K\}$ are the kth eigenvalue and eigenvector, respectively, $\mathbf{u}_k\triangleq\sqrt{\lambda_k}\mathbf{q}_k$ is a $P\times 1$ vector, $\mathbf{v}_k$ equals $-\mathbf{u}_k$ when λ_k is negative, otherwise it is the same as $\mathbf{u}_k$, and K is the rank of Γ. After some derivations, the first component of the sum in (4) can be further expressed as

$$\sum_{p=1}^{2P}\left\|\mathbf{Y}^{\mathrm{H}}\big((\mathbf{a}_p\mathbf{a}_p^{\mathrm{H}})\odot\mathbf{\Gamma}\big)\mathbf{Y}\right\|^2=\sum_{p=1}^{2P}\sum_{k=1}^{K}\sum_{k'=1}^{K}\big(\mathbf{y}^{\mathrm{H}}\big((\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}})\odot\mathbf{\Gamma}_{kk'}^{\mathrm{r}}\big)\mathbf{y}\big)^2+\big(\mathbf{y}^{\mathrm{H}}\big((\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}})\odot\mathbf{\Gamma}_{kk'}^{\mathrm{i}}\big)\mathbf{y}\big)^2 \tag{6}$$

where $\mathbf{A}_p\triangleq\mathbf{I}_M\otimes\mathbf{a}_p$, $\mathbf{y}\triangleq\mathrm{vec}(\mathbf{Y})=\left[\mathbf{y}_1^{\mathrm{T}},\ldots,\mathbf{y}_M^{\mathrm{T}}\right]^{\mathrm{T}}$ is the $MP\times 1$ vectorized version of $\mathbf{Y}$, $\mathbf{\Gamma}_{kk'}^{\mathrm{r}}\triangleq\mathbf{I}_M\otimes\big(\mathbf{u}_k\mathbf{v}_{k'}^{\mathrm{H}}+\mathbf{v}_{k'}\mathbf{u}_k^{\mathrm{H}}\big)/2$ and $\mathbf{\Gamma}_{kk'}^{\mathrm{i}}\triangleq\mathbf{I}_M\otimes i\big(\mathbf{u}_k\mathbf{v}_{k'}^{\mathrm{H}}-\mathbf{v}_{k'}\mathbf{u}_k^{\mathrm{H}}\big)/2$.

Ignoring the constant summations for the latter two components of the sum in (4), the waveform design problem (2) can be rewritten as

$$\min_{\mathbf{y}}\ \sum_{p=1}^{2P}\sum_{k=1}^{K}\sum_{k'=1}^{K}\big(\mathbf{y}^{\mathrm{H}}\big((\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}})\odot\mathbf{\Gamma}_{kk'}^{\mathrm{r}}\big)\mathbf{y}\big)^2+\big(\mathbf{y}^{\mathrm{H}}\big((\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}})\odot\mathbf{\Gamma}_{kk'}^{\mathrm{i}}\big)\mathbf{y}\big)^2 \tag{7a}$$
$$\text{s.t.}\quad |y_m(p)|=1,\ m=1,\ldots,M;\ p=1,\ldots,P. \tag{7b}$$

The objective function (7a) takes a quartic form with respect to $\mathbf{y}$, and it can be transformed to the following form

$$\begin{aligned}\textbf{Obj}&=\sum_{p=1}^{2P}\sum_{k=1}^{K}\sum_{k'=1}^{K}\mathrm{tr}^2\big\{\tilde{\mathbf{Y}}^{\mathrm{H}}\big((\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}})\odot\mathbf{\Gamma}_{kk'}^{\mathrm{r}}\big)\big\}+\mathrm{tr}^2\big\{\tilde{\mathbf{Y}}^{\mathrm{H}}\big((\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}})\odot\mathbf{\Gamma}_{kk'}^{\mathrm{i}}\big)\big\}\\&=\mathrm{vec}^{\mathrm{H}}\big(\tilde{\mathbf{Y}}\big)\tilde{\mathbf{\Phi}}\mathrm{vec}\big(\tilde{\mathbf{Y}}\big)\end{aligned} \tag{8}$$

