MATRIX COMPLETION BASED MIMO RADARS WITH CLUTTER AND INTERFERENCE MITIGATION VIA TRANSMIT PRECODING

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ABSTRACT

In this paper, we propose a new matrix completion based MIMO radar (MIMO-MC) using a random unitary matrix as the waveform matrix. We show that the corresponding data matrix has a good incoherent property, which guarantees accurate reconstruction of the data matrix from partial entries. The derived performance guarantees hold for any random unitary waveforms and radar precoder. This indicates that the proposed MIMO-MC approach can dynamically adapt its waveform/precoding for the purpose of waveform security or interference suppression in low SINR conditions, without affecting the performance of data matrix completion. We further investigate the application of MIMO-MC transmit precoding for clutter mitigation and spectrum sharing with wireless communications.

Index Terms— Collocated MIMO radar, matrix completion, transmit precoding, spectrum sharing, clutter

1. INTRODUCTION

MIMO radars transmit different waveforms from their transmit (TX) antennas, and their receive (RX) antennas forward their measurements to a fusion center for further processing. Based on the forwarded data, the fusion center populates a matrix, referred to as the "data matrix", which is then used by standard array processing schemes for target estimation. For a relatively small number of targets, the data matrix is low-rank [1–3], thus allowing one to fully reconstruct it (under certain conditions) based on a small, uniformly sampled set of its entries. This observation is the basis of MIMO-MC radars; the RX antennas forward to the fusion center a small number of pseudo-randomly sub-Nyquist sampled values of the target returns, along with their sampling scheme, each RX antenna partially filling a row of the data matrix. Note that the signal power will be retained after matrix completion. Therefore, there is no SNR loss due to subsampling. The subsampling at the antennas avoids the need for high rate analog-to-digital converters, and the reduced amount of samples translates into power and bandwidth savings in the antenna-fusion center link. Compared to the compressive sensing (CS) based MIMO radars, MIMO-MC radars achieve data reduction while avoiding the basis mismatch issues inherent in CS-based approaches [4].

However, it was shown in [3] that the matrix completion performance degrades severely when the SINR drops to 10dB. Usually, the echoes from interfering clutter, e.g., ground, sea and rain, are much stronger than those returned from targets of interest. With the emergence of radar-communication spectrum sharing [5–8], large interference from wireless communication systems is also unavoidable. In addition, the radar waveforms are required to be orthogonal and have flat spatial power spectrum in order to have good incoherence properties [9]. The search for such waveforms in [9] involves high computational complexity. On the other hand, radar waveforms need to be updated frequently as security against adversaries. It is of significant interest to propose a MIMO-MC radar framework which can operate under low SINR conditions, and supports waveform agility.

In this paper, we propose a MIMO-MC radar framework using a random unitary matrix [10] as the waveform matrix. We show that the data matrix has a good incoherence property, which guarantees accurate reconstruction of the data matrix from its partial entries. The derived performance guarantees hold for any waveform matrix that is random unitary and any radar precoder. This indicates that the proposed MIMO-MC system can periodically change its waveforms, which would be good for security reasons, without affecting the matrix completion performance. Also, we can design the radar precoder, without affecting the incoherence property of the data matrix, for the purpose of transmit beamforming or interference suppression. We further investigate the application of MIMO-MC transmit precoding for clutter mitigation and spectrum sharing with wireless communications.

The paper is organized as follows. Section 2 introduces the signal model of MIMO-MC radars. The proposed MIMO-MC radar framework is given in Section 3 and its application in clutter mitigation and spectrum sharing with wireless communication systems is given in Section 4. Numerical results and conclusions are provided respectively in Sections 5 and 6. *Notation:* $\mathcal{CN}(\mu, \Sigma)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and covariance matrix Σ . $|\cdot|$ and $\text{Tr}(\cdot)$ denote the matrix determinant and trace respectively. The set \mathbb{N}_{L}^{+} is defined as $\{1, \ldots, L\}$. δ_{ij} denotes the Kronecker delta. $\lfloor x \rfloor$ denotes the largest integer not larger than x. \mathbf{A}^{T} and \mathbf{A}^{H} respectively denote the transpose and Hermitian transpose of \mathbf{A} .

