

ENHANCED INDOOR LOCALIZATION THROUGH CROWD SENSING

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ABSTRACT

In localization tasks, one typically assumes a statistical model of the observations, where the model quantifies the observations by exploiting interrelationships based on geometry. These models might incorporate unknown parameters that, in general, are functions of space. In this article, we propose a crowd sensing method for estimating a spatial field of a quantity (e.g., ranging biases due to line-of-sight/non-line-of-sight or path-loss parameter) allowing for improved indoor localization. Our method takes advantage of the information provided by various users that navigate the area of interest. The proposed learning approach is based on Gaussian processes and its computational cost does not increase with the number of measurements. We present numerical results that show how the proposed method estimates a spatial field of biases and how these estimates lead to much improved performance in estimation of user positions.

Index Terms— indoor localization, crowd sourcing, Gaussian processes, spatial field, biases

1. INTRODUCTION

Indoor localization systems are a key technology for enabling Location-based Services (LBS) [1]. Due to the importance of the technology, it is not surprising that the literature on addressing the main challenges associated with indoor localization is quite rich [2, 3]. There are several methodologies for localization including fingerprinting and model-based methods. Here, we are interested in the latter, where localization algorithms are designed so that a user computes its location in real-time based on its own past and current measurements (for instance, through Bayesian filtering). In this context, there is the need to specify a model (usually statistical) that maps the measurements to geometric quantities (distance, angle, position, etc.). The adopted model often depends critically on some (potentially unknown) parameters which are typically location-dependent such as the line-of-sight (LOS)/non-line-of-sight (NLOS) condition, ranging bias and variance, or the path loss exponent in the received signal strength indicator (RSSI). When a subset of model parameters is unknown, a number of approaches for their improved

estimation have been investigated, including methods based on cooperation and learning [4–11].

The aforementioned learning schemes typically require a huge amount of measurements to be reliable, which in turn must come from expensive training sessions that become out-of-date after a short time. Alternatively, due to the widespread use of mass market devices with sensing capabilities, mobile crowd sensing has emerged as an appealing paradigm to enable large-scale applications [12]. In many practical scenarios one may have measurements acquired by hundreds or thousands of people navigating through an indoor environment in different hours, days and months and which can be exploited for improved localization [13]. In this context one challenging aspect is the “Big data” issue. Considering that in crowd sensing scenarios the number N of measurements could grow exponentially, we want to investigate learning approaches whose memory and computational burden do not increase with N .

The contribution of this article is in proposing a methodology for indoor localization which relies on estimating a spatial field of unknowns of the environment defining the observation model. It is assumed that the estimated position of a user can be improved considerably if one has good estimates of these unknowns. Specifically, we propose a method for learning the spatial field by exploiting the *power of the crowd*. After a user crosses the area of interest, it takes advantage of the available estimated field obtained from measurements acquired by previous users. In turn, the estimate of this field is updated by the measurements of this user. Thereby, subsequent users can also benefit by using the field for their own localization. This can be termed as an *indirect cooperation* approach.

We propose a learning and spatial representation scheme whose memory and computational burden do not increase with the number of measurements. Furthermore, our scheme has an important feature in that it is capable of accounting for spatial correlation. In the simulation section, we provide numerical results of a ranging model with unknown bias due to NLOS conditions caused by an obstacle. We demonstrate how our method can construct the field biases via the crowd sensing approach and how it gets updated. We also show how the field estimate can improve the estimated positions of the users.

2. PROBLEM STATEMENT

Consider a scenario wherein a potentially high number K of mobile users move in an indoor environment not necessarily at the same time. For example, the scenario could be a shopping mall where crowds of users roam during different hours of a day or on different

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days. Without loss of generality, we consider that the users enter and move in the mall and collect noisy measurements. There are N_A nodes (anchors) in the environment, and they are located at fixed and known positions $\mathbf{I}^{(l)}$, $l = 1, 2, \dots, N_A$. We denote the measurements of user k by $\mathbf{y}_{1:N^{(k)}}^{(k)} = \{\mathbf{y}_1^{(k)}, \mathbf{y}_2^{(k)}, \dots, \mathbf{y}_{N^{(k)}}^{(k)}\}$, where $N^{(k)}$ is the number of measurements, and $\mathbf{y}_n^{(k)} = \{y_n^{(k,l)}\}$ with $y_n^{(k,l)}$ being the measurement of the k th user with the l th anchor at time step n . We denote with $\mathbf{x}_n^{(k)}$ the position (i.e., state) of the k th mobile user at time step n , $n = 1, 2, \dots, N^{(k)}$, relative to the starting instant of the user's path.