where $\tilde{\mathbf{Y}}\triangleq\mathbf{y}\mathbf{y}^{\mathrm{H}}$ and $\tilde{\mathbf{\Phi}}$ is defined as $\tilde{\mathbf{\Phi}}\triangleq\bar{\mathbf{\Phi}}\odot\bar{\mathbf{\Gamma}}$ with

$$\bar{\mathbf{\Phi}}\triangleq\sum_{p=1}^{2P}\mathrm{vec}\big(\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}}\big)\mathrm{vec}^{\mathrm{H}}\big(\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}}\big) \tag{9}$$

$$\bar{\mathbf{\Gamma}}\triangleq\sum_{k=1}^{K}\sum_{k'=1}^{K}\mathrm{vec}\big(\mathbf{\Gamma}_{kk'}^{\mathrm{r}}\big)\mathrm{vec}^{\mathrm{H}}\big(\mathbf{\Gamma}_{kk'}^{\mathrm{r}}\big)+\mathrm{vec}\big(\mathbf{\Gamma}_{kk'}^{\mathrm{i}}\big)\mathrm{vec}^{\mathrm{H}}\big(\mathbf{\Gamma}_{kk'}^{\mathrm{i}}\big). \tag{10}$$

Then, the problem (7) can be rewritten as

$$\min_{\tilde{\mathbf{Y}}}\quad \mathrm{vec}^{\mathrm{H}}\big(\tilde{\mathbf{Y}}\big)\tilde{\mathbf{\Phi}}\mathrm{vec}\big(\tilde{\mathbf{Y}}\big) \tag{11a}$$
$$\text{s.t.}\quad \tilde{\mathbf{Y}}=\mathbf{y}\mathbf{y}^{\mathrm{H}} \tag{11b}$$
$$|\mathbf{y}(p')|=1,\ p'=1,\ldots,MP. \tag{11c}$$

Before applying majorization to (11a), we present the following result to be used later.

Given a set of N-dimension arbitrary complex vectors $\{\mathbf{d}_k\}_{k=1}^{K}$ and an $N\times N$ arbitrary Hermitian matrix $\mathbf{H}$, the following generalized inequality $\sum_{k=1}^{K}\big(\mathbf{d}_k\mathbf{d}_k^{\mathrm{H}}\big)\odot\mathbf{H}\preceq\lambda_{\max}(\mathbf{H})\mathbf{D}$ holds, where $\mathbf{D}\triangleq\mathrm{diag}\big\{\sum_{k=1}^{K}|\mathbf{d}_k(1)|^2,\ldots,$

$\sum_{k=1}^{K}|\mathbf{d}_k(N)|^2\}$ and $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue of a matrix. The result that $\mathbf{x}^{\mathrm{H}}\mathbf{Q}\mathbf{x}$ is majorized by the function $\mathbf{x}^{\mathrm{H}}\mathbf{G}\mathbf{x} + 2\Re(\mathbf{x}^{\mathrm{H}}(\mathbf{Q}-\mathbf{G})\mathbf{x}_0) + \mathbf{x}_0^{\mathrm{H}}(\mathbf{G}-\mathbf{Q})\mathbf{x}_0$ at $\mathbf{x}_0$ when $\mathbf{G} \succeq \mathbf{Q}$ is also used in the following. They can be found in the associate full-size journal paper [24].

It can be shown that $\lambda_{\max}(\bar{\mathbf{\Gamma}})$ is a constant value, and the value of diagonal elements of the first component within the Hadamard product in $\bar{\mathbf{\Phi}}$ is either zero or $2P$. Therefore, we can select $\mathbf{G} \triangleq \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{I}_{M^2P^2}$ (here$\lambda_{\tilde{\mathbf{\Phi}}} \triangleq 2P\lambda_{\max}(\bar{\mathbf{\Gamma}})$) to satisfy the generalized inequality $\mathbf{G} \succeq \tilde{\mathbf{\Phi}}$. Using the above-mentioned majorization results, the objective function (11a) can be majorized as

