A LATTICE METHOD FOR RESOLVING RANGE AMBIGUITY IN DUAL-FREQUENCY RFID TAG LOCALISATION

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ABSTRACT

The Radio Frequency Identification (RFID) is a rapidly developing technology with growing applications in several fields. One of the key applications is the localisation of tagged objects using signal phase difference information via dual-frequency technology. In this application, unwrapping signal phases to acquire the range between the reader and RFID tag is the major issue that has typically been addressed in the literature using either the Chinese Remainder Theorem or Lattice Theory. In this paper, a lattice-based method robust to phase measurement noise is presented to resolve the wrapped range. The proposed algorithm is shown to be more robust and efficient than existing approaches in terms of the reconstruction probability. Simulations are presented to illustrate the performance of the proposed algorithm.

Index Terms— Lattice theory, Wrapped phase, Phase ambiguity, RFID

1. INTRODUCTION

Radio Frequency Identification (RFID) is an emerging communication technology with many applications across a range of engineering fields, such as identification, localization, and tracking[1, 2, 3, 4]. RFID involves transmission of radio signals to communicate data between the reader and electronic tags attached to objects. RFID is becoming one of the primary approaches to localization because of its remote operation capability, low cost, simple structure, and high accuracy.

The RFID with dual-frequency technology measures the phase difference of arrival to obtain the time-of-flight of the round trip signal. This requires small bandwidth to obtain the range between the tag and reader with high accuracy [5, 6, 7]. In considering real applications, where the signal wavelength is much less than the actual range to be measured, the phase difference measurement is wrapped by 2π, in other words, the "measured" range from the reader to RFID tag is folded by an integer multiple of the signal wavelengths. Such a phase ambiguity problem can be efficiently addressed by either a closed-form algorithm using the Chinese Remainder Theorem (CRT) [8, 9] or a lattice-based algorithm [10, 11, 12].

In the lattice-based approach, the problem is formulated as a state estimation problem, requiring simultaneous estimation of the wrapped signal phase and the unknown integer for the number of times that the range is folded by the signal wavelength. Practically, if the unknown integer is estimated incorrectly, the range estimation error is significantly large. As a result, the reconstruction probability of the wrapped integer becomes the key measure of algorithm capability for resolving ambiguity.

In this paper, we propose an improved lattice-based algorithm for the RFID localization problem that uses dual-frequency RFID tags. We show analytically that the proposed algorithm has better performance in terms of reconstruction probability than the one in [10], though with a slightly high computational complexity.

The rest of the paper is organised as follows. In Section 2, the RFID localization system using dual-frequency and a conventional lattice-based algorithm are introduced. The proposed new algorithm is presented in Section 3, together with an analysis of it. The computer simulations to highlight the performance of the proposed methods are presented in Section 5. Finally, we give a conclusion in Section 6.

2. FORMULATION OF DUAL-FRQUENCY RFID RANGING

Consider a pure sine wave signal of dual-frequency, f_0, f_1 with $f_1 > f_0$, transmitted by an RFID reader to communicate with an RFID tag. In the ideal case, the received signal is defined by [5], i.e. $s_i(t) = a_i \exp(-j\phi_i)s(t),\ i = 0, 1,$ where a_i is amplitude, $s(t)$ is the transmitted signal and ϕ_i is the noise-free phase measured at the transmitter.

Let r_0 be the distance between the transmitter and tag,

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then the phase, $\phi_i,\ i = 0, 1$, can be written as $\phi_i = 2\frac{2\pi f_i r_0}{c}$, where c is the speed-of-light and the constant 2 is because of the round trip of the signal. The observed phase difference without noise is defined as $\phi_{0,1}^{(true)} = \phi_1 - \phi_0$. Then $\phi_{0,1}^{(true)}$ can be written into following form corresponding to the frequency difference $f_{0,1} = f_1 - f_0$,

$$\phi_{0,1}^{(true)} = 2\frac{2\pi f_{0,1} r_0}{c} \Rightarrow r_0 = \frac{c\phi_{0,1}^{(true)}}{4\pi f_{0,1}} \Rightarrow r_0 = \frac{1}{2}\phi_{0,1}^{(true)}\lambda_{0,1}$$

where $\lambda_{0,1} = c/(2\pi f_{0,1})$. The technique is illustrated in Fig. 1.