Often in theory and practice, a probabilistic state-space Markovian model with additive noise is adopted. It characterizes the evolution of the state considering the uncertainty about the system as follows [3]:

$$\mathbf{x}_n^{(k)} = g\left(\mathbf{x}_{n-1}^{(k)}\right) + \mathbf{w}_n^{(k)}, \quad (1)$$

$$\mathbf{y}_n^{(k,l)} = h\left(\mathbf{x}_n^{(k)}; f^{(l)}\left(\mathbf{x}_n^{(k)}\right)\right) + \nu_n^{(k,l)}, \quad (2)$$

where the function $g(\cdot)$ models the dynamics of the user, the function $h(\cdot)$ maps the state to measurements, and $\mathbf{w}_n^{(k)}$ and $\nu_n^{(k,l)}$ are the state process perturbation and measurement noise, respectively. Here, both $\mathbf{w}_n^{(k)}$ and $\nu_n^{(k,l)}$ are assumed to be independent, identically distributed (i.i.d.) Gaussian random variables (RVs). The unknown function $f^{(l)}(\mathbf{x})$ in (2) represents a spatial field that models position-dependent uncertainties about the observation model of anchor l . For instance, $f^{(l)}(\mathbf{x})$ could include NLOS / LOS conditions, path-loss parameters, time-of-arrival (TOA) biases, etc. Recall that there are N_A anchors and that each anchor has its own spatial field. We denote the set of all fields by $\mathbf{f}(\mathbf{x}) = \{f^{(1)}(\mathbf{x}), f^{(2)}(\mathbf{x}), \dots, f^{(N_A)}(\mathbf{x})\}$.

The state-space model in (1)-(2) can be efficiently solved by Bayesian filtering. With Bayesian filtering we recursively estimate the marginal posterior distribution $p\left(\mathbf{x}_n^{(k)} | \mathbf{y}_{1:n}^{(k)}\right) = \text{BF}\left(\mathbf{y}_n^{(k)}, \mathbf{x}_{n-1}^{(k)}; \mathbf{f}(\cdot)\right)$ of the current state $\mathbf{x}_n^{(k)}$ given all the past measurements $\mathbf{y}_{1:n}^{(k)}$ [3]. Here $\text{BF}\left(\mathbf{y}_n^{(k)}, \mathbf{x}_{n-1}^{(k)}; \mathbf{f}(\cdot)\right)$ symbolizes the Bayesian filter with inputs $\mathbf{y}_n^{(k)}$, $\mathbf{x}_{n-1}^{(k)}$ and $\mathbf{f}(\cdot)$. It is important to point out that the method requires a constant number of computations at each time instant n .

There are several approaches to implement the recursive Bayesian filtering step $\text{BF}\left(\mathbf{y}_n^{(k)}, \mathbf{x}_{n-1}^{(k)}; \mathbf{f}(\cdot)\right)$. They include extended Kalman filter (EKF) and particle filtering (PF) methods [3]. Once the marginal posterior distribution $p\left(\mathbf{x}_n^{(k)} | \mathbf{y}_{1:n}^{(k)}\right)$ of the current state $\mathbf{x}_n^{(k)}$ is computed, we can obtain a point estimate $\hat{\mathbf{x}}_n^{(k)}$ of $\mathbf{x}_n^{(k)}$ by using the minimum mean-square error (MMSE) or the maximum a posteriori (MAP) criteria.

In (2), it is explicitly specified that the measurement $\mathbf{y}_n^{(k,l)}$ is a function of the spatial field $f^{(l)}\left(\mathbf{x}_n^{(k)}\right)$. How do we construct $f^{(l)}(\mathbf{x})$ and how do we update it by using measurements of new users? How do we update the field in an efficient way so that we minimize the computational and memory burden of the method? How do we exploit the estimated field for improved localization?

3. MODEL PARAMETERS ESTIMATION THROUGH CROWD SENSING

In this section we provide answers to the above listed questions. For notation convenience, we omit the indexes l and k .