$$\begin{aligned}\tilde{g}_1(\tilde{\mathbf{Y}},\tilde{\mathbf{Y}}^{(k)}) = {} & \lambda_{\tilde{\mathbf{\Phi}}}\mathrm{vec}^{\mathrm{H}}(\tilde{\mathbf{Y}})\mathrm{vec}(\tilde{\mathbf{Y}}) \\ & + 2\Re\Big\{\mathrm{vec}^{\mathrm{H}}(\tilde{\mathbf{Y}})(\tilde{\mathbf{\Phi}} - \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{I}_{M^2P^2})\mathrm{vec}(\tilde{\mathbf{Y}}^{(k)})\Big\} \\ & + \mathrm{vec}^{\mathrm{H}}(\tilde{\mathbf{Y}}^{(k)})(\lambda_{\tilde{\mathbf{\Phi}}}\mathbf{I}_{M^2P^2} - \tilde{\mathbf{\Phi}})\mathrm{vec}(\tilde{\mathbf{Y}}^{(k)}) \qquad (12)\end{aligned}$$

where $\tilde{\mathbf{Y}}^{(k)} \triangleq \mathbf{y}^{(k)}(\mathbf{y}^{(k)})^{\mathrm{H}}$. Note that $\mathrm{vec}^{\mathrm{H}}(\tilde{\mathbf{Y}})\mathrm{vec}(\tilde{\mathbf{Y}}) = \|\mathbf{y}\|^4 = M^2P^2$. Hence, both summations for the first and third components in (12) are immaterial for optimization. The problem (11) can therefore be rewritten as

$$\begin{aligned}\min_{\tilde{\mathbf{Y}}} \quad & \mathrm{vec}^{\mathrm{H}}(\tilde{\mathbf{Y}})\Big(\tilde{\mathbf{\Phi}} - \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{I}_{M^2P^2}\Big)\mathrm{vec}(\tilde{\mathbf{Y}}^{(k)}) \\ \text{s.t.} \quad & \tilde{\mathbf{Y}} = \mathbf{y}\mathbf{y}^{\mathrm{H}} \\ & |\mathbf{y}(p')| = 1,\ p' = 1,\ldots,MP. \qquad (13)\end{aligned}$$

Using the explicit expression of $\tilde{\mathbf{\Phi}}$, the properties $\mathrm{vec}(\tilde{\mathbf{Y}}) = (\mathbf{y}^{\mathrm{T}} \otimes \mathbf{I}_{MP})^{\mathrm{H}}\mathbf{y}$ and $\mathrm{vec}(\mathbf{A}_p\mathbf{A}_p^{\mathrm{H}}) = (\mathbf{A}_p^{\mathrm{T}} \otimes \mathbf{I}_{MP})^{\mathrm{H}}\mathrm{vec}(\mathbf{A}_p)$, and performing some manipulations between Hadamard and Kronecker products, the objective of (13) can be derived as

$$\begin{aligned}&\mathrm{vec}^{\mathrm{H}}(\tilde{\mathbf{Y}})\Big(\tilde{\mathbf{\Phi}} - \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{I}_{M^2P^2}\Big)\mathrm{vec}(\tilde{\mathbf{Y}}^{(k)}) \\ &= \mathbf{y}^{\mathrm{H}}\left(\sum_{p=1}^{2P}(\tilde{\mathbf{a}}_p\tilde{\mathbf{a}}_p^{\mathrm{H}}) \odot \mathbf{\Delta}_p^{(k)} - \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{y}^{(k)}(\mathbf{y}^{(k)})^{\mathrm{H}}\right)\mathbf{y} \qquad (14)\end{aligned}$$

where $\tilde{\mathbf{a}}_p \triangleq \mathbf{1}_M \otimes \mathbf{a}_p$ and $\mathbf{\Delta}_p^{(k)}$ is an $MP \times MP$ matrix whose (i,j)th element is expressed as

$$\big[\mathbf{\Delta}_p^{(k)}\big]_{i,j} = \bar{\gamma}_{i,j}^{\mathrm{T}}\Big((\mathbf{y}^{(k)} \odot \tilde{\mathbf{a}}_p^*) \otimes ((\mathbf{y}^{(k)})^* \odot \tilde{\mathbf{a}}_p)\Big) \qquad (15)$$