2. THE MIMO-MC RADARS

Consider a collocated MIMO radar system with $M_{t,R}$ TX antennas and $M_{r,R}$ RX antennas arranged as uniform linear arrays (ULA) with inter-element spacing d_t and d_r , respectively. The radar is pulse based with pulse repetition interval T_{PRI} and carrier wavelength λ_c . The K far-field targets are with distinct angles $\{\theta_k\}$, target reflection coefficients $\{\beta_k\}$ and Doppler shifts $\{\nu_k\}$ and are assumed to fall in the same range bin. Following the clutter-free model of [2, 3, 9], the data matrix at the fusion center can be formulated as

$$\mathbf{Y}_R = \mathbf{V}_r \mathbf{\Sigma} \mathbf{V}_t^T \mathbf{P} \mathbf{S} + \mathbf{W}_R, \tag{1}$$

where the *m*-th row of $\mathbf{Y}_R \in \mathbb{C}^{M_{r,R} \times L}$ contains the *L* samples forwarded by the *m*-th antenna; the waveforms are given in $\mathbf{S} = [\mathbf{s}(1), \cdots, \mathbf{s}(L)]$, with $\mathbf{s}(l) = [s_1(l), \cdots, s_{M_{t,R}}(l)]^T$ being the *l*-th snapshot across the transmit antennas; the transmit waveform-

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s are assumed to be orthogonal, *i.e.*, it holds that $\mathbf{SS}^{H} = \mathbf{I}_{M_{t,R}}$ [3]; \mathbf{W}_{R} denotes additive noise; and $\mathbf{P} \in \mathbb{C}^{M_{t,R} \times M_{t,R}}$ denotes the transmit precoding matrix. $\mathbf{V}_{t} \triangleq [\mathbf{v}_{t}(\theta_{1}), \dots, \mathbf{v}_{t}(\theta_{K})]$ and $\mathbf{V}_{r} \triangleq [\mathbf{v}_{r}(\theta_{1}), \dots, \mathbf{v}_{r}(\theta_{K})]$ respectively denote the transmit and receive steering matrix and $\mathbf{v}_{r}(\theta) \in \mathbb{C}^{M_{r,R}}$ is the receive steering vector defined as

$$\mathbf{v}_{r}(\theta) \triangleq \left[e^{-j2\pi 0\vartheta^{r}}, \dots, e^{-j2\pi (M_{r,R}-1)\vartheta^{r}}\right]^{T}, \qquad (2)$$

where $\vartheta^r = d_r \sin(\theta)/\lambda_c$ denotes the spatial frequency w.r.t. the receive array. $\mathbf{v}_t(\theta) \in \mathbb{C}^{M_{t,R}}$ is the transmit steering vector and is respectively defined. Matrix Σ is defined as $\Sigma \triangleq$ diag $([\beta_1 e^{j2\pi\nu_1}, \ldots, \beta_K e^{j2\pi\nu_K}])$. Note that Doppler shifts might be neglected in the model since we are considering a fast-time processing. $\mathbf{D} \triangleq \mathbf{V}_r \Sigma \mathbf{V}_t^T$ is also called the target response matrix. After matched filtering at the fusion center, target estimation can be performed based on \mathbf{Y}_R via standard array processing schemes [11].

When K is smaller than $M_{r,R}$ and L, the noise-free data matrix $\mathbf{M} \triangleq \mathbf{DPS}$ is low-rank and can be provably recovered based on a subset of its entries. This observation gave rise to MIMO-MC radars [2,3,9], where each RX antenna sub-samples the target returns and forwards the samples to the fusion center. The partially filled data matrix at the fusion center can be mathematically expressed as follows (see [3] Scheme I)

$$\mathbf{\Omega} \circ \mathbf{Y}_R = \mathbf{\Omega} \circ (\mathbf{M} + \mathbf{W}_R), \tag{3}$$

where \circ denotes the Hadamard product; Ω is the sub-sampling matrix containing 0's and 1's. The sub-sampling rate p equals $\|\Omega\|_0/(LM_{r,R})$. When p = 1, the Ω matrix is filled with 1's, and the MIMO-MC radar is identical to the traditional MIMO radar. At the fusion center, the completion of M can be achieved by the following nuclear norm minimization problem [12]