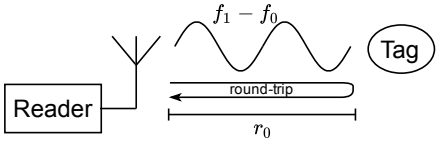

Fig. 1: The illustration of the dual-frequency RFID system

Since normally the $\lambda_{0,1}$ is not large enough to cover the possible range of values of r_0, then $\phi_{0,1}^{(true)}$ is wrapped, so that we have

$$r_0 = \frac{1}{2}\left(n_{0,1}\lambda_{0,1} + \phi_{0,1}^{(true)}\lambda_{0,1}\right) \tag{1}$$

where $n_{0,1}$ is an unknown integer.

Since (1) is a Diophantine equation for r_0, we need multiple measurements in different pairs of dual-frequency to determine the unknown integer $n_{0,1}$, and we assume that the RFID reader in question supports this operation. Let f_0 be the reference frequency then, for m frequency pairs, we have,

$$r \triangleq 2r_0 = n_{0,i}\lambda_{0,i} + \phi_{0,i}^{(true)}\lambda_{0,i} \qquad (i = 1, \cdots, m) \tag{2}$$

where $n_{i,0}$ is unknown integer and $\lambda_{0,i} = \frac{c}{2\pi(f_i - f_0)}$, $f_0 < f_i$. To guarantee that it is possible to solve this system of equations uniquely for $r \triangleq 2r_0$, we assume that $r < \text{LCM}(\lambda_{0,1}, \cdots, \lambda_{0,m})$, where $\text{LCM}(\cdot)$ is the least common multiple function.

In practice, the measurement is corrupted by noise, then we define the noisy measurement by $\phi_{0,i} = \phi_{0,i}^{(true)} + \omega'_{0,i}$, where $\omega'_{0,i} \sim \mathcal{N}(0, \delta^2)$ is the observation noise.

Thus (2) can be written into

$$\tilde{r} = n_{0,1}\lambda_{0,i} + y_{0,i} \qquad (i = 1, \cdots, m) \tag{3}$$

where $\tilde{r}$ is possible value of r, $y_{0,i} = (\phi_{0,i}^{(true)} + \omega'_{0,i})\lambda_{0,i}$. Let $\omega_{0,i} = \omega'_{0,i}\lambda_{0,i}$, then it is easy to find that $\omega_{0,i} \sim \mathcal{N}(0, \lambda_i^2\delta^2)$. Therefore, our job is to estimate r using (3).

We write the noise terms of Equation (3) in vector form $\boldsymbol{\omega} \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$, where $\boldsymbol{\omega} = [\omega_{0,1}, \cdots, \omega_{0,m}]$, $\mathbf{0} = [0, \cdots, 0]$, $\mathbf{C} = \text{diag}\{\lambda_{0,1}^2\delta^2, \cdots, \lambda_{0,m}^2\delta^2\}$. Let $\mathbf{y} = [y_{0,1}, \cdots, y_{0,m}]$, $\mathbf{n} = [n_{0,1}, \cdots, n_{0,m}]$, $\boldsymbol{\lambda} = [\lambda_{0,1}, \cdots, \lambda_{0,m}]$. Then the likelihood $p(\mathbf{y}|\mathbf{n}, r)$ is proportional to

$$-\left(r\mathbf{1} - \mathbf{n}\cdot\boldsymbol{\lambda} - \mathbf{y}\right)\mathbf{C}^{-1}\left(r\mathbf{1} - \mathbf{n}\cdot\boldsymbol{\lambda} - \mathbf{y}\right)^T$$

where $\mathbf{1} \triangleq [1, \cdots, 1]$ and $(\cdot)$ is the dot product.