with the $M^2P^2 \times 1$ vector $\bar{\gamma}_{i,j}$ defined as

$$\begin{aligned}\bar{\gamma}_{i,j} \triangleq \Big[&[\bar{\mathbf{\Gamma}}]_{i,j}, [\bar{\mathbf{\Gamma}}]_{i,j+MP}, \ldots, [\bar{\mathbf{\Gamma}}]_{i,j+(MP-1)MP}, \\ &\ldots, [\bar{\mathbf{\Gamma}}]_{i+(MP-1)MP,j+(MP-1)MP}\Big]^{\mathrm{T}}. \qquad (16)\end{aligned}$$

Therefore, the optimization problem (14) can be rewritten as

$$\begin{aligned}\min_{\mathbf{y}} \quad & \mathbf{y}^{\mathrm{H}}\left(\sum_{p=1}^{2P}(\tilde{\mathbf{a}}_p\tilde{\mathbf{a}}_p^{\mathrm{H}}) \odot \mathbf{\Delta}_p^{(k)} - \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{y}^{(k)}(\mathbf{y}^{(k)})^{\mathrm{H}}\right)\mathbf{y} \\ \text{s.t.} \quad & |\mathbf{y}(p')| = 1, \quad p' = 1,\ldots,MP. \qquad (17)\end{aligned}$$

Applying the majorization result (see the last paragraph on the previous page) to the component $((\tilde{\mathbf{a}}_p\tilde{\mathbf{a}}_p)^{\mathrm{H}}) \odot \mathbf{\Delta}_p^{(k)}$ in (17), we obtain that $((\tilde{\mathbf{a}}_p\tilde{\mathbf{a}}_p)^{\mathrm{H}}) \odot \mathbf{\Delta}_p^{(k)} \preceq \lambda_{\max}(\mathbf{\Delta}_p^{(k)})\mathbf{I}_{MP}$. Thus by selecting $\mathbf{G} \triangleq \sum_{p=1}^{2P}\lambda_{\max}(\mathbf{\Delta}_p^{(k)})\mathbf{I}_{MP}$, the objective of (17) can be majorized as

$$\begin{aligned}&\tilde{g}_2(\mathbf{y},\mathbf{y}^{(k)}) \\ &= \sum_{p=1}^{2P}\lambda_{\max}(\mathbf{\Delta}_p^{(k)})\mathbf{y}^{\mathrm{H}}\mathbf{y} + 2\Re\Bigg\{\mathbf{y}^{\mathrm{H}}\Bigg(\sum_{p=1}^{2P}(\tilde{\mathbf{a}}_p\tilde{\mathbf{a}}_p^{\mathrm{H}}) \odot \mathbf{\Delta}_p^{(k)} \\ &\quad - \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{y}^{(k)}(\mathbf{y}^{(k)})^{\mathrm{H}} - \sum_{p=1}^{2P}\lambda_{\max}(\mathbf{\Delta}_p^{(k)})\mathbf{I}_{MP}\Bigg)\mathbf{y}^{(k)}\Bigg\} \\ &\quad + (\mathbf{y}^{(k)})^{\mathrm{H}}\Bigg(\sum_{p=1}^{2P}\lambda_{\max}(\mathbf{\Delta}_p^{(k)})\mathbf{I}_{MP} + \lambda_{\tilde{\mathbf{\Phi}}}\mathbf{y}^{(k)}(\mathbf{y}^{(k)})^{\mathrm{H}} \\ &\quad - \sum_{p=1}^{2P}(\tilde{\mathbf{a}}_p\tilde{\mathbf{a}}_p^{\mathrm{H}}) \odot \mathbf{\Delta}_p^{(k)}\Bigg)\mathbf{y}^{(k)} \qquad (18)\end{aligned}$$

where the summations for the first and third components of the sum in (18) do not need to be considered for optimization since they are constant. Therefore, (17) can be finally simplified into the following optimization problem