Consider a generic spatial field $f(\mathbf{x})$, for instance, the ranging bias with respect to a specific anchor node. Suppose the spatial field has been observed so far in N locations $\{\mathbf{x}_n\}$, $n = 1, 2, \dots, N$, where the observed field, z_n , follows the model

$$z_n = f(\mathbf{x}_n) + \epsilon_n \quad n = 1, 2, \dots, N \quad (3)$$

with the random terms $\{\epsilon_n\}$ treated as samples of i.i.d. zero-mean Gaussian RVs with variance σ_n^2 .

We want to obtain the maximum statistical knowledge of the spatial field, $f(\mathbf{x}_T)$, at a given location \mathbf{x}_T , where \mathbf{x}_T is not in the set $\{\mathbf{x}_n | n = 1, 2, \dots, N\}$. This will allow a new user to estimate its position by using the model in (2) with respect to a specific anchor node. We assume that the user has a rough initial estimate of its position (e.g., by means of prediction using the dynamic model (1)).

In absence of specific and accurate models for $f(\mathbf{x})$, in non-parametric regression, one common approach is to assume that $f(\mathbf{x})$ is a sample from a Gaussian process (GP)

$$f \sim \mathcal{GP}(\mu(\mathbf{x}), \kappa(\mathbf{x}, \mathbf{x}')) \quad (4)$$

with mean $\mu(\mathbf{x})$ and covariance function $\kappa(\mathbf{x}, \mathbf{x}')$.

The solution of this regression problem is well-known [14]. However, its formal simplicity is hampered by the computational cost needed to invert the corresponding $N \times N$ Gram matrix at each step which, in general, requires $\mathcal{O}(N^3)$ operations. Several methods have been proposed to overcome this issue including methods based on the approximation of the Gram matrix with a matrix having a smaller rank [15]. For regular grids, fast solutions can be obtained through FFT-based approaches [16]. An alternative is to approximately describe the GP through state-space models, thereby making the complexity of the method independent on N under certain conditions but still requiring matrix inversions at each step [17]. Other methods are referenced in [15].

3.1. A Combined GP-State Space Method

In the following we generalize the non-parametric regression method by combining GPs and state-space descriptions of the field proposed in [18].

Without loss of generality, consider a square area of $L \times L$ square meters centered at the origin of the coordinate system, where the interest is to estimate the field $f(\mathbf{x})$. Given an appropriate 2D orthogonal basis with functions $\{\psi_m(\mathbf{x})\}$, $m = 0, 1, \dots, M-1$, $f(\mathbf{x})$ can be approximated at time instant $n = 0$ (i.e., before measurements are collected) as

$$f(\mathbf{x}) \cong \sum_{m=0}^{M-1} c_{m,0} \psi_m(\mathbf{x}), \quad (5)$$

where $\{c_{m,0}\}$ are the coefficients at time instant $n = 0$. These coefficients are Gaussian RVs.

To avoid the need to calculate matrix inversion for any new location of interest, we propose the following combined GP-state space method.

Consider the conditional RVs $\mathbf{c}_n = [c_{0,n}, c_{1,n}, \dots, c_{M-1,n}]^T$, where $c_{m,n} = c_{0,n} | \mathbf{y}_{1:n}$ represents the RV $c_{0,n}$ conditioned on the field observations $\mathbf{z}_{1:n} = [z_1, z_2, \dots, z_n]$ collected at the first n

locations. It is easy to show that the conditional GP can be expressed from (5) as

$$f(\mathbf{x})|\mathbf{z}_{1:n} \cong \mathbf{H}(\mathbf{x}) \mathbf{c}_n, \quad (6)$$

where $\mathbf{H}(\mathbf{x}) = [\psi_0(\mathbf{x}), \psi_1(\mathbf{x}), \dots, \psi_{M-1}(\mathbf{x})]$.

