ON TOA ESTIMATION OF VIBRATION SIGNALS FOR LOCALIZING IMPACTS ON SOLID SURFACES

Nguyen Quang Hanh, V. G. Reju, and Andy W. H. Khong

School of Electrical and Electronic Engineering Nanyang Technological University, Singapore Email: {quanghan001, reju, andykhong}@ntu.edu.sg

ABSTRACT

We propose a TDOA-based algorithm for source localization on rigid surfaces. This allows the conversion of readily available large surfaces into touch interfaces using surface-mounted vibration sensors. To achieve this, we characterize the arrival of each sensorreceived signal by the arrival times of its frequency components. To estimate the arrival time of each frequency component, we first model each component as a harmonic random process. A kurtosis sequence, which exhibits a sharp rising edge when the signal begins to deviate from the background noise, can then be obtained for each component. By accurately estimating the starting point of the rising edge, our algorithm can avoid the uncertainty due to gradual noise-to-signal transition. Experiment results show that the proposed algorithm achieves better localization performance than existing techniques on large surfaces.

Index Terms— Human-computer interface, source localization on solids, TOA, TDOA, kurtosis

1. INTRODUCTION

In the emerging era of the internet of things (IoT), the way human interacts with computer has been being revolutionized in various aspects. New human-computer interfaces (HCI) are replacing traditional input devices such as keyboards and mice. In this work, we propose an impact-localization algorithm that works on rigid surfaces, utilizing the vibration signals generated by users on the surface. This provides a cost-effective solution which enables the use of daily objects such as glass panels and tabletops as HCIs [1–5]. In these applications, vibration signals are captured by low-cost sensors mounted on the surface. The location of the user impact is then obtained from processing the sensor-received signals.

A popular approach for impact localization is based on the timedifference-of-arrival (TDOA) information across the sensors [6–12]. Challenges of TDOA estimation on rigid surfaces include dispersion and multipath. In the presence of dispersion, wave propagation velocity is dependent on its frequency, causing signal components of different frequencies to arrive at a sensor at different times [13]. Multipath, on the other hand, results in signal distortions due to reflections from the medium boundaries [14]. The traditional TDOA estimation technique using the generalized cross-correlation (GCC) [15] suffers from the combined effects of these two phenomena.

Recent methods circumvent the effect of dispersion by performing TDOA estimation on component(s) of a single frequency or within a narrow frequency band. In addition, the effect of multipath is mitigated by focusing on the onset of each signal, where only its initial arrival but no reflection is present [8–12]. Authors in [11] extract the initial part of each signal as where the first dip and peak are observed before application of GCC. Other techniques avoid direct estimation of TDOA and adopt the indirect approach of first estimating the time-of-arrival (TOA) at each sensor. The TDOA is subsequently obtained as the difference between the estimated TOAs. The TOA of a signal can be associated with its phase transit point which corresponds to the maximum change in frequency distribution of the received signal [8]. However, for a relatively long noise-to-signal transition period, the frequency distribution varies slowly, resulting in erroneous phase transit point estimates. The TiF-HA method [10] converts the short-time Fourier transform (STFT) coefficients of the signal into Hermitian angles (HA) and it was noted that the standard deviation of HA across frequency bins decreases abruptly as the signal arrives. Techniques such as in [8] and [10] are shown to be suitable only for small surfaces where the noise-to-signal transition is abrupt [12]. For gradual transition, the transition can be modeled using the four-parameter logistic function and instead of explicitly estimating the TOA, TDOA estimates are obtained as the difference in the translation parameters of the models fitted for each sensor-received signal [12]. While the use of translation parameter partially compensates the TDOA estimation uncertainty due to gradual transitions, it does not directly address the problem of gradual transition.

In this work, we employ a statistical method to estimate the starting point of the noise-to-signal transition. While the use of translation parameter in [12] inherits the uncertainty of a long transition period, our approach of estimating the starting point of the transition period does not suffer from this uncertainty. It however requires good signal-to-noise ratio (SNR) to achieve good estimation. We also propose in this work procedures to improve TOA/TDOA estimation accuracy in the presence of noise, which relies on constructing a statistical TOA profile for each sensor-received signal from its frequency components. It will be shown later in the experiment results that our proposed technique can achieve better localization of the impact source compared to existing techniques.