$$\begin{aligned}\min_{\mathbf{y}} \quad & \mathbf{y}^{\mathrm{H}}\mathbf{z}^{(k)} \\ \text{s.t.} \quad & |\mathbf{y}(p')| = 1, \quad p' = 1,\ldots,MP \qquad (19)\end{aligned}$$

where the $MP \times 1$ vector $\mathbf{z}^{(k)}$ is defined as

$$\begin{aligned}\mathbf{z}^{(k)} \triangleq {} & \left(\sum_{p=1}^{2P}\lambda_{\max}(\mathbf{\Delta}_p^{(k)}) + MP\lambda_{\tilde{\mathbf{\Phi}}}\right)\mathbf{y}^{(k)} \\ & - \sum_{p=1}^{2P}\Big((\tilde{\mathbf{a}}_p\tilde{\mathbf{a}}_p^{\mathrm{H}}) \odot \mathbf{\Delta}_p^{(k)}\Big)\mathbf{y}^{(k)}. \qquad (20)\end{aligned}$$

Due to the constant modulus property of $\mathbf{y}$, (19) is equivalent to the following optimization problem

$$\begin{aligned}\min_{\mathbf{y}} \quad & \|\mathbf{y} - \mathbf{z}^{(k)}\| \\ \text{s.t.} \quad & |\mathbf{y}(p')| = 1, \quad p' = 1,\ldots,MP \qquad (21)\end{aligned}$$

which leads to the following closed-form solution

$$\mathbf{y}(p') = e^{j\arg(\mathbf{z}^{(k)}(p'))}, \quad p' = 1,\ldots,MP. \qquad (22)$$

Stacking $\mathbf{y}$ into a $P \times M$ matrix, we obtain the final waveform matrix $\mathbf{Y}$. Based on the above derivations, we propose an original algorithm concluded in Algorithm 1 for solving (2). Note that the matrix $\mathbf{\Delta}_p^{(k)}$ can be efficiently obtained in each iteration, for example, if needed, parallel computation can be used. We refer interested reader to the literature for accelerated schemes, for example, the SQUAREM scheme [25] used in [19], which can speed up the proposed Algorithm 1 as well.

4. SIMULATION RESULTS

We compare the performance of our proposed waveform design algorithm with that of the WeCAN algorithm (see [12]) and the method in [23] (named as WISLSong) accelerated by

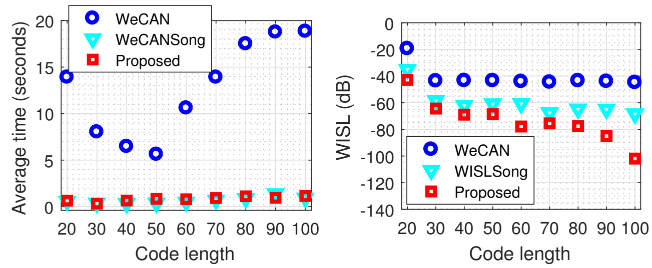
Algorithm 1 WISL minimization via MaMi

$k \leftarrow 0$, $\mathbf{y} \leftarrow$ unimodular sequence with random phases.
repeat
  procedure WISLMAMI$(\mathbf{y}^{(k)})$
    Calculate $\{\mathbf{\Delta}_p^{(k)}\}_{p=1}^{2P}$ using (15).
    Calculate $\mathbf{z}^{(k)}$ using (20).
    $\mathbf{y}^{(k+1)}(p) = e^{j\arg\left(\mathbf{z}^{(k)}(p')\right)},\ p' = 1, \ldots, MP.$
    $k \leftarrow k + 1$
  end procedure
until convergence

the SQUAREM scheme. We generate unimodular sequences with random phases as the initialization for each design, and the same initialized sequence is used for comparison. The SQUAREM scheme is also used to accelerate the proposed Algorithm 1. We select the absolute WISL difference between the current and previous iterations normalized to the initial WISL as the stopping criterion, and the tolerance is set to be 10^{-8}. All simulations for the tested methods are conducted using the same hardware and software platforms.