The vector of RVs \mathbf{c}_n can be viewed as the state at discrete time n of the following state-space model¹

$$\begin{aligned} \mathbf{c}_n &= \mathbf{c}_{n-1}, \\ z_n &= \mathbf{H}(\mathbf{x}_n) \mathbf{c}_n + \epsilon_n, \end{aligned} \quad (7)$$

where z_n is the measurement taken at location \mathbf{x}_n . The state-space model (7) is linear and Gaussian and, therefore, it can be converted into a special case of the Kalman filtering problem, which admits the following simple recursive solution [19]:

$$\begin{aligned} s_n &= \mathbf{H}(\mathbf{x}_n) \mathbf{P}_{n-1} \mathbf{H}(\mathbf{x}_n)^T + \sigma_n^2, \\ \mathbf{W}_n &= \mathbf{P}_{n-1} \mathbf{H}(\mathbf{x}_n)^T s_n^{-1}, \\ \mathbf{m}_n &= \mathbf{m}_{n-1} + \mathbf{W}_n (z_n - \mathbf{H}(\mathbf{x}_n) \mathbf{m}_{n-1}), \\ \mathbf{P}_n &= \mathbf{P}_{n-1} - \mathbf{W}_n s_n \mathbf{W}_n^T, \end{aligned} \quad (8)$$

where \mathbf{m}_n and \mathbf{P}_n are, respectively, the $M \times 1$ mean and $M \times M$ covariance matrix of the state at time n conditioned on all the measurements collected until time n . Note that s_n is a scalar and, thus, no matrix inversion is required, which results in a computational advantage. The initial values of \mathbf{m}_0 and \mathbf{P}_0 represent the a priori knowledge about the field.

Once a new measurement z_n becomes available, one step of (8) is performed. Due to the nature of the problem, the state \mathbf{c}_n is a set of joint Gaussian RVs with mean \mathbf{m}_n and covariance \mathbf{P}_n . Then \mathbf{m}_n and \mathbf{P}_n provide a full statistical description of the RVs $\{c_{m,n}\}$, and hence of the spatial field, at time n . Then the mean and variance of the field at any location of interest \mathbf{x}_T can be evaluated as follows:

$$\begin{aligned} \mu_n(\mathbf{x}_T) &= \mathbb{E}\{f(\mathbf{x}_T)|\mathbf{z}_{1:n}\} \cong \mathbf{H}(\mathbf{x}_T) \mathbf{m}_n, \\ \kappa_n(\mathbf{x}_T, \mathbf{x}_T) &\cong \mathbf{H}^*(\mathbf{x}_T) \mathbf{P}_n \mathbf{H}^T(\mathbf{x}_T) - \mu_n^2(\mathbf{x}_T). \end{aligned} \quad (9)$$

A standard point estimate of $\hat{f}(\mathbf{x})$, given the past n measurements, would be $\hat{f}(\mathbf{x}) = \mu_n(\mathbf{x})$.

Note that only $2M$ numbers are sufficient to completely describe the acquired information about the spatial field in the entire area, which is much less than the memory necessary to store the full set of N measurements. In fact, in real crowd sensing applications, N could grow up to thousands whereas M remains fixed. In the numerical results presented in the next section, the 2D Fourier series expansion of the periodical repetition of $f(\mathbf{x})$ with a period L in each dimension is used as in [18].

4. CROWD-ENHANCED LOCALIZATION ALGORITHM: CASE STUDY

We illustrate how the proposed approach is used within the tracking operation by considering a specific case study. In this study, the spatial field $f^{(l)}(\mathbf{x})$ represents the ranging bias due to NLOS channel conditions of the following observation model in (2):

$$\mathbf{y}_n^{(k,l)} = \left| \mathbf{x}_n^{(k)} - \mathbf{I}^{(l)} \right| + f^{(l)}(\mathbf{x}_n^{(k)}) + \nu_n^{(k,l)}, \quad (10)$$

¹The first equation implies that we consider time-invariant fields. Our approach can be readily extended to time-variant fields by including a dynamic model here.

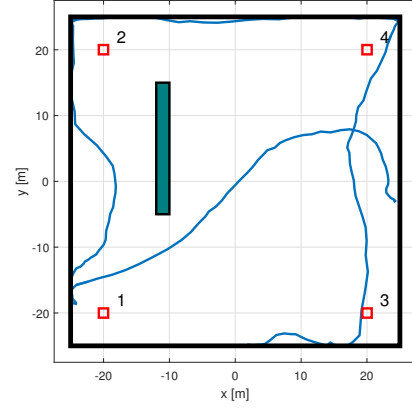


Fig. 1. Floor plan, with a random trajectory of a user.