The maximum likelihood solution is given by

$$(\hat{r}, \hat{\mathbf{N}}) = \arg\min_{(r,\mathbf{n})\in\mathbb{R}\times\mathbb{Z}^m} \|r\bar{\boldsymbol{\lambda}} - \mathbf{n} - \bar{\mathbf{y}}\|, \tag{4}$$

where $\bar{\boldsymbol{\lambda}} \triangleq [1/\lambda_{0,1}, \cdots, 1/\lambda_{0,m}]$, $\bar{\mathbf{y}} \triangleq [y_{0,1}/\lambda_{0,1}, \cdots, y_{0,m}/\lambda_{0,m}]$ and $\hat{\mathbf{N}} = [\hat{N}_{0,1}, \cdots, \hat{N}_{0,m}]$ is the estimate of ground truth $\mathbf{N} = [N_{0,1}, \cdots, N_{0,m}]$. Estimating either $\mathbf{N}$ or r solves the (4).

Before solving (4), we define two matrices:

$$\mathbf{A}(x_1, \cdots, x_m) = \begin{pmatrix} \prod_{i=1}^{m-1} x_i & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \prod_{i=1}^{m-1} x_i & 0 \\ -\frac{\prod_{i=1}^{m} x_i}{x_1} & \cdots & -\frac{\prod_{i=1}^{m} x_i}{x_{m-1}} & 0 \end{pmatrix} \tag{5}$$

and,

$$\mathbf{B}(x_1, \cdots, x_m) = \text{diag}\left(\left\{\frac{\prod_{i=1}^{m-1} x_i}{x_1}, \cdots, \frac{\prod_{i=1}^{m-1} x_i}{x_{m-1}}, 0\right\}\right) \tag{6}$$

Obviously, $\mathbf{A}(x_1, \cdots, x_m)$ and $\mathbf{B}(x_1, \cdots, x_m)$ are multivariate functions of $(x_1, \cdots, x_m)$.

Suppose that $\lambda_{0,1}, \cdots, \lambda_{0,m}$ are co-prime, then (4) can be converted [10] into

$$\hat{\mathbf{N}} = \arg\min_{\mathbf{n}\in\mathbb{Z}^m} \left\|\mathbf{n}\mathbf{B}(\lambda_{0,1}, \cdots, \lambda_{0,m}) + \bar{\mathbf{y}}\mathbf{A}(\lambda_{0,1}, \cdots, \lambda_{0,m})\right\|. \tag{7}$$

Formula (7) is typically the closest point searching problem in lattices and can be solved via Babai's algorithm efficiently [10, 13] if the wavelengths are co-prime. It only needs m times computations. This algorithm is referred as the conventional lattice algorithm in this paper.

3. IMPROVEMENT OVER THE CONVENTIONAL LATTICE ALGORITHM

Define $\boldsymbol{\lambda} = [\lambda_{0,1}, \cdots, \lambda_{0,m}]$, and assume that $\lambda_{0,1} < \cdots < \lambda_{0,m}$ and that $[\lambda_{0,1}, \cdots, \lambda_{0,m}]$ are co-prime. As in [10], when the variance of the measurement noise is proportional to the wavelength, the probability of correct reconstruction of the integer vector $\mathbf{N}$, $Pr(\hat{\mathbf{N}} = \mathbf{N})$, is largely impacted by the minimum wavelength in the set $[\lambda_{0,1}, \cdots, \lambda_{0,m}]$.

We define the circular shift function $S(\cdot, \cdot)$ of an array as

$$\begin{aligned} S(\boldsymbol{\lambda}, 0) &= [\lambda_{0,1}, \cdots, \lambda_{0,m}] \\ S(\boldsymbol{\lambda}, 1) &= [\lambda_{0,m}, \lambda_{0,1}, \cdots, \lambda_{0,m-1}] \\ &\vdots \\ S(\boldsymbol{\lambda}, m-1) &= [\lambda_{0,2}, \lambda_{0,3}, \lambda_{0,4}, \cdots, \lambda_{0,m}, \lambda_{0,1}] \end{aligned}$$

Accordingly, recalling the definition of $\mathbf{A}(\cdot)$ and $\mathbf{B}(\cdot)$ in (5) and (6), we let $\mathbf{A}_j \triangleq \mathbf{A}(S(\boldsymbol{\lambda}, j)), \mathbf{B}_j \triangleq \mathbf{B}(S(\boldsymbol{\lambda}, j)), j = 0, 1, \cdots, m-1$.

