ARIMA-GARCH MODELING FOR EPILEPTIC SEIZURE PREDICTION

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ABSTRACT

This paper provides a procedure to analyze and model EEG (electroencephalogram) signal as a time series using ARIMA-GARCH to predict an epileptic attack. The heteroskedasticity of EEG signal is examined through the ARCH or GARCH, (Autoregressive conditional heteroskedasticity, Generalized autoregressive conditional heteroskedasticity) test. The best ARIMA-GARCH model in AIC sense is utilized to measure the volatility of the EEG from epileptic canine subjects, to forecast the future values of EEG. ARIMA-only model can perform prediction but the ARCH or GARCH model acting on the residuals of ARIMA attains a considerable improved forecast horizon. First, we estimate the best ARIMA model, then different orders of ARCH and GARCH modelings are examined to determine the best heteroskedastic model of the residuals of the mentioned ARIMA. Using the simulated conditional variance of selected ARCH or GARCH model, we suggest the procedure to predict the oncoming seizures. The results indicate that GARCH modeling determines the dynamic changes of variance well before the onset of seizure. It can be inferred that the prediction capability comes from the ability of the combined ARIMA-GARCH modeling to cover the heteroskedastic nature of EEG signal changes.

Index Terms— Epileptic seizure, prediction, ARIMA, ARCH and GARCH modeling, heteroskedasticity

1. INTRODUCTION

Epileptic seizure prediction is of a great importance due to its applications in the quality of lives of about one percent of world population suffering from epilepsy especially those with uncontrollable seizures with drugs [1, 2]. Different methods have been proposed to predict epileptic seizures; one has a glance at these methods, can find the variety of algorithms oriented towards longer prediction with high sensitivity and low false alarm [3, 4]. But only a few number of them are oriented to model mathematically the probable changes related to basic nature of seizures in EEG signals. Mathematical model not only can support the longer prediction, but also let to quantitatively know how the factors of model affect the prediction time, sensitivity and false alarm. A central hypothesis in the field of seizure prediction is based on the postulated existence of an altered brain state that is measurably different from a normal state, via EEG time series analyses. The earlier studies have been largely based on algorithmic time series analysis of EEG. A major drawback in the time series analysis methods is the high level of abstraction of the features of the signals from the mechanisms that lead to seizures [5], unless these features are representative of EEG *process*. The key point in this paper is to note that an epileptic seizure is associated by fluctuations of EEG process, in a sense, a future seizure is to occur if the ranges of EEG time series values experience consistent fluctuations, large changes are followed by large changes. This interpretation from the observations of EEG signals corresponds with the *volatility* notion. Associating with a time series, the volatility is a subjective term in contrast to the variance which is an objective term, i.e., given the data the variance can be computed, while volatility cannot be evaluated just by having the data. Volatility is associated with the process, and not with the data, volatility is not an observable characteristics of a process. Heteroskasticity modeling through ARCH or GARCH approach have proven effective in modeling non-stationary time series that exhibit time-varying volatility clustering, i.e. periods of high oscillations distributed with periods of relative calm [6, 7]. In predicting the epileptic seizures, a plausible mathematical modeling can be beneficial to represent the basic details related to the nature of seizure to better understand the underlying mechanisms of seizures [8]. From a mathematical point of view, any process in the nature, can be modeled by a differential equation [9]. Some of the proposed methods, computationally model the seizures, for instance, using differential equation [10], this is a motivation for seeking a statistical modeling of seizure incorporating the volatility associated with onset of seizure. This kind of modeling can be treated as a wide modeling arena, from micro and macro scale cellular models to EEG signal models [11]. In this paper, we use the heteroskedasticity property of EEG signal that is a characteristic phenomenon of seizure attack to mathematically model and predict the behavior of the EEG signal before the seizure onset.

The rest of this paper is organized as follows. Section 2 formulates the ARIMA-ARCH-GARCH modeling of EEG while Section 3 gives the main contribution of our proposed method in EEG prediction using ARCH or GARCH modeling of the residual models of ARIMA model to forecast the volatility of EEG conditional on the past observed information, in this section GARCH modeling of the residuals of ARIMA model of EEG via AIC analysis precedes in performance. Performance assessment via simulations on epileptic EEG records is presented in Section 4. Section 5 concludes the paper.

2. PROBLEM FORMULATION

A variety of models including autoregressive (AR), autoregressive moving average (ARMA), time-varying ARMA are proposed and performed on EEG signals, that the latest one is appeared better in modeling different characteristics of EEG signals like non-stationary nature. It is important to note that most of the earlier works assumed Gaussianity for the EEG signal, while recently non-Gaussian models have shown a better performance than the Gaussian models in the modeling of EEG signals [12]. In time series analysis, an ARIMA model is utilized as a generalization of ARMA model; usually for forecasting applications. ARIMA models are applied in some cases where data show evidence of non-stationarity, where an initial differencing step, corresponding to the integrated part of the model, is applied to reduce the non-stationarity. Assuming EEG time series is denoted by X_t where t is an integer index and theX_t are real numbers, the ARIMA(p; d; q) is described by

$$\left(1-\sum_{k=1}^{p} a_k L^k\right)(1-L)^d X_t = \left(1+\sum_{k=1}^{q} b_k L^k\right)\varepsilon_t \quad (1)$$

where L is the lag operator, the a_k are the parameters of the autoregressive part of the model, the b_k are the parameters of the moving average part and the ε_t are error terms. The error terms ε_t are generally assumed to be independent, identically distributed variables sampled from a normal distribution with zero mean. Here p is the AR, and q the order of MA, and d is the order of differencing, respectively. Differencing has to be done with care as it may unnecessarily introduce correlation; for example if X_t is correlated, $L\{X_t\} = X_t - X_{t-1}$ is uncorrelated [13, 14]. The first step in modeling time index EEG data by (1) is to convert the non-stationary time series to almost stationary one, the series is lagged 1 step and subtracted from the original series. This is important for the fact that the non-stationary EEG time series are erratic and unpredictable while stationarity results in a mean-reverting series, i.e, the series fluctuates around a constant mean with constant variance. The conversion to stationarity is evident in the autocorrelation and partial autocorrelation which are the core of ARIMA model. Also, in fitting ARIMA model, the idea of parsimony plays an important role to reduce the numerical errors that can be introduced into the model and hence the standard deviation of ARIMA, the parsimonious(best) ARIMA model is obtained via AIC_C(corrected AIC) [15].

2.1. GARCH model

Almost all natural time series possess time-varying variance; in order to predict the future ranges of values of a process, volatility which is an unobservable quantity or quality of a process is usually being considered by formal models such as ARCH, GARCH [6], or stochastic volatility; therefore, it can be inferred that modeling volatility plays an important role in in predicting approachning fluctuations in a time series. In ARCH process, conditional variance changes over time as a function of past errors on conditional variance constant. If the changes occur simultaneously with more flexible lag structure, the process is treated as a GARCH process. For a given time series, y_t, we can model the time series with the GARCH(p,q), if $\text{E}\{\text{y}_t\} = 0$ and

$$y_t = \sqrt{h_t}\epsilon_t \quad (2)$$

$$h_t = a_0 + \sum_{i=1}^{p} a_i {y^2}_{t-i} + \sum_{j=1}^{q} b_j h_{t-j} \quad (3)$$

where h_t [1] is the conditional variance of $\{\text{y}_t\}$, $a_0 > 0$, $a_i \geq 0$, $b_j \geq 0$ and $\{\epsilon_t\}$ is a sequence of independent identically distributed zero mean with variance one random variables, in practice, $\{\epsilon_t\}$ is often assumed to be a standard normal random variate [6]. From (1), it can be seen that ARIMA is a linear model and although provides best linear forecast for the EEG series, it cannot reflect nonlinear characteristics, i.e. volatility, of EEG data. Consequently, ARCH or GARCH method, with an ability to model volatility, comes into play. If there are clusters of volatility, i.e. episode of attack is to start, ARCH or GARCH should be used to model the volatility of the series to reflect more recent changes and fluctuations in the EEG series, in this paper, ARCH or GARCH act on the residuals of the ARIMA process of EEG. Finally, autocorrelation and partial correlation of the squared residuals will help to confirm if the residuals (noise term) are not independent and can be predicted, this fact is demonstrated in Fig. (1). Statistically speaking, we note that a linear or non-linear approach cannot predict a white noise, but, a non-linear approach such as ARCH or GARCH can be successfully applied to predicting an almost white noise. If the residuals of ARIMA-alone modeled EEG are almost a white noise, they cannot be used to predict the onset of an attack, but, using a GARCH model, i.e., a non-linear modeling, on the residuals of ARIMA possible large fluctuations in volatility can be predicted, hence, the onset of an attack. For checking ARCH(heteroskedasticity) effects we include ARCH(0) or whether the residuals are independent.

[1]Volatility is the conditional standard deviation of a time series.

3. SEIZURE PREDICTION METHODOLOGY

In this section, after introducing the dataset, the method for modeling the EEG signals, is proposed. The stepwise description is as follows: First the data segment is preprocessed. Then using the AIC, best ARIMA model is specified by the orders of p , d and q. Then the best model will be determined and the residuals of the fitted model, is extracted. Next we fit different orders of ARCH or GARCH models to the residuals and again use the AIC to evaluate the goodness of fit for the models. After selecting the best ARCH or GARCH model, using the conditional variance of the model, the pre-seizure dynamic changes is determined. After all, inspired by the methodology of "1000-period future horizon" in econometric models [16], we adopt the "5000-period future horizon" in epilepsy.

3.1. Dataset

A dataset [2] from American epilepsy society seizure prediction challenge is used to check the modeling. Pre-seizure and non-seizures EEG signals for five canine subjects are available in the dataset. The data is organized into ten minute EEG clips labeled "Preictal" for pre-seizure data segments, or "Interictal" for non-seizure data segments. Preictal data segments are provided covering 65 to 5 minutes prior to the seizure onset.

3.2. Preprocessing and ARIMA models

Data segments for canine subjects are downsampled and normalized between values -1 and 1. Then, different ARIMA(p, d, q) models are fitted and the corresponding AIC_C are computed as in Table 1. Since ARIMA is a linear modeling, when the signal is modeled by ARIMA, we expect that linear characteristics of the signal is being manifested in the model and therefore what remained is the non-linear behavior in the residuals. In the next step, this non-linearity is captured by using the non-linear modeling through ARCH or GARCH. In Table 1, ARIMA modeling on a typical epileptic EEG data is performed where ARIMA(2,1,2) achieves the best AIC_C.

3.3. ARCH or GARCH models

We note that a previous work [12] ARCH test is used to verify the ARCH or GARCH the existence of heteroskedasticity in the data, in Table 2 we also obtain ARCH(0) to verify the presence of heteroskedasticity in EEG data too. AIC_C is our criterion to find the best order of the model, furthermore, using the determined ARIMA-GARCH model for the EEG signal, the simulated conditional variance of this model is used to forecast the seizure using

[2]https://www.kaggle.com/c/seizure-prediction/data

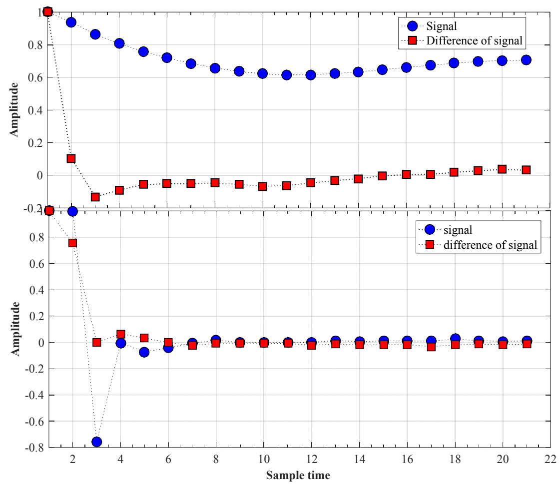

Fig. 1: Autocorrelation (top) and partial correlation (down), EEG is a highly correlated signal, differencing results in an almost uncorrelatedness.

Table 1: Parameters of the ARIMA models; N is the EEG signal sample size.

Model	N	Log likelihood	AIC$_C$
ARIMA(1,1,1)	143860	2.4821e+05	-4.9643e+05
ARIMA(2,1,1)	143860	2.4832e+05	-4.9663e+05
ARIMA(2,1,2)	143860	2.4828e+05	-4.9685e+05
ARIMA(1,1,2)	143860	2.4834e+05	-4.9668e+05
ARIMA(2,2,2)	143860	2.4612e+05	-4.9224e+05
ARIMA(2,2,1)	143860	2.3593e+05	-4.7186e+05
ARIMA(1,2,2)	143860	2.4567e+05	-4.9133e+05
ARIMA(1,2,1)	143860	2.3142e+05	-4.6283e+05

the pre-seizure the simulated variance of the ARIMA-GARCH. As it is shown, in Fig. 3 the pre-seizure changes is detected (19.24+5) minutes before the onset.

4. RESULTS AND PERFORMANCE EVALUATION

4.1. Prediction results

In this section, the results from proposed modeling, is presented. At the first, when we intend to specify the best ARIMA model, difference the signal and compare the correlation and partial correlation of signal and difference of signal to verify the true use of the differencing. Next AIC is computed to select the best orders of p, d and q of ARIMA. In Table 1, loglikelihood and corresponding AIC$_C$, for each model is computed and the best model for the this typical segment of signal is determined as ARIMA(2,1,2) but also for other segments of preictal signal, the best orders of model is calculated, and we conclude that ARIMA(2,1,2) is appropriate. At the next step, ARCH and GARCH modeling are implemented on the residuals of ARIMA(2,1,2), and the best orders ARCH or GARCH are determined in Table 1. In Fig 3, the resultant simulated conditional variance for a the segment, covering sixty five minutes prior to seizure onset is demonstrated. Having a look at this simulated figure, it can be inferred that the sudden pre-seizure fluctuations have started up to several minutes before the attack onset, specifically, in this figure, the prediction time is 24.24 minutes before the seizure onset.

Table 2: Parameters of the ARCH and GARCH models, GARCH(1,1) attains the smallest AIC$_C$.

Model	N	Log likelihood	AIC$_C$
ARCH(0)	143860	2.4860e+05	-4.9721e+05
ARCH(1)	143860	2.5283e+05	-5.0567e+05
ARCH(2)	143860	2.5425e+05	-5.0849e+05
ARCH(3)	143860	2.5514e+05	-5.1028e+05
ARCH(4)	143860	2.5563e+05	-5.1126e+05
ARCH(5)	143860	2.5594e+05	-5.1187e+05
ARCH(6)	143860	2.5619e+05	-5.1238e+05
ARCH(7)	143860	2.5633e+05	-5.1267e+05
ARCH(8)	143860	2.5645e+05	-5.1291e+05
ARCH(9)	143860	2.5656e+05	-5.1312e+05
GARCH(1,1)	143860	2.5755e+05	-5.1509e+05

4.2. 5000-period future horizon

In econometric modeling, there is a forecasting methodology termed as "1000-period future horizon;" under this approach a forecast to compute the expected conditional variance for 1000-period future horizon is estimated. [7, 17]. In order to do that, they use the observed returns and inferred residuals and conditional variances as presample data. Adopting this methodology in epileptic seizure prediction, we define and propose a 5000-period future horizon. This horizon, is checked and implemented for canine subjects. In Fig. 2, three 5000-period future horizons are depicted repectively for ten, twenty five and forty minutes before the seizure onset. Having a look at these figures in the time axis approaching to seizure, we can find out that the 95 percent confidence interval for signal, is getting wider and wider. Respectively, for forty minutes, twenty five minutes and ten minutes before the onset of seizure, this 95% interval is 0.1, 0.14 and 0.18.

4.3. Performance evaluation

We implemented our approach on several segments of pre-seizure and non-seizure EEG data from five subjects, including 18 seizure attacks and several hours of non-seizure data segments. In seizure prediction, two of the most important characteristic for evaluating the performance, are the seizure prediction horizon(SPH) and the sensitivity. SPH is the time interval between the seizure alarm and the onset of an epileptic seizure. The sensitivity is defined as the ratio of the number of correct predictions to all registered seizures [18]. In this paper, we were able to obtain the average SPH of 29.44 minutes and the sensitivity of 100% for the dataset of five canine subjects. Comparing to the recent related research on seizure prediction, by the approach presented in this paper, an improved extended time(extended SPH) of sensitivity to the onset of an attack is reached. Furthermore, it should be noted that almost, all other existing real time or nonreal time seizure prediction methods provide an alarm based on the