$$\min_{\mathbf{M}} \|\mathbf{M}\|_{*} \quad \text{s.t.} \|\mathbf{\Omega} \circ \mathbf{M} - \mathbf{\Omega} \circ \mathbf{Y}_{R}\|_{F} \le \delta, \tag{4}$$

where $\delta > 0$ is a parameter determined by the sampled entries of the noise matrix, *i.e.*, $\Omega \circ W_R$. The recovery error of M is bounded with high probability, given that M is incoherent with parameters (μ_0, μ_1) , and Ω corresponds to uniformly at random sub-sampling operation [12]. It is important to note that the stable reconstruction of the data matrix M with high accuracy implies that almost all the received target echo power is retained.

The incoherence parameters (μ_0, μ_1) are given by $\mu_0 \geq \max(\mu(U), \mu(V)), \ \mu_1 \sqrt{\frac{K}{M_{r,R}L}} \geq \|\sum_{k=1}^{K} \mathbf{U}_{\cdot k} \mathbf{V}_{\cdot k}^{H}\|_{\infty}$, where $\mathbf{U} \in \mathbb{C}^{M_{r,R} \times K}$ and $\mathbf{V} \in \mathbb{C}^{L \times K}$ contain the left and right singular vectors of \mathbf{M} ; the coherence of subspace V spanned by basis matrix \mathbf{V} is defined as

$$\iota(V) \triangleq \frac{L}{K} \max_{1 \le l \le L} \left\| \mathbf{V}_{l \cdot} \right\|^2 \in \left[1, \frac{L}{K} \right].$$

The upper bounds on the incoherence parameters of \mathbf{M} are given in the following theorem [2,9].

Theorem 1. ([9, Theorem 2] Coherence of **M** when $\mathbf{P} = \mathbf{I}_{M_{t,R}}$) Let the minimum spatial frequency separation of the K targets be ξ_t and ξ_r w.r.t. the transmit and receive arrays. On denoting the Fejér kernel by $F_n(x)$, and for $d_t = d_r = \lambda_c/2$ and

$$K \le \min\left\{\sqrt{M_{r,R}/F_{M_{r,R}}(\xi_r)}, \sqrt{M_{t,R}/F_{M_{t,R}}(\xi_t)}\right\},$$

it holds that

$$\mu(U) \le \frac{\sqrt{M_{r,R}}}{\sqrt{M_{r,R}} - (K-1)\sqrt{F_{M_{r,R}}(\xi_r)}} \triangleq \mu_0^r.$$

Further, if every snapshot of the waveforms satisfies that $|\mathbf{S}_{l}^{T}\mathbf{v}_{t}(\theta)|^{2} = \frac{M_{t,R}}{2}, \ \forall l \in \mathbb{N}_{L}^{+}, \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],$

$$\mathbf{S}_{l}^{I}\mathbf{v}_{t}(\theta)|^{2} = \frac{m_{t,n}}{L}, \ \forall l \in \mathbb{N}_{L}^{+}, \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \tag{5}$$

then $\mu(\mathbf{V})$ is upper bounded by

$$\iota(V) \le \frac{\sqrt{M_{t,R}}}{\sqrt{M_{t,R}} - (K-1)\sqrt{F_{M_{t,R}}(\xi_t)}} \triangleq \mu_0^t$$

Consequently, the matrix **M** is incoherent with parameters $\mu_0 \triangleq \max\{\mu_0^r, \mu_0^t\}$ and $\mu_1 \triangleq \sqrt{K}\mu_0$.

In the following we discuss two points that motivate the contribution of this paper.

- In [9], the condition in (5) and the orthogonality property was used to design waveforms with good incoherence properties. However, radar waveforms need to be updated frequently for security against adversaries, which subsequently brings us the issue of computational complexity. The work of [9] involves numerical optimization on the complex Stiefel manifold [9], which has high computational complexity.
- 2. In radar system design, the adaptability of transmit waveforms and/or precoder is critical for the suppression of interference, including noise, clutter and jamming. In particular for MIMO-MC radars, the matrix completion performance will degrade severely when the SINR drops to as low as 10dB [3], which in turn emphasizes the importance of waveform and/or precoder design for MIMO-MC radar noise and interference mitigation. However, the results in Theorem 1 cannot be easily extended for a nontrivial transmit precoding matrix.