As shown in Lemma 1, in the conventional lattice algorithm, the use of different basis $\mathbf{A}_j, j = 0, \cdots, m-1$ implies different reconstruction probability $Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, j)\right)$.

Lemma 1 *Define $\boldsymbol{\lambda} = [\lambda_{0,1}, \cdots, \lambda_{0,m}]$. Suppose $\lambda_{0,1} < \cdots < \lambda_{0,m}$ and are co-prime. Then the probabilities of correct reconstruction of the integer set $\mathbf{N}$ using the conventional lattice algorithm[10] satisfies $Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 0)\right) > Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, j)\right), j = 1, \cdots, m-1$.*

Proof 1 *We only prove the case $Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 0)\right) > Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 1)\right)$. The proofs of other cases are similar.*

As indicated in [10], for $S(\boldsymbol{\lambda}, 0)$, the lattice algorithm will return the true value if $|\omega_{0,i} - \omega_{0,m}| < \frac{1}{2}$ for $i = 1, \cdots, m-1$, and this yields

$$Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 0)\right) = Pr\left(\cap_{i=1}^{m-1} |\omega_{0,i} - \omega_{0,m}| < 1/2\right)$$

Similarly, for $S(\boldsymbol{\lambda}, 1)$, we have

$$\begin{aligned} & Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 0)\right) \\ = & Pr\Big(\cap_{i \in \{1, \cdots, m\} \backslash \{m-1\}} |\omega_{0,i} - \omega_{0,m-1}| < 1/2\Big) \end{aligned} \quad (8)$$

Since $\omega_{0,i} \sim \mathcal{N}(0, \delta^2 \lambda_{0,i}^2), i = 1, \cdots, m$, indicating that the variance of $\omega_{0,i}$ is proportional to the value of wavelengths $\lambda_{0,i}$, it is not hard to see that

$$Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 0)\right) > Pr\left(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 1)\right) \quad (9)$$

The result in Lemma 1 is from a statistical perspective. However, in each realization, it is possible that the conventional lattice algorithm may return wrong estimation of $\mathbf{N}$ using $\mathbf{A}_0$ while the algorithm returns correct estimation using $\mathbf{A}_j, j = 1, \cdots, m-1$. Thus it is useful to draw information not only from $\mathbf{A}_0$, as used in conventional lattice algorithm in [10], but also from $\mathbf{A}_j, j = 1, \cdots, m-1$ before a decision is made. Based on the above idea, we present following estimator. The performance of the estimator is analysed in the next section.

$$\begin{aligned} \hat{\mathbf{N}} = & \hat{\mathbf{N}}_{j_0} \quad (10) \\ \text{s.t. } j_0 = & \arg \min_{j=0,\cdots,m-1} \left| \mathbb{E}\left[(\hat{\hat{r}}_j - \hat{\mathbf{r}}_j) \cdot \bar{\boldsymbol{\lambda}}_j \right] \right| \\ \hat{\mathbf{N}}_j = & \arg \min_{\mathbf{n} \in \mathbb{Z}^m} \|\mathbf{n}\mathbf{B}_j + \bar{\mathbf{y}}_i \mathbf{A}_j\|, j = 0, \cdots, m-1 \\ \hat{\mathbf{r}}_j = & \hat{\mathbf{N}}_j \boldsymbol{\lambda} + \mathbf{y}_j \\ \hat{\hat{r}}_j = & E\left[r | \hat{\mathbf{r}}_j\right] \end{aligned}$$

where $\mathbb{E}[\cdot]$ is the arithmetic mean operator, $\bar{\boldsymbol{\lambda}}_j \triangleq S(\bar{\boldsymbol{\lambda}}, j)$, $\mathbf{y}_j \triangleq S(\mathbf{y}, j)$, $\bar{\mathbf{y}}_j = S(\bar{\mathbf{y}}, j)$, and $\hat{\hat{r}}_j = E\left[r | \hat{\mathbf{r}}_j\right]$ is the optimal estimator of r given $\hat{\mathbf{r}}_j$, as can be found in [10].