where $\mathbf{y}_n^{(k,l)}$ is the range measurement of the l th anchor at time n , $\mathbf{x}_n^{(k)}$ is the location of the k th mobile user at time n . According to the method described in Sec. 3.1, we model $f^{(l)}(\mathbf{x})$ as a realization of a GP with mean $\mu^{(l)}(\mathbf{x})$ and variance $V^{(l)}(\mathbf{x}) = \kappa^{(l)}(\mathbf{x}, \mathbf{x})$ for each anchor node. The LOS locations are characterized by $\mu^{(l)}(\mathbf{x}) = 0$ and, typically, by a small variance $V^{(l)}$, whereas the NLOS locations by $\mu^{(l)}(\mathbf{x}) > 0$ and a larger variance. As before, $\mu_0^{(l)}(\mathbf{x})$ and $\kappa_0^{(l)}(\mathbf{x}, \mathbf{x}')$ represent the a priori information about $f^{(l)}(\mathbf{x})$. Such information is refined as soon as other users' path estimates become available.

The proposed approach can be summarized as follows, where $\hat{\mathbf{f}}^{(k)}(\cdot)$ is the estimate of $\mathbf{f}(\cdot)$, according to the information collected up to the k th user, that can be exploited by the $k+1$ th user: For each new user k do:

1. $n = 1$
 - (a) $p(\mathbf{x}_n^{(k)}|\mathbf{y}_{1:n}^{(k)}) = \text{BF}(\mathbf{y}_n^{(k)}, \mathbf{x}_{n-1}^{(k)}; \hat{\mathbf{f}}^{(k-1)}(\cdot))$;
 - (b) compute the state estimate $\hat{\mathbf{x}}_n^{(k)}$;
 - (c) $n = n + 1$; Iterate from *a* until $n = N^{(k)}$;
2. update the maps
$$\hat{\mathbf{f}}^{(k)} = \text{UPDATE}(\hat{\mathbf{x}}_{1:N^{(k)}}^{(k)}, \mathbf{y}_{1:N^{(k)}}^{(k)}; \hat{\mathbf{f}}^{(k-1)}(\cdot))$$
;
3. $k = k + 1$ (new user). Go to 1.

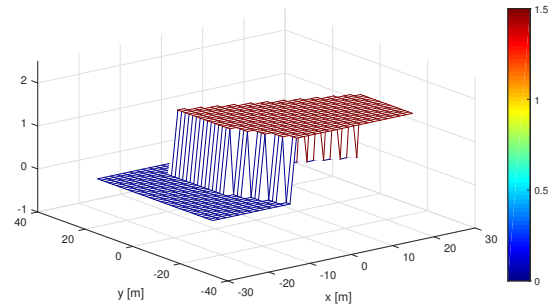


Fig. 2. True field (bias) for anchor 2.

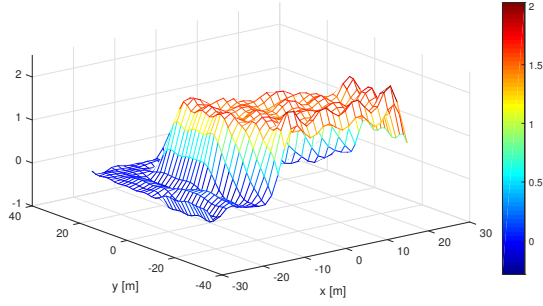


Fig. 3. Estimated field (bias) for anchor 2 after 1000 users.

The UPDATE step is performed by a central unit by iterating $N = N^{(k)}$ times the mapping algorithm in Sec. 3.1 with the measurements $\mathbf{y}_{1:N^{(k)}}^{(k)}$, thus obtaining an updated estimate $\hat{\mathbf{f}}^{(k)}(\mathbf{x})$ at the k th path of fields $\mathbf{f}(\mathbf{x}) = \{f^{(l)}(\mathbf{x})\}$, for $l = 1, 2, \dots, N_A$. This step requires a model that expresses the relationship between $y_n^{(k,l)}$ and $z_n^{(k,l)}$ in (3). In our case study, the measurement $z_n^{(k,l)}$ is given by

$$z_n^{(k,l)} = y_n^{(k,l)} - \left| \hat{\mathbf{x}}_n^{(k)} - \mathbf{I}^{(l)} \right|, \quad (11)$$

and $y_n^{(k,l)}$ is the range measurement collected by the l th anchor in that position at time n . That is, $z_n^{(k,l)}$ is extracted from the filter's innovations. As a result, the random term in (3) has a variance given by the innovation's variance of the filter.