To address the above two issues, we propose to use a random unitary matrix [10] as the waveform matrix \mathbf{S} . This choice is motivated by the simulations in [9], which show that the random unitary matrix performs almost the same as the optimally designed waveform.

3. THE PROPOSED MIMO-MC RADARS USING RANDOM UNITARY MATRIX

A random unitary matrix [10] can be obtained through performing the Gram-Schmidt orthogonalization on a random matrix with entries distributed as i.i.d Gaussian. This means that we can generate waveform candidates easily. The following theorem provides an upper bound on the incoherence parameter $\mu(U)$ and $\mu(V)$ of **M** when the random unitary waveform is used.

Theorem 2. (Bounding $\mu(U)$ and $\mu(V)$) Consider the MIMO-MC radar presented in Section 2 with **S** being random unitary. For any transmit precoder **P** such that the rank of **M** is $K_0 \leq K$, and arbitrary transmit array geometry and target angles, the coherence of subspace V obeys the following:

$$\mu(V) \le \frac{K_0 + 2\sqrt{3K_0 \ln L} + 6\ln L}{K_0} \triangleq \tilde{\mu}_0^t$$

with probability $1 - L^{-2}$, and the coherence of subspace U obeys $\mu(U) \leq \frac{K}{K_0} \mu_0^r$, where μ_0^r is defined in Theorems 1.

Proof. The proof can be found in our extended journal version [13]. \Box

Based on Theorem 2, we have the following theorem for the incoherence parameters of \mathbf{M} .

Theorem 3. (Coherence of **M** with random unitary waveform matrix) Consider the MIMO-MC radar presented in Section 2 with **S** being random unitary. For $d_r = \lambda_c/2$, arbitrary transmit array geometry, and $K \leq \sqrt{M_{r,R}/F_{M_{r,R}}(\xi_r)}$, the matrix **M** is incoherent with parameters $\mu_0 \triangleq \max\{\frac{K}{K_0}\mu_0^r, \tilde{\mu}_0^t\}$ and $\mu_1 \triangleq \sqrt{K}\mu_0$ with

probability $1 - L^{-2}$, where μ_0^r and $\tilde{\mu}_0^t$ are defined in Theorems 1 and 2, respectively. The incoherence property of **M** holds for any precoding matrix **P** such that the rank of **M** is K_0 .

Proof. The theorem can be proven by combining the bounds on $\mu(U)$ and $\mu(V)$ in Theorems 1 and 2, respectively.

Remark 1. Some comments are in order. First, if K_0 is $\mathcal{O}(\ln L)$, the upper bound $\tilde{\mu}_0^t > 1$ is a small constant. Therefore, M has a good incoherence property. A similar bound was provided on the coherence of the subspaces spanned by random orthogonal basis in [14]. Second, unlike the results in Theorem 1, the probabilistic bound on $\mu(V)$ is independent of the target angles and array geometry. Third, the above results hold for any random unitary matrix S. The radar waveform can be changed periodically, which would be good for security reason, without affecting the matrix completion performance. Finally, the probabilistic bound on $\mu(V)$ in Theorem 2 is independent of P. This means that we can design P, without affecting the incoherence property of M, for the purpose of transmit beamforming and interference suppression. This key observation validates the feasibility of radar precoding for clutter mitigation and spectrum sharing approaches for MIMO-MC radar and communication systems in the sequel.