It is not hard to solve (10). One could first solve $\hat{\mathbf{N}}_j, j = 0, \cdots, m-1$ according to $\mathbf{A}_j = \mathbf{A}(S(\boldsymbol{\lambda}, j))$ using the conventional lattice algorithm, and then compute the $\hat{\mathbf{r}}_j$. After calculating $\hat{\mathbf{N}}_j$ and $\hat{\mathbf{r}}_j$ for all j, the estimation of $\hat{\mathbf{N}}$ is easy to find by computing $\left| \mathbb{E}\left[(\hat{\hat{r}}_j - \hat{\mathbf{r}}_j) \cdot \bar{\boldsymbol{\lambda}}_j \right] \right|$.

The algorithm can be divided into two parts: the off-line part and the on-line part. The matrices used are constructed and stored in memory in the off-line part. In the on-line part, the unknown range is estimated using measurements and the stored matrices. The algorithm is set out as Algorithm 1.

Algorithm 1: Proposed algorithm to find $\hat{\mathbf{N}}$ with different constructions of $\mathbf{A}_j$

Data: $\boldsymbol{\lambda} = [\lambda_{0,1}, \cdots, \lambda_{0,m}]$,
$\lambda_{0,1} < \cdots < \lambda_{0,m}$, $\{\lambda_{0,1}, \cdots, \lambda_{0,m}\}$ are co-prime,
$\mathbf{y} = [y_{0,1}, \cdots, y_{0,m}]$
Result: Estimation of $\mathbf{N}$, i.e. $\hat{\mathbf{N}}$
Off-line part:
for $j = 0 : m-1$ **do**
  $\mathbf{A}_j = \mathbf{A}(S(\boldsymbol{\lambda}, j))$;
  Compute $\mathbf{B}_j$ and other associated matrices;
On-line part:
$\bar{\mathbf{y}} = [y_{0,1}/\lambda_{0,1}, \cdots, y_{0,m}/\lambda_{0,m}]$;
for $j = 0 : m-1$ **do**
  $\mathbf{y}_j = S(\mathbf{y}, j), \bar{\mathbf{y}}_j = S(\bar{\mathbf{y}}, j)$;
  Calculate $\hat{\mathbf{N}}_j$ using the conventional lattice algorithm with data $\bar{\mathbf{y}}_j \mathbf{A}_j$, $\mathbf{B}_j$ and other associated matrix;
  $\hat{\hat{r}}_j = E\left[r | \hat{\mathbf{r}}_j\right]$, $\text{Obj}_j = \left| \mathbb{E}\left[(\hat{\hat{r}}_j - \hat{\mathbf{r}}_j) \cdot \bar{\boldsymbol{\lambda}}_j \right] \right|$;
$j_0 = \arg\min_{j=0,\cdots,m-1} \text{Obj}_j$;
return $\hat{\mathbf{N}} = \hat{\mathbf{N}}_{j_0}$

4. THE COMPUTATIONAL COMPLEXITY AND PERFORMANCE ANALYSIS

In this section, we aim to analyse the algorithm described in Algorithm 1 in terms of computational complexity and reconstruction performance. The computational complexity is measured by the required on-line computation time, while the reconstruction performance is measured by the probability of correct reconstruction of $\mathbf{N}$; that is, $Pr(\hat{\mathbf{N}} = \mathbf{N})$.

The required on-line computation has two parts, 1) the processes to calculate the candidate estimations, i.e. $\hat{\mathbf{N}}_j, j = 0, \cdots, m-1$, and 2) the outer loop from 0 to $m-1$. Since, for each j, the first part needs m times computations and therefore, the total computation complexity is $m \cdot m = m^2$.

