# NON-SEPARABLE QUADRUPLE LIFTING STRUCTURE FOR FOUR-DIMENSIONAL INTEGER WAVELET TRANSFORM WITH REDUCED ROUNDING NOISE

Fairoza Amira Binti Hamzah<sup>1</sup>, Taichi Yoshida<sup>1</sup> and Masahiro Iwahashi<sup>1</sup>

<sup>1</sup>Department of Electrical Engineering, Nagaoka University of Technology, Niigata, Japan

# ABSTRACT

The Wavelet Transform (WT) in JPEG 2000 is using a 'separable' lifting structure, where the one-dimensional (1D) transform is put into multidimensional image signal of its spatial and temporal dimensions. A 'non-separable' threedimensional (3D) structure as the existing method is used to minimize its lifting steps. The 'non-separable' 3D structure in the (5,3) type transform for lossless coding is proved to reduce the rounding noise inside it. However, in the (9,7) type transform for lossy coding, the rounding noise inside the 'non-separable' 3D structure has increased. This paper proposed a new 'non-separable' two-dimensional (2D) structure for integer implementation of a four-dimensional (4D) quadruple lifting WT. Since the order of the original lifting step is preserved, the total amount of the rounding noise observed in pixel values of the decoded image is significantly reduced, and the lossy coding performance for 4D input signal is increased.

Index Terms- Wavelet, lifting, 4D, rounding, coding

## **1. INTRODUCTION**

Recent advances in multidimensional image data have caused the study of the most suitable compression method for them to be very important. JPEG 2000, which has been approved as an international standard for the compression of still digital pictures, is currently wavelet-based unlike JPEG which use discrete cosine transform (DCT) [1]. JPEG 2000 restricts the user's choice to two wavelet transforms: Daubechies (9,7) for lossy compression [2] and the (5,3) LeGall wavelet [3], which has rational coefficients, for the reversible or lossless compression. It also specifies that these should be executed using the lifting scheme [4].

A class of 'separable' 2D wavelet transform based on JPEG 2000 has been broadly developed for various applications. Then, they are extended to 3D transform to be applied in various data such as the videos, hyper spectral images and medical volumetric data [5, 6, 7]. The 4D image compression based on 4D transforms, based on JPEG 2000 has recently been studied [8, 9]. However, all of them are

using the 'separable' 4D WT that has lots of rounding noise and lifting steps.

Therefore, the 'non-separable' 3D WT is proposed in [10] to overcome its limitations. Unfortunately, it was limited to 'double' lifting WT especially applied for lossless coding. Later, a 'non-separable' quadruple 3D DWT is proposed in [11] for lossy coding. But, unlike the double lifting WT, the variance of rounding noise is increased in the quadruple lifting WT even though the number of lifting steps is decreased.

This paper proposed a non-separable structure of 2D quadruple lifting structure for 4D input signal to deal with its picture quality degradation problem due to its integer implementation. It has an advantage that its output signals, apart from the rounding noise, are exactly the same as a conventional transform whose transfer function is expressed as a product of 1D transfer function.

#### 2. PREVIOUS WORKS

### 2.1 Separable 4D structure (Existing I)

Fig. 1 illustrates the (9,7) type separable 4D wavelet transform. In JPEG 2000 standard, the 1D processing is applied to a 4D signal in *x*, *y*, *z* and *t* dimension, where *x* and *y* denotes two spatial dimensions within a slice, the variable *z* denotes the third spatial dimension. But, the 'separable' 4D structure increases total number of lifting steps as well as total delay from input to output. This structure has 16 lifting steps and 192 rounding operators.

For a 4D input signal  $X(\mathbf{z})$ , the transform splits the input signal into 16 channels, as shown in Fig. 1. In JPEG 2000 standard, applying the 1<sup>st</sup> to 4<sup>th</sup> lifting steps in the spatial dimension, *x* with

 $\begin{bmatrix} A_1(z) & A_3(z) \\ A_2(z) & A_4(z) \end{bmatrix} = \begin{bmatrix} h_1(1+z_A^{+1}) & h_3(1+z_A^{+1}) \\ h_2(1+z_A^{-1}) & h_4(1+z_A^{-1}) \end{bmatrix}, \quad (1)$ the 5<sup>th</sup> to 8<sup>th</sup> lifting steps in the spatial dimension, y, the 9<sup>th</sup> to

the 5<sup>th</sup> to 8<sup>th</sup> lifting steps in the spatial dimension, y, the 9<sup>th</sup> to  $12^{th}$  lifting steps in the spatial dimension, z and the  $13^{th}$  to  $16^{th}$  lifting steps in temporal dimension, t.