4. APPLICATIONS FOR CLUTTER MITIGATION AND SPECTRUM SHARING

We consider the coexistence scenario in [6], where a MIMO-MC radar system and a MIMO communication system operate using the same carrier frequency. Consider the same target scene in a particular range bin as in Section 2 but with clutter. The signal received by the radar RX antennas during L symbol durations can be respectively expressed as

$$\mathbf{\Omega} \circ \mathbf{Y}_{R} = \mathbf{\Omega} \circ \left(\underbrace{\mathbf{DPS}}_{\text{signal}} + \underbrace{\mathbf{CPS} + \mathbf{X}_{\text{int}}}_{\text{interference}} + \underbrace{\mathbf{W}_{R}}_{\text{noise}} \right), \tag{6}$$

where \mathbf{Y}_R , \mathbf{D} , \mathbf{P} , \mathbf{S} , \mathbf{W}_R , and $\boldsymbol{\Omega}$ are defined in Section 2. The waveform-dependent interference CPS contains interferences from point scatterers (clutter or interfering objects). Suppose that there are K_c point clutters with angles $\{\theta_k^c\}$, reflection coefficients $\{\beta_k^c\}$ in the same range bin as the targets. $\mathbf{C} \triangleq \sum_{k=1}^{K_c} \beta_k^c \mathbf{v}_r(\theta_k^c) \mathbf{v}_t^T(\theta_k^c)$ is the clutter response matrix. \mathbf{X}_{int} denotes the interference from the wireless communication system which coexists with the MIMO-MC radar. It was shown in [6-8] that the cooperation on the joint design transmit schemes significantly boosts the spectrum efficiency for the co-existence of traditional MIMO radars and wireless communications. In this paper, we also assume that the radar and communication systems cooperate with each other to estimate and share the interference channels. Specifically, we assume that the covariance matrices of the columns in \mathbf{X}_{int} are identical and are known to the MIMO-MC radar, saying \mathbf{R}_x . In addition, $\mathbf{G} \in \mathbb{C}^{M_{r,C} imes M_{t,R}}$ denotes the interference channel from between the radar transmitter and the communication receiver. We also assume that \mathbf{G} \in $\mathbb{C}^{M_{r,C} \times M_{t,R}}$ is known to the radar system and is flat fading and remain the same over L symbol intervals [15–18]. We assume that \mathbf{W}_R contains i.i.d random entries distributed as $\mathcal{CN}(0, \sigma_R^2)$. For a more detailed co-existence model and joint designs of the two systems, we refer the readers to our extended journal version [13].

The MIMO-MC radar only partially samples \mathbf{Y}_R . Therefore, only the sampled target signal and sampled interference determine the matrix completion performance. Based on this observation, we define the *effective signal power* (ESP) and *effective interference*

power (EIP) at the radar RX node as follows:

$$\begin{split} \mathsf{ESP} &\triangleq \mathbb{E} \left\{ \mathsf{Tr} \left(\mathbf{\Omega} \circ \mathbf{M} \left(\mathbf{\Omega} \circ \mathbf{M}^{H} \right) \right) \right\} = pLM_{r,R} \mathsf{Tr} \left(\mathbf{\Phi} \mathbf{D}_{t} \right), \\ \mathsf{EIP} &\triangleq \mathbb{E} \left\{ \mathsf{Tr} \left(\mathbf{\Omega} \circ \left(\mathbf{CPS} + \mathbf{X}_{\mathsf{int}} \right) \left(\mathbf{\Omega} \circ \left(\mathbf{CPS} + \mathbf{X}_{\mathsf{int}} \right) \right)^{H} \right) \right\} \\ &= pLM_{r,R} \mathsf{Tr} \left(\mathbf{\Phi} \mathbf{C}_{t} + \mathbf{R}_{x} \right), \end{split}$$

where $\mathbf{\Phi} \triangleq \mathbf{PP}^H/L$ is positive semi-definite; $\mathbf{D}_t = \sum_{k=1}^K \sigma_{\beta_k}^2 \mathbf{v}_t^*(\theta_k) \mathbf{v}_t^T(\theta_k)$, $\mathbf{C}_t = \sum_{k=1}^{K_c} \sigma_{\beta_k}^2 \mathbf{v}_t^*(\theta_k^c) \mathbf{v}_t^T(\theta_k^c)$, σ_{β_k} and $\sigma_{\beta_k}^c$ denote the standard deviation of β_k and β_k^c , respectively. The derivation can be found in [13] and is omitted here for brevity. Incorporating the expressions for effective target signal, interference and additive noise, the *effective* radar SINR is given as

$$\text{ESINR} = \frac{\text{Tr}(\mathbf{\Phi}\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}\mathbf{C}_t) + \text{Tr}(\mathbf{R}_x) + \sigma_R^2}$$