In the first example, we evaluate the convergence properties (running time versus code length) and the correlation properties (WISL versus time lags) of waveforms generated by the three designs tested for a single-waveform scenario, i.e., $M = 1$. The code length P is set from 20 to 100 with stepsize 10, and the controlling ISL weights are $\gamma_0 = 1$, $\gamma_p = 0.1991$, $p \in \{-9, \ldots, -1\} \cup \{1, \ldots, 9\}$, while the others are zeros ($\mathbf{\Gamma}$ is positive semi-definite as required by WeCAN). The results are averaged over 50 trials. It can be seen that our proposed design significantly outperforms the WeCAN design in convergence speed with respect to the consumed time and shows as better convergence property (within 1.3 seconds for all tested code length) as the method in [23] with SQUAREM acceleration (see Fig. 1(a)). Indeed, the WeCAN algorithm converges more slowly when the set of weights is not significantly sparse, while our algorithm is not sensitive to sparsity. Moreover, our proposed algorithm always achieves much lower WISL when it reaches the stopping tolerance (see Fig. 1(b)). This is mainly because our proposed design deals with the true objective of the corresponding WISL minimization problem, while the WeCAN design deals with a surrogate of it. For a fixed code length, the largest gaps between the achieved WISL by our proposed algorithm and the other two designs have reached about 40 dB and 55 dB, respectively.

In the second example, we present the correlations of $M = 2$ waveforms obtained by the three tested methods with code length of 32 for a multiple-waveform scenario. The controlling ISL weights are $\gamma_0 = 1$, $\gamma_p = 0.311$, $p \in \{-5, \ldots, -1\} \cup \{1, \ldots, 5\}$, while the others are zeros. The four sub figures in Fig. 2 stand for the auto- and cross-correlations of the two sets of waveforms generated by the methods tested. It can be seen that the auto-correlations associated with time lags $[-5, -1] \cup$



(a) Iterations versus code length. (b) WISL versus code length.

Fig. 1. Evaluations of consumed time and achieved WISL.

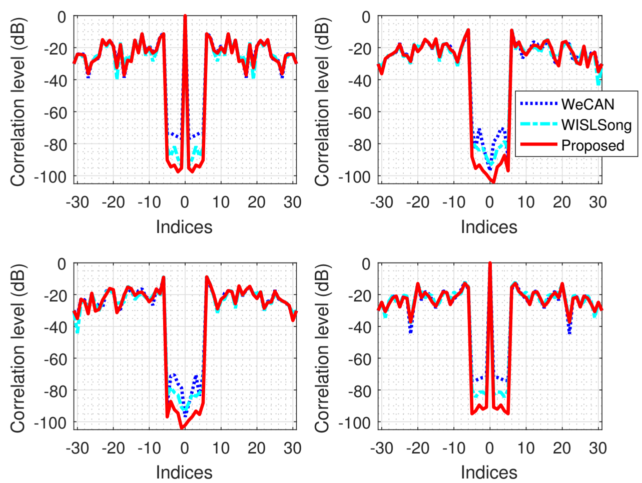


Fig. 2. Evaluation of correlation properties.

$[1, 5]$ and cross-correlations associated with time lags $[-5, 5]$ for the three generated sets of waveforms are controlled, while waveform correlations associated with other time lags are not controlled, and therefore, show much higher correlation levels. Under the condition of the same convergence tolerance, the correlation levels corresponding to the time lags of interest by the proposed waveform design are better than those by the other two methods. The largest gaps between the obtained correlations by the proposed method and the other two have reached about 15 dB and 20 dB, respectively, and the WeCAN method shows the worst weighted correlations.

5. CONCLUSION

We have developed an efficient algorithm for designing single or multiple unimodular waveforms with good weighted correlations. WISL metric has been employed as the criterion for designing waveforms, and the waveform design has been formulated as a non-convex quartic problem where Hadamard product of matrices is involved. This quartic optimization problem has been converted into a quadratic form and then solved by means of MaMi technique where majorized objective functions are properly selected. The proposed algorithm has shown better weighted correlations of designed waveforms and faster convergence as compared to its counterparts.

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