It has long delay from input to output since it has 16 lifting steps. Since a lifting step must wait for a result of the previous lifting step, many steps mean long delay. As the 'separable' structure also has lots of rounding operations,



Fig. 1 'Separable' 4D structure for integer 4D WT (Existing I)

there are also lots of rounding noise inside the transform. Therefore, the 'non-separable' 3D structure is proposed in [10]. However, the non-separable 3D structure, when it is used in the 4D signal for (9,7) type of WT, the rounding noise inside it has greatly increased compared to the 'separable' 4D structure. Thus, the coding performance of it is also greatly reduced due to the rounding noise generated inside it.

# 2.2 Non-separable 3D Structure (Existing II)

Fig. 2 shows the non-separable 3D structure of integer WT for 4D input signal designed in (9,7) type of transform. In 1<sup>st</sup>

to  $4^{\text{th}}$  lifting steps, the 4D input signal, after it is being decomposed into 16 channels, it is applied in spatial dimension, *x* as in equation (1), and

$$\begin{bmatrix} X_{0000}^{(B)}(z) \\ X_{0001}^{(B)}(z) \\ \vdots \\ X_{1111}^{(B)}(z) \end{bmatrix} = \begin{bmatrix} R \left[ k^{-1} X_{0000}^{(A)}(z) \right] \\ R \left[ k^{-1} X_{0001}^{(A)}(z) \right] \\ \vdots \\ R \left[ k^{+1} X_{1111}^{(A)}(z) \right] \end{bmatrix},$$
(2)

Then, from 5<sup>th</sup> to 12<sup>th</sup> lifting steps, the signals are transformed simultaneously in the spatial dimension, y and z, and the temporal dimension, t, by using the non-separable 3D



Fig. 2 'Non-separable' 3D structure for 4D integer WT (Existing II)

structure. In this step, a 3D filtering with 3D memory accessing  $B_1(\mathbf{z})C_1(\mathbf{z})D_1(\mathbf{z})$  is used. R[] denotes the rounding operation on a signal value. Similarly, prediction of other channels is also independent. As a result, the total number of liftings steps and rounding operators of the 'non separable' structure is reduced from 16 to 12 and from 192 to 96, respectively, comparing to the 'separable' structure in Fig. 2. However, the quality of the decoded image is degraded by the rounding noise inside the transform in its integer implementation. This problem is overcome by the proposed method as explained in the next section.

#### **3. PROPOSED WORK**

## 3.1 Non-separable 2D Structure (Proposed)

To overcome the problems of increased rounding noise inside the transform, the non-separable 2D structure for 4D input signal is proposed. Fig. 3 illustrated the proposed method. Unlike the Existing I and Existing II method, it is composed of non-separable 2D structure only. The original order of lifting steps for each dimensions is also maintained in this structure.

The 1<sup>st</sup> and 2<sup>nd</sup> lifting steps are composed of 1D structures. Those are expressed as

$$\begin{cases} X_{1c_{1}c_{2}c_{3}}^{(1)}(z) = X_{1c_{1}c_{2}c_{3}}(z) + R[A_{1}X_{0c_{1}c_{2}c_{3}}(z)] \\ X_{0c_{1}c_{2}c_{3}}^{(1)}(z) = X_{0c_{1}c_{2}c_{3}}(z) + R[A_{2}X_{1c_{2}c_{3}}(z)] \end{cases}, (3)$$
  
where  $c_{1}, c_{2}, c_{3} \in \{0, 1\}.$ 

The 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> lifting steps are consisted of the non-

separable 2D structures. The same goes to the 5<sup>th</sup> to 11<sup>th</sup> lifting steps. Finally, the 12<sup>th</sup> and 13<sup>th</sup> lifting steps are composed of the separable 1D structure. For example, the 3<sup>rd</sup> lifting step is expressed as

$$X_{11c_{1}c_{2}}^{(2)}(\mathbf{z}) = X_{11c_{1}c_{2}}^{(1)}(\mathbf{z}) + R \left[ A_{3}B_{1}X_{00c_{1}c_{2}}^{(1)}(\mathbf{z}) + A_{3}X_{01c_{1}c_{2}}^{(1)}(\mathbf{z}) + B_{1}X_{10c_{1}c_{2}}^{(1)}(\mathbf{z}) \right], \qquad (4)$$

where  $c_1, c_2 \in \{0, 1\}$ .