Let us consider a scenario in which the radar searches in particular directions of interest, given by set $\{\theta_k\}$ for targets with unknown RCS variances [19,20]. For the unknown $\{\sigma_{\beta_k}^2\}$, we instead use the worst possible target RCS variance $\{\sigma_0^2\}$, which is the smallest target RCS variance that could be detected by the radar. In practice, the prior on $\{\theta_k\}$ could be obtained in tracking applications, where the target parameters obtained from previous tracking cycles are provided to focus the transmit power onto directions of interest. We assume that $\{\sigma_{\beta_k}^2\}$ and $\{\theta_k^c\}$ are known. In practice, these clutter parameters could be estimated when target is absent [21].

Based on Theorem 3, the radar precoder \mathbf{P} can be designed without affecting the incoherence property of \mathbf{M} . In the following, we present a design of the radar precoding matrix so that the clutter and communication interference at the radar RX antennas are minimized for successful matrix completion, with constraints on the radar transmit beampattern and the interference to the communication system, i.e.,

$$(\mathbf{P}_{1}) \max_{\boldsymbol{\Phi} \succeq 0} \text{ESINR}(\boldsymbol{\Phi}), \text{Tr}(\boldsymbol{\Phi}) \leq P_{R},$$
(7a)

$$\text{.t. } \operatorname{Tr}(\mathbf{G}\boldsymbol{\Phi}\mathbf{G}^{H}) \leq \eta, \tag{7b}$$

$$\operatorname{Tr}(\mathbf{\Phi}\mathbf{V}_k) \ge \xi \operatorname{Tr}(\mathbf{\Phi}), \forall k \in \mathbb{N}_K^+,$$
 (7c)

where $\mathbf{V}_k \triangleq \mathbf{v}_t^*(\theta_k) \mathbf{v}_t^T(\theta_k)$. The constraint of (7a) restricts the total radar transmit power to be no larger than P_R . The constraint of (7b) restricts the interference to the communication system to be at most η , in order to support reliable communication. The constraints of (7c) restrict that the power of the radar probing signal at interested directions must be not smaller than that achieved by the uniform precoding matrix $\frac{\text{Tr}(\Phi)}{M_{t,R}}$ **I**, *i.e.*, $\mathbf{v}_t^T(\theta_k) \Phi \mathbf{v}_t^*(\theta_k) \ge \xi \mathbf{v}_t^T(\theta_k) \frac{\text{Tr}(\Phi)}{M_{t,R}} \mathbf{I} \mathbf{v}_t^*(\theta_k) = \xi \text{Tr}(\Phi). \ \xi \ge 1$ is a parameter used to control the beampattern at the interested target angles. Problem (\mathbf{P}_1) is a constrained fractional SDP problem, which can be transformed into an equivalent SDP problem via Charnes-Cooper Transformation [21, 22] and be solved efficiently. It can be shown by the Karush-Kuhn-Tucker (KKT) conditions that any optimal solution Φ of (\mathbf{P}_1) has rank at most K. Therefore, the constraint on the rank of $\mathbf{P} = \sqrt{L} \Phi^{1/2}$ in Theorem 3 is satisfied. We omit the proof here for brevity. The readers are referred to [13] for more details.

5. NUMERICAL RESULTS

In this section, we provide simulation examples to quantify the performance of the proposed transmit precoding design for clutter mit-



Fig. 1: The radar transmit beampattern and the MUSIC spatial pseudo-spectrum for MIMO-MC radar with different precoding schemes. The true positions of the targets and clutters are labeled using solid and dashed vertical lines, respectively.