2. THE PROPOSED TOA ESTIMATION ALGORITHM

2.1. Analysis of impact-induced signals

Due to velocity dispersion, for a sensor-received signal x(n), different frequency components arrive at the sensor at different time instants. Therefore, the TOA can be determined only for each frequency component, rather than for the whole signal. We first extract components of x(n) corresponding to a set of selected frequencies $f_k, k = 1, \ldots, K$. The arrival of x(n) is then characterized by the set that includes the TOA of each component.



Fig. 1: Frequency components extracted from a typical vibration signal induced by a finger tap on glass surface where (a) the sensor-received signal is not modified and (b) white Gaussian noise is added to the signal at SNR of 10 dB. The black solid lines represent the real parts of the frequency components while the dashed lines represent their respective magnitudes. The thick gray lines represent the kurtosis sequences $\gamma_k(n)$ obtained from (9). The black dots represent the TOAs estimated using $\gamma_k(n)$.

Extraction of the component $\underline{z}_k(n)$ corresponding to frequency f_k can be achieved using a narrow bandpass filter centered at f_k . Here, according to the Heisenberg uncertainty principle, there is a trade-off between time and frequency resolutions [16]. While various filtering technique can be applied, we employ Gabor filters for their well-known property of providing the best time-frequency resolution [17]. The component $\underline{z}_k(n)$ is then obtained as

$$\underline{z}_k(n) = x(n) * \underline{\Psi}_k(n), \tag{1}$$

where * denotes the convolution operator and the Gabor function is given as

$$\underline{\Psi}_{k}(n) = \frac{1}{\sqrt[4]{\pi}} \sqrt{\frac{\omega_{k}}{\varsigma}} \exp\left[-\frac{1}{2} \left(\frac{\omega_{k}n}{\varsigma}\right)^{2}\right] \exp\left(-j\omega_{k}n\right), \quad (2)$$

with $\omega_k = 2\pi f_k/f_s$, f_s being the sampling frequency. The parameter ς determines the spread of $\underline{\Psi}_k(n)$ in time as well as in frequency. The real part and the magnitude of each extracted complex signal $\underline{z}_k(n)$ are illustrated in Fig. 1(a) for a typical sensor-received signal due to a finger tap on a glass surface.

In this work, the vibration signals are captured by sensors whose frequency responses roll off beyond 10 kHz. In such a low frequency range, only the flexural mode of vibration is present [18], where the wave propagation velocity is proportional to the square root of the vibration frequency. This velocity can be expressed as [19]

$$c(f) = \sqrt[4]{\frac{E}{12\rho(1-\nu^2)}}\sqrt{2\pi f L_z},$$
(3)

where the modulus of elasticity E, the flexural rigidity ρ , and the Poisson's ratio ν are material-dependent properties, and L_z is the thickness of the surface. The time required for the component of frequency f to arrive at the sensor is given by

$$\tau(f) = \eta / \sqrt{f},\tag{4}$$

where $\eta = d/\sqrt[4]{\sqrt{\pi^2 L_z^2 E/[3\rho(1-\nu^2)]}}$, d being the distance from the impact location to the sensor. Considering only signal components within a frequency band centered at f_c , i.e., $f_k \in (f_c - \Delta_f, f_c + \Delta_f)$, $k = 1, \ldots, K$, the variation among the TOAs τ_k of $\underline{z}_k(n)$ is limited to $\Delta_\tau \approx \left| \frac{d\tau}{df}(f_c) \right| \Delta_f = \eta \Delta_f / (2\sqrt{f_c^3})$. By selecting a sufficiently narrow bandwidth Δ_f , Δ_τ becomes negligibly small so that all τ_k for a particular sensor can be considered equal. Such an effort to reduce the effect of dispersion is illustrated in Fig. 1(a), where f_k are selected within 3 kHz to 5 kHz. The TOAs τ_k (estimated using the method proposed in the next section), represented by a dot (•) in each subplot, can be seen relatively aligned at a specific time instant. The arrival of the signal can now be characterized by the TOA set $\tau = \{\tau_1, \ldots, \tau_K\}$. While a common TOA can be estimated as $\hat{\tau}_= \sum_{k=1}^K \tau_k/K$, the spread $\sigma_\tau = \left[\sum_{k=1}^K (\tau_k - \hat{\tau})^2 / K\right]^{1/2}$ represents the quality of the estimation, where large σ_τ implies high uncertainty of the estimate. In addition, outliers in τ can be removed to obtain a more accurate estimate.