Thus, the proposed method is a combination of nonseparable 2D structures and a product of separable 1D structure. It has fewer lifting steps than the Existing I method, which has reduced from 16 to 13, and fewer rounding operators, which is decreased from 192 to 96 compared to the Existing I structure.

### 3.2 Comparison of the structures

As illustrated in Fig. 1, the Existing I method is composed of the lifting steps  $A_1, A_2, ..., D_4$  which is expressed as

Separable 4D  $A_1A_2A_3A_4B_1B_2B_3B_4C_1C_2C_3C_4D_1D_2D_3D_4$  (5) In the Existing I method, firstly, the order of lifting steps for spatial dimension, x is remained the same, but the order of lifting steps for spatial dimension y and z, and the temporal dimension, t is changed as

Separable 3D'  $B_1B_2C_1C_2D_1D_2B_3B_4C_3C_4D_3D_4$  (6) And then a part of it is implemented in the non-separable 3D structure (Existing II). Non-separable 3D

$$A_1 A_2 A_3 A_4 (B_1 B_2 C_1 C_2 D_1 D_2)_{3D} (B_3 B_4 C_3 C_4 D_3 D_4)_{3D}$$
(7)

In order to maintain the original structure of Existing I



Fig. 3 'Non-separable' 2D structure for 4D integer WT (Proposed)

structure, the proposed structure is expressed as *Non-separable 2D* 

 $A_1A_2(A_3A_4B_1B_2)_{2D}(B_3B_4C_1C_2)_{2D}(C_3C_4D_1D_2)_{2D}D_3D_4$  (8) The detailed comparison of the structures is explained in Table 1.

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Structure	Lifting steps	Rounding operators	Memory accessing
Separable 4D (Existing I)	16	192	1D
Non-separable 3D (Existing II)	12	96	1D & 3D
Non-separable 2D (Proposed)	13	96	2D

#### **4. EXPERIMENTAL RESULTS**

In the following experiments, a set of 4D MRI data provided in [12] is tested. Results of WT for the data is shown in Fig. 4. The image has 50x224x224x16 pixels, which is expressed in  $x \times y \times z \times t$  sequence. Note that each frequency band signals are normalized to the range [0,255] for display purpose in this figure. In this paper, the variance of the noise in frequency domain is investigated.

The input signal is not limited to the MRI data, but also a random 4D input signal with  $128 \times 128 \times 32 \times 16$  pixels and a 4D auto-regressive (AR) model with  $256 \times 256 \times 32 \times 16$  pixels are included in our experiments. The AR auto-correlation coefficient is set to  $\rho = 0.9$ . The range of signal values is set to [-128,127], namely 8-bit depth.



Fig. 4 Results of wavelet transform of 4D MRI data.



Fig. 5 Average rounding noise for each data in each structure

Fig. 5 shows the average variance of rounding error in each frequency bands for random input signal, AR model and MRI data, respectively. Total amount of rounding errors between Proposed and Existing I structure for random input signal, AR model and MRI data is reduced to 39.22[%], 28.31[%] and 0.11[%], respectively. Comparing to the total amount of rounding errors, which has been greatly increased in Existing II structure, it has been reduced to 75.03[%], 74.82[%] and 86.61[%] for random input signal, AR model and MRI data, respectively.



Fig. 6 Coding performance in lossy mode

Fig. 6 illustrates the rate-distortion curve, which compares performance of the methods in lossy coding mode tested on AR model. The horizontal and vertical axis represents the entropy rate measured in bit rate per pixel [bpp] and PSNR of the reconstructed signal which is the quality of its, respectively. It was observed that under the same bitrate, the proposed method has a PSNR of 2.12 [dB] higher than the existing I method and 23.69 [dB] higher than the existing II method. Thus, the proposed structure performed the best among others.

#### 5. CONCLUSION

In this paper, the non-separable 2D structure for 4D input signal in quadruple lifting WT is introduced instead of the non-separable 3D structure. The total number of lifting steps and rounding operators is reduced compared to the existing separable 4D structure. However, the total amount of rounding noise due to integer implementation of signal values inside the transform has increased in existing non-separable 3D structure, due to the change of its original lifting structure. Therefore, by maintaining its original lifting structure, the proposed non-separable 2D structure has reduced the total amount of rounding noise inside it, as well as increasing the quality of reconstructed signal in lossy coding.

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