We have conducted simulations in which the users move by following a random path inside a square area. The side size of the area is $L = 50$ m, it contains a single obstacle, and it has four anchors, as shown in Fig. 1. For each user k , we consider that $N^{(k)} = 200$ measurements are taken, at intervals of 5 s. The state process perturbation and measurement variances in (1)-(2) are, respectively, 0.0156 m^2 and 0.01 m^2 . For each anchor and a given location, the sight condition can be classified as LOS or NLOS. For simplicity we have considered constant bias values: 0 for the LOS case and 1.5 m for NLOS. Fig. 2 illustrates the corresponding field $f^{(2)}(\mathbf{x})$ for anchor 2.

We have used the EKF, as Bayesian filtering technique, combined with three alternative approaches: the crowd-enhanced localization algorithm, truncating the series expansion of the field to $M = 169$ terms; a naive method assuming always LOS, and a genie-aided method with perfect knowledge of the true sight condition.

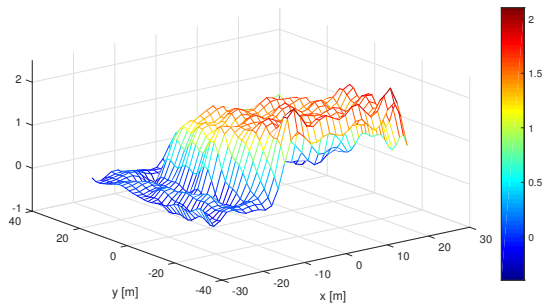


Fig. 4. Estimated field (bias) for anchor 2 after 100 users.

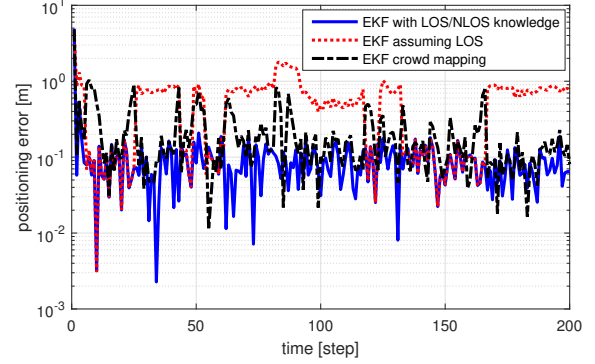


Fig. 5. Positioning error (distance between estimated and true locations) as a function of time for the 100th user in the second setup.

We consider two setups for testing the crowd enhanced method. In the first one, the true location is used in Eq. (11). This is useful for testing the potential of the method. Fig. 3 shows the resulting estimated field for anchor 2 as seen by the 1000th user.

We now consider a second setup with $K = 100$ users, of which the first 20 travel along prescribed trajectories to provide bootstrap positions to the learning algorithm. For the remaining users, the current location estimate is used in Eq. (11). The resulting estimated field for anchor 2 as seen by the 100th user is shown in Fig. 4, which is similar to Fig. 3 in spite of the reduced number of users. Fig. 5 shows the positioning error as a function of the time step, for the 100th user, whose trajectory is the one shown in Fig. 1. The improvement in terms of root mean squared error (RMSE), where the average is performed along the trajectory, of the crowd enhanced method with respect to the method assuming LOS is 41%, for the 100th user of the second setup, and 64%, for the 1000th user of the first setup. It can also be observed that the proposed method approaches the performance of the method with perfect knowledge of the spatial field.

5. CONCLUSIONS

In this paper, we have presented an approach for improved localization in indoor environments based on crowd sourcing capable of learning spatially-distributed unknown parameters (spatial field). This field can represent the distribution of relevant parameters over space defining the observation model, such as NLOS/LOS conditions and path-loss parameters. We have proposed a combined GP-state space model to characterize this field so that its representation and maintenance do not grow with additional measurements. Besides relying on locally received measurements for estimating its location, a mobile has at its disposal estimates of spatial fields obtained from the measurements of the previous mobiles. We have demonstrated by computer simulations how the localization and tracking performance of this method get close to that obtained by a genie-aided approach after a reasonable number of previous users.

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