2.2. Kurtosis for TOA estimation

In this section, we estimate the arrival time for each f_k -component of x(n). While the noise-to-transition in $\underline{z}_k(n)$ may occur gradually, we avoid the uncertainty of the long transition period by estimating its starting point. The starting point is defined as the time instant when the energy of $\underline{z}_k(n)$ deviates from that of the background noise. To capture the appearance of the signal on the background noise, we compute the kurtosis sequence for a sliding length-L frame $\underline{z}_k(n) = [\underline{z}_k(n), \dots, \underline{z}_k(n + L - 1)]^T$, where T is the transposition operator. Due to the sensitivity of the kurtosis to outliers, kurt $\{\underline{z}_k(n)\}$ exhibits a large value when the energy of $\underline{z}_k(n)$ exceeds that of the noise. This results in a sharp rising edge in the kurtosis sequence whenever an energy deviation occurs. However, due to the high variation in energy of $\underline{z}_k(n)$ after its arrival, direct computation of kurt $\{\underline{z}_k(n)\}$ results in a non-smooth sequence with multiple peaks, as illustrated in Fig. 2(b). This makes it challenging to determine the exact instant when $\underline{z}_k(n)$ arrives. To address this problem, we model $\underline{z}_k(n)$ as a realization from a harmonic random process with frequency f_k defined as

$$\underline{Z}_k(n) = a_k(n)e^{j2\pi f_k n + \varphi_k(n)},$$
(5)

where $a_k(n) = |\underline{z}_k(n)|$ and $\varphi_k(n)$ is a random variable uniformly distributed on $[-\pi, \pi]$. Now, instead of kurt $\{\underline{z}_k(n)\}$, we compute the kurtosis for each $\underline{Z}_k(n) = [\underline{Z}_k(n), \dots, \underline{Z}_k(n+L-1)]^{\mathrm{T}}$ as [20]

$$\gamma_k(n) = \kappa_4 \left\{ \underline{\mathbf{Z}}_k(n) \right\} / \kappa_2^2 \left\{ \underline{\mathbf{Z}}_k(n) \right\},\tag{6}$$

where $\kappa_2 \{ \underline{Z}_k(n) \}$ and $\kappa_4 \{ \underline{Z}_k(n) \}$ are the second and fourth cumulants of $\underline{Z}_k(n)$, respectively.

It can easily be seen from (5) that $E\{\underline{Z}_k(n)\} = 0$, which yields $E\{\underline{Z}_k(n)\} = \sum_{i=n}^{n+L-1} E\{\underline{Z}_k(i)\} = 0$. The second and fourth cumulants can therefore be respectively reduced to

$$\kappa_{2} \left\{ \underline{Z}_{k}(n) \right\} = \mathbb{E} \left\{ \underline{Z}_{k}(n) \underline{Z}_{k}^{*}(n) \right\}$$
$$= \frac{1}{L} \sum_{i=n}^{n+L-1} \frac{1}{2\pi} \int_{-\pi}^{\pi} \underline{z}_{k}(n) \underline{z}_{k}^{*}(n) d\varphi_{k}(n)$$
$$= \frac{1}{L} \sum_{i=n}^{n+L-1} |\underline{z}_{k}(n)|^{2}, \qquad (7)$$

and κ_4

$$\{ \underline{Z}_{k}(n) \} = \mathbf{E} \{ \underline{Z}_{k}(n) \underline{Z}_{k}^{*}(n) \underline{Z}_{k}(n) \underline{Z}_{k}^{*}(n) \}$$

$$= \frac{1}{L} \sum_{i=n}^{n+L-1} \frac{1}{2\pi} \int_{-\pi}^{\pi} \underline{z}_{k}(n) \underline{z}_{k}^{*}(n) \underline{z}_{k}(n) \underline{z}_{k}^{*}(n) d\varphi_{k}(n)$$