Since the performance of the proposed algorithm is measured by $Pr(\hat{\mathbf{N}} = \mathbf{N})$, as indicated in the algorithm, $\hat{\mathbf{N}} = \mathbf{N}$ happens if and only if there exists at least one $j_0 \in [0, \cdots, m-1]$ such that following event happens

$$\{\hat{\mathbf{N}}_{j_0} = \mathbf{N}\} \cap \{j_0 = \arg \min_{j=0,\cdots,m-1} \text{Obj}_j\}, \qquad (11)$$

where $\text{Obj}_j = \left| \mathbb{E}\left[(\hat{\hat{r}}_j - \hat{\mathbf{r}}_j) \cdot \bar{\boldsymbol{\lambda}}_j \right] \right|$. Therefore,

$$\begin{aligned} & Pr\left(\hat{\mathbf{N}} = \mathbf{N}\right) \\ =& Pr\left(\{\hat{\mathbf{N}}_{j_0} = \mathbf{N}\} \cap \{j_0 = \arg \min_{j=0,\cdots,m-1} \text{Obj}_j\} \right) \\ =& Pr\left(j_0 = \arg \min_{j=0,\cdots,m-1} \text{Obj}_j \middle| \hat{\mathbf{N}}_{j_0} = \mathbf{N} \right) Pr\left(\hat{\mathbf{N}}_{j_0} = \mathbf{N}\right) \end{aligned}$$

Since, when $\hat{\mathbf{N}}_{j_0} = \mathbf{N}$, we have $\hat{\hat{r}}_{j_0} \simeq r$,

$$(\hat{\hat{r}}_{j_0} - \hat{\mathbf{r}}_{j_0}) \cdot \bar{\boldsymbol{\lambda}}_{j_0} \simeq (r - \hat{\mathbf{r}}_{j_0}) \cdot \bar{\boldsymbol{\lambda}}_{j_0} = \boldsymbol{\omega}_{j_0} \cdot \bar{\boldsymbol{\lambda}}_{j_0},$$

where $\boldsymbol{\omega}_{j_0} \triangleq S(\boldsymbol{\omega}, j_0)$.

According to the definition of $\boldsymbol{\omega_j}$ and $\bar{\boldsymbol{\lambda}}_{j_0}$, we know that

$$\boldsymbol{\omega}_{j_0} \cdot \bar{\boldsymbol{\lambda}}_{j_0} \sim \mathcal{N}(0, \delta^2 \mathbf{I}_m)$$

where $\mathbf{I}_m$ is the m-dimensional identity matrix.

If $\hat{\mathbf{N}}_{j_0} = \mathbf{N}$, then $\left| \mathbb{E}\left[(\hat{\hat{r}}_{j_0} - \hat{\mathbf{r}}_{j_0}) \cdot \bar{\boldsymbol{\lambda}}_{j_0} \right] \right| \simeq 0$, and it will normally include the minimum value for $j = 0, \cdots, m-1$ if m is large. This implies that

$$\begin{aligned} & Pr\left(j_0 = \arg \min_{j=0,\cdots,m-1} \text{Obj}_j \middle| \hat{\mathbf{N}}_{j_0} = \mathbf{N} \right) \\ \simeq & Pr\left(j_0 = \arg \min_{j=0,\cdots,m-1} \text{Obj}_j \middle| \hat{\mathbf{N}}_{j_0} = \mathbf{N}, \text{Obj}_{j_0} = 0 \right) \\ = & 1 \end{aligned}$$