Precoding	ESINR	MC Relative
schemes	(dB)	Recovery Errors
Joint-design	31.3	0.038
Uniform	-44.3	1.00
NSP based	-46.3	1.00

 Table 1: The radar ESINR and MC relative recovery errors for MIMO-MC radar and communication spectrum sharing.

igation and the coexistence of the MIMO-MC radars and communication systems. The MIMO radar system consists of collocated $M_{t,R} = 16$ TX and $M_{r,R} = 16$ RX antennas, respectively forming transmit and receive half-wavelength uniform linear arrays. The radar waveforms are chosen from the rows of a random orthonormal matrix [5]. We set the length of the radar waveforms to L = 16. The wireless communication system consists of collocated $M_{t,C} = 4$ TX and $M_{r,C} = 4$ RX antennas, respectively forming transmit and receive half-wavelength uniform linear arrays. The radar and communication transmit power budget are 64000 and 64 (the power is normalized by the power of the radar waveform), respectively. The additive white Gaussian noise variance is $\sigma_R^2 = 0.01$. There are three stationary targets with RCS variance $\sigma_{\beta 0}^2 = 0.5$, located in the far-field with pathloss 10^{-3} , and clutter is generated by four point scatterers. All scatterers RCS variances are set to be identical and are denoted by σ_{β}^2 , which is decided by the prescribed clutter to noise ratio (CNR) $10 \log \sigma_{\beta}^2 / \sigma_R^2$. The interference channel G is modeled as Rician fading. The power in the direct path is 0.1, and the variance of Gaussian components contributed by the scattered paths is 10^{-3} . The performance metrics considered in this paper include the following: 1) The matrix completion relative recovery error, defined as $\|\mathbf{M} - \hat{\mathbf{M}}\|_F / \|\mathbf{M}\|_F$, where $\hat{\mathbf{M}}$ is the completed data matrix at the radar fusion center; 2) The radar transmit beampattern, i.e., the transmit power for different azimuth angles $\mathbf{v}_t^T(\theta) \mathbf{P} \mathbf{v}_t^*(\theta)$; and 3) The MUSIC pseudo-spectrum obtained using the completed data matrix M.

We present an example to show the advantages of the proposed radar precoding scheme as compared to the trivial uniform precoding, *i.e.*, $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{I}$, and null space projection (NSP) precoding, *i.e.*, $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{V}\mathbf{V}^H$, where **V** contains the basis of the null space of \mathbf{G}_1 [17]. For the proposed joint-design based scheme in (7), we choose $\xi = \lfloor \xi_{\text{max}} \rfloor$. The target angles w.r.t. the array are respectively -10° , 15° , and 30° ; the four point scatterers are at angles -45° , -30° , 10° , and 45° . The CNR is 30 dB. In this simulation, the direct path in \mathbf{G}_1 is generated as $\sqrt{0.1}\mathbf{v}_t(\phi)\mathbf{v}_t^H(\phi)$, where $\phi = 15^{\circ}$, with $\mathbf{v}_t(\phi)$ is defined in (2). In other words, the communication receiver is taken at the same azimuth angle as the second target.

The radar transmit beampattern and the spatial pseudo-spectrum obtained using the MUSIC algorithm are shown in Fig. 1. The correspondingly achieved ESINR and MC relative recovery error are listed in Table 1. From Fig. 1, we observe that the proposed jointdesign based precoding scheme successfully focuses the transmit power towards the three targets and nullifies the power towards the point scatterers. The three targets can be accurately estimated from the pseudo-spectrum obtained by the proposed scheme. As expected, the uniform precoding scheme just spreads the transmit power uniformly in all directions. The NSP precoding scheme results in a similar beampattern as the uniform precoding scheme except the deep null at the direction of the communication receiver. This means that the transmit power towards the second target is severely attenuated by the NSP precoding scheme. It is highly possible that the second target will be missed. In addition, both the uniform and N-SP precoding schemes have no capability of clutter mitigation. As shown in Fig. 1 and Table 1, the proposed joint-design based precoding scheme achieves significant improvement in ESINR and MC relative recovery error.

6. CONCLUSION

In this paper, we have proposed a MIMO-MC radar approach which uses random unitary matrices as waveforms and supports transmit precoding. We have shown that the recoverability of the data matrix is always guaranteed for any random unitary waveform and precoding matrix. This indicates that the proposed MIMO-MC radar supports waveform agility and precoding for interference mitigation. We have investigated the application of transmit precoding in the spectrum sharing framework of MIMO-MC radars and wireless communications. Simulation results have shown that the proposed MIMO-MC radars could achieve accurate data matrix completion, high output SINR, and good target angle estimation, under strong clutter conditions and communication interference.

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