$$= \frac{1}{L} \sum_{i=n}^{n+L-1} |\underline{z}_{k}(n)|^{4} ,$$

$$(8)$$

where $\underline{Z}_{k}^{*}(n)$ and $\underline{z}_{k}^{*}(n)$ are the complex conjugates of $\underline{Z}_{k}(n)$ and $\underline{z}_{k}(n)$, respectively. As a result, the kurtosis sequence is obtained as

$$\gamma_k(n) = \frac{\frac{1}{L} \sum_{i=n}^{n+L-1} |\underline{z}_k(i)|^4}{\left(\frac{1}{L} \sum_{i=n}^{n+L-1} |\underline{z}_k(i)|^2\right)^2}.$$
(9)

We note from (9) that $\gamma_k(n)$ is an energy-based measure. Therefore, similar to existing energy-based TOA techniques such as TiF-HA and STFT-Logistic [10, 12], utilization of $\gamma_k(n)$ also requires high signal-to-noise ratio (SNR). The effect of SNR on TOA estimation is shown in Fig. 1(b), where x(n) is contaminated with white Gaussian noise. While the overall SNR is 10 dB, when considering each component individually, the SNR decreases with frequency. This is because high-frequency components of x(n) are of less power than low-frequency components. As can be seen in Fig. 1, empirically, an empirically determined SNR of at least 10 dB is necessary for $\gamma_k(n)$ to be well conditioned so that an accurate TOA estimate can be obtained. We can, however, perform TOA estimation only on $\underline{z}_k(n)$ that satisfies the SNR requirement.

To estimate the SNR for each $\underline{z}_k(n)$, the silent period is first estimated from the signal x(n). Applying kurtosis on the sliding frame $\mathbf{x}(n) = [x(n), \dots, x(n+L-1)]^{\mathrm{T}}$, we obtain

$$\gamma(n) = \kappa_4 \{\mathbf{x}(n)\} / \kappa_2^2 \{\mathbf{x}(n)\} \\ = \frac{\frac{1}{L} \sum_{i=n}^{n+L-1} [x(i) - \overline{x}(n)]^4}{\left(\frac{1}{L} \sum_{i=n}^{n+L-1} [x(n+i) - \overline{x}(n)]^2\right)^2},$$
(10)



Time index n

Fig. 2: The kurtosis sequences and their respective curvatures. The sequence $\gamma_k(n)$ in (a) is the kurtosis of $\underline{Z}_k(n)$, computed as in (9), while the kurtosis sequence in (b) is computed from $\underline{z}_k(n)$. The curvature of $\gamma_k(n)$ in (a) can be utilized to determine the starting point of the rising edge in $\gamma_k(n)$. The starting point (•) corresponds to the peak of $\xi_k(n)$, denoted by \circ .

where $\overline{x}(n) = \sum_{i=n}^{n+L-1} x(i)/L$. An initial estimate of the onset time separating the silent period and the signal period is then given as

$$\tau_{\rm on} = \operatorname*{argmax}_{n} \gamma(n). \tag{11}$$

The silent period is subsequently determined as $[1, \tau_{on} - \Delta]$, where Δ is a buffer length chosen empirically to guarantee that the signal content is absent during the silent period. Similarly, the signal period is determined as $[\tau_{on} + \Delta', N]$ for some empirical value Δ' . The SNR of each $\underline{z}_k(n)$ is then estimated as

$$\operatorname{SNR}_{k} = \frac{\sum_{i=\tau_{\text{on}}+\Delta'}^{N} \underline{z}_{k}(n) \underline{z}_{k}^{*}(n) / (N - \tau_{\text{on}} - \Delta' + 1)}{\sum_{i=1}^{\tau_{\text{on}}-\Delta} \underline{z}_{k}(n) \underline{z}_{k}^{*}(n) / (\tau_{\text{on}} - \Delta)}.$$
 (12)

2.3. Arrival time estimation using curvature

While $\gamma_k(n)$ exhibits a sharp rising edge on noise-to-signal transition, there is no knowledge of a correct threshold value that $\gamma_k(n)$ would exceed at the transition point. Thresholding is therefore not an appropriate approach for determining the starting point of the rising edge. Here, we determine the point by utilizing the curvature of $\gamma_k(n)$. The curvature of a function measures how fast a function changes its direction, and a sharp rising edge corresponds to such a change being sudden. The curvature of $\gamma_k(n)$ at point *n* is given by

$$\xi_k(n) = \Delta^2 \gamma_k(n) \left[1 + (\Delta \gamma_k(n))^2 \right]^{-3/2},$$
 (13)

where $\Delta \gamma_k(n) = \gamma_k(n+1) - \gamma_k(n)$ and $\Delta^2 \gamma_k(n) = \Delta \gamma_k(n+1) - \Delta \gamma_k(n) = \gamma_k(n+2) - 2\gamma_k(n+1) + \gamma_k(n)$ are, respectively, the first and second discrete difference of $\gamma_k(n)$. The arrival time of $\underline{z}_k(n)$ is estimated as

$$\tau_k = \operatorname*{argmax}_n \xi_k(n). \tag{14}$$

It is worth noting that for a noisy function, its curvature fluctuates significantly since $\xi_k(n)$ is sensitive to the the sign of $\Delta^2 \gamma_k(n)$, and therefore is not a good measure for edge detection. However, the envelope $|\underline{z}_k(n)|$ is a relatively low frequency signal with its maximum frequency negligibly small compared to f_k . Therefore, $\xi_k(n)$ can be utilized as shown in Fig. 2(a). This is also the advantage of $\gamma_k(n)$ over other non-smooth measures such as the kurtosis directly computed from $\underline{z}_k(n)$, denoted by $\gamma'_k(n)$, whose curvature $\xi'_k(n)$ fluctuates significantly as illustrated in Fig. 2(b).

2.4. TDOA estimation

Consider the signals $x_1(n)$, ..., $x_R(n)$ captured by R surfacemounted sensors. For each $x_i(n)$, i = 1, ..., R, denote by τ_i^k the TOA estimated for its f_k -component $\underline{z}_i^k(n)$ using (14). The TDOA between $\underline{z}_i^k(n)$ and $\underline{z}_j^k(n)$ for each sensor pair (i, j) is given by

$$\tau_{i,j}^k = \tau_i^k - \tau_j^k. \tag{15}$$

Since the frequencies f_k are chosen within a narrow band where velocity dispersion is negligible, $\tau_{i,j}^k$ are all equal to a common value $\tau_{i,j}$. In practice, $\tau_{i,j}^k$ will be randomly distributed in a neighborhood of $\tau_{i,j}$. The TDOA between $\{\underline{z}_i^k\}$ and $\{\underline{z}_j^k\}$ can be estimated as

$$\widehat{\tau}_{i,j} = \frac{1}{K} \sum_{k=1}^{K} \tau_{i,j}^k.$$
(16)

Note that the quality of the estimate $\hat{\tau}_{i,j}$ depends on the consistency of $\boldsymbol{\tau}_{i,j} = \{\tau_{i,j}^1, \ldots, \tau_{i,j}^K\}$. To achieve better accuracy in source localization, we only utilize $\boldsymbol{\tau}_{i,j}$ that exhibits sufficiently low spread. This results in a binary weighting matrix $[w_{i,j}]$ given by

$$w_{i,j} = \begin{cases} 1, & \text{if } \sigma_{\tau_{i,j}} < \overline{\tau}, \\ 0, & \text{otherwise,} \end{cases}$$
(17)

where $\sigma_{\tau_{i,j}} = \left[\sum_{k=1}^{K} \left(\tau_{i,j}^k - \hat{\tau}_{i,j}\right)^2 / K\right]^{1/2}$, and $\overline{\tau}$ is an empirical threshold.

Denoting by f_c the center frequency of the band where f_k are selected from, since dispersion is negligible within the band, all f_k -components propagate at the same speed $c(f_c)$. This speed can be estimated during calibration as described in [10]. The source location (u_s, v_s) can then be estimated by minimizing the error function

$$(\hat{u}_{s}, \hat{v}_{s}) = \underset{(u,v)}{\operatorname{argmin}} \sum_{i,j} \left(\frac{d_{i}(u,v) - d_{j}(u,v)}{c(f_{c})} - \frac{\hat{\tau}_{i,j}}{f_{s}} \right)^{2} w_{i,j},$$
(18)

where $d_i(u, v) = \sqrt{(u - u_i)^2 + (v - v_i)^2}$ is the distance from an arbitrary location (u, v) to the *i*th sensor positioned at (u_i, v_i) . Existing optimization algorithms such as Levenberg-Marquardt [21] or Nelder-Mead [22] can be utilized for such minimization.