It follows that $Pr\left(\hat{\mathbf{N}} = \mathbf{N}\right) \simeq Pr\left(\hat{\mathbf{N}}_{j_0} = \mathbf{N}\right)$. Define the event $\mathcal{E} \triangleq \{\hat{\mathbf{N}}_{j_0} = \mathbf{N}, \exists j_0 \in \{0, \cdots, m-1\}\}$, then $\mathcal{E}^c$ states that $\{\hat{\mathbf{N}}_j \neq \mathbf{N}, \forall j = 0, \cdots, m-1\}$. In consequence,

$$\begin{aligned} Pr(\mathcal{E}) = 1 - Pr(\mathcal{E}^c) =& 1 - Pr\left(\cap_{j=0}^{m-1} \{\hat{\mathbf{N}}_j \neq \mathbf{N}\} \right) \\ >& 1 - Pr\left(\hat{\mathbf{N}}_j \neq \mathbf{N} \right) \\ =& Pr\left(\hat{\mathbf{N}}_j = \mathbf{N} \right) \quad \forall j = 0, \cdots, m-1 \end{aligned}$$

By Lemma 1, this implies that the proposed algorithm is more robust than the conventional lattice algorithm in terms of the probability of correct reconstruction.

5. SIMULATION

The RFID ranging performance using the proposed algorithm listed in Algorithm 1 is demonstrated using Monte Carlo simulations. We take the US UHF band, 902 $\sim$ 928 MHz, as an example. The frequency 902 MHz is selected as the reference frequency; that is, $f_0 = 902$ MHz, and therefore, the available frequency differences are within $1 \sim 26$ MHz with 1MHz increment[5] and wavelengths are within $300 \sim 12$ m. In the simulation, the following co-prime wavelengths are selected $\boldsymbol{\lambda} = [21,\ 19,\ 17,\ 13]$m.

The distance r is randomly selected between 0 and the LCM of all used wavelengths. The parameter δ in the measurement noise variance $\delta^2 \lambda_i^2$ is chosen so that $-20 \log_{10} \delta = 34 : 2 : 50$, which provides an indication for both noise level and signal-to-noise ratio in the simulation. All simulation results illustrated are averaged over 5000 Monte Carlo runs. The algorithm performance is evaluated in terms of the reconstruction probability $Pr(\mathbf{N} = \hat{\mathbf{N}})$ versus the phase measurement noise level.

The simulation results are shown in Figure 2. For a comparison, the performance of the conventional lattice algorithm using the lattice basis $\mathbf{A}_0$ is presented as well. As indicated in Lemma 1, the reconstruction probability of the conventional lattice algorithm satisfies $Pr(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, 0)) > Pr(\hat{\mathbf{N}} = \mathbf{N} | S(\boldsymbol{\lambda}, j)), j = 1, \cdots, m-1$ and $S(\boldsymbol{\lambda}, 0)$ corresponds to the lattice basis $\mathbf{A}_0$. Thus it is sufficient to compare the performance of the new algorithm and conventional lattice algorithm using the lattice basis $\mathbf{A}_0$.

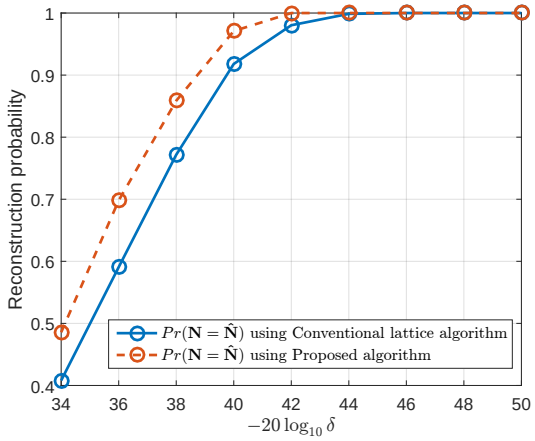

Fig. 2: The reconstruction probability comparison between the conventional lattice [10] algorithm and proposed algorithm in Section 3.

Figure 2 shows that the proposed algorithm outperforms the conventional lattice algorithm in that it has a high probability of correct reconstruction for a given noise level.

6. CONCLUSIONS

In this work, we have presented an improved lattice algorithm for estimating the distance between a RFID reader and a tag using multiple pairs of dual frequencies. The proposed algorithm is more robust than the conventional lattice algorithm in terms of the probability of correct reconstruction of the wrapped integers at the cost of only slightly higher complexity. The performance of the proposed algorithm is analysed. This algorithm is suitable for other situations when wrapped measurements arise.

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