3. EXPERIMENT RESULTS

We evaluate the performance of the proposed algorithm using different sets of real data collected on a glass plate of dimension $1.2 \text{ m} \times 1.0 \text{ m} \times 5.0 \text{ mm}$. On the plate surface, impacts are generated by either a finger or a metal stylus at the locations (0.3, 0.3), (0.3, 0.5),(0.3, 0.7), (0.5, 0.3), (0.6, 0.5), (0.6, 0.7), (0.9, 0.3), (0.9, 0.5),and (0.9, 0.7) (all the dimensions are in meters). At each location, a set of five impacts are generated, and hence a total of forty-five test cases are performed for each experiment setup. The induced vibrations are captured by eight surface-mounted Murata PKS1-4A10 piezoelectric shock sensors mounted at the corners and the midpoints of the edges, 0.1 m away from the plate boundaries. The sensor outputs are subsequently digitized at a sampling frequency of $f_s = 96$ kHz. In order to quantify the overall performance of each algorithm on a set of data, the root-mean-square error (RMSE)



Fig. 3: Performance comparison of the proposed algorithm with that of TiF-HA [10] and STFT-Logistic [12] using (a) RMSE and (b) standard deviation of the localization errors (STDE).

and the standard deviation (STDE) of the localization errors for all impacts in the set are employed.

Here we compare the performance of our proposed algorithm with that of TiF-HA [10] and STFT-Logistic [12], which have been shown to outperform other methods such as the KLID-based [8] or scalogram-based [23] by significant margins. It can be seen from Fig. 3 that the proposed algorithm and STFT-Logistic outperform the TiF-HA in both cases. This is because TiF-HA assumes abrupt noise-to-signal transitions which is unlikely the case for vibration signals captured on large plates. In addition, STFT-Logistic only modestly outperforms TiF-HA for taps generated by fingers. For taps generated by fingers, which may include vibrations due to flesh, nail and bone, the transition becomes complex and is not correctly described by the logistic function. This is reflected in the modest margin of STFT-Logistic against TiF-HA. Our proposed algorithm, however, captures the starting instant of the transition period, and therefore is not affected by the complex transition. As a result, our proposed algorithm outperforms both STFT-Logistic and TiF-HA by decent margins.

4. CONCLUSION

In our proposed TDOA-based algorithm for HCI applications, frequency components of each sensor-received signal are first modeled as harmonic random processes. The theoretical kurtosis is then derived for each process, which exhibits a sharp rising edge when the signal energy deviates from that of the background noise. Due to the smoothness of the derived kurtosis sequence, its curvature can be utilized to determine the starting point of the rising edge as the arrival time of the signal. While the noise-to-signal transition may be gradual, the proposed algorithm captures the starting point of the transition period and therefore avoids the complication of transition modeling. Experiment results show that the proposed algorithm outperforms existing methods, especially when the transition is gradual and complicated as in the case for impacts generated by fingers.

5. REFERENCES

 D. T. Pham, M. Al-Kutubi, Z. Ji, M. Yang, Z. B. Wang, and S. Catheline, "Tangible acoustic interface approaches," in *Proc. Int. Virtual Conf. IPROMs*, 2005.

- [2] K. Poletkin, X. Yap, and A. W. H. Khong, "A touch interface exploiting the use of vibration theories and infinite impulse response filter modeling based localization algorithm," in *Proc. IEEE Int. Conf. Multimedia and Expo (ICME)*, 2010, pp. 286– 291.
- [3] K. R. Arun, X. Yap, and A. W. H. Khong, "A touch interface exploiting time-frequency classification using Zak transform for source localization on solids," *IEEE Trans. Multimed.*, vol. 13, pp. 487–497, 2011.
- [4] K. R. Arun, V. G. Reju, and A. W. H. Khong, "Source localization on solids utilizing time-frequency analysis of parameterized warped signals," in *Proc. IEEE Int. Conf. Acoustics*, *Speech and Signal Processing (ICASSP)*, May 2014, pp. 689– 693.
- [5] Q. H. Nguyen, V. G. Reju, and A. W. H. Khong, "Noise robust source localization on solid surfaces," in *Proc. 2015 10th Int. Conf. Information, Communications and Signal Processing (ICICS)*, pp. 1–5.
- [6] S. M. Ziola and M. R. Gorman, "Source location in thin plates using cross-correlation," J. Acoust. Soc. Amer., vol. 90, pp. 2551–2556, 1991.
- [7] W. Rolshofen, D. T. Pham, M. Yang, Z. B. Wang, Z Ji, and M. Al-Kutubi, "New approaches in computer-human interaction with tangible acoustic interfaces," in *Proc. Int. Virtual Conf. IPROMs*, 2005.
- [8] K. R. Arun, E. Ong, and A. W. H. Khong, "Source localization on solids using Kullback-Leibler discrimination information," in *Proc. 2011 8th Int. Conf. Information, Communications and Signal Processing (ICICS)*, pp. 1–5.
- [9] P. Sedlak, Y. Hirose, and M. Enoki, "Acoustic emission localization in thin multi-layer plates using first-arrival determination," *Mech. Syst. Sig. Process.*, vol. 36, no. 2, pp. 636 – 649, 2013.
- [10] V. G. Reju, A. W. H. Khong, and A. B. Sulaiman, "Localization of taps on solid surfaces for human-computer touch interfaces," *IEEE Trans. Multimed.*, vol. 15, pp. 1365–1376, Oct. 2013.
- [11] H. Nakatani, T. Kundu, and N. Takeda, "Improving accuracy of acoustic source localization in anisotropic plates," *Ultrasonics*, vol. 54, no. 7, pp. 1776 – 1788, 2014.
- [12] Q. H. Nguyen, V. G. Reju, and A. W. H. Khong, "Source localization on solids utilizing logistic modeling of energy transition in vibration signals," in *Proc. 2016 IEEE Int. Conf. Acoustics, Speech and Signal Processing (ICASSP).*
- [13] Y. T. Chan, T. K. C. Lo, and H. C. So, "Passive range-difference estimation in a dispersive medium," *IEEE Trans. Signal Process.*, vol. 58, pp. 1433–1439, 2010.
- [14] F. Foroozan and A. Asif, "Time reversal based active array source localization," *IEEE Trans. Signal Process.*, vol. 59, pp. 2655–2668, Jun. 2011.
- [15] C. Knapp and G. C. Carter, "The generalized correlation method for estimation of time delay," *IEEE Trans. Acoust.*, *Speech, Signal Process.*, vol. 24, no. 4, pp. 320–327, 1976.
- [16] K. Gröchenig, *Foundations of Time-Frequency Analysis*. Birkhäuser Boston, 2001.
- [17] H. Jeong and Y. Jang, "Fracture source location in thin plates using the wavelet transform of dispersive waves," *IEEE Trans. Ultrason. Ferroelectr., Freq. Control*, vol. 47, no. 3, pp. 612– 619, 2000.

- [18] S. A. H. Sabzevari and M. Moavenian, "Locating the acoustic source in thin glass plate using low sampling rate data," *Ultrasonics*, vol. 70, pp. 1 – 11, 2016.
- [19] I. A. Viktorov, Ralyleigh and Lamb Waves: Physical Theory and Applications. Plenum Press, 1967.
- [20] J. M. Mendel, "Tutorial on higher-order statistics (spectra) in signal processing and system theory: Theoretical results and some applications," *Proc. IEEE*, vol. 79, no. 3, pp. 278–305, Mar 1991.
- [21] D. W. Marquardt, "An algorithm for least-squares estimation of nonlinear parameters," *J. Soc. Ind. Appl. Math.*, vol. 11, no. 2, pp. 431–441, 1963.
- [22] J. A. Nelder and R. Mead, "A simplex method for function minimization," *Comput. J.*, vol. 7, no. 4, pp. 308–313, 1965.
- [23] L. Gaul and S. Hurlebaus, "Identification of the impact location on a plate using wavelets," *Mech. Syst. Signal Process.*, vol. 12, no. 6, pp. 783–795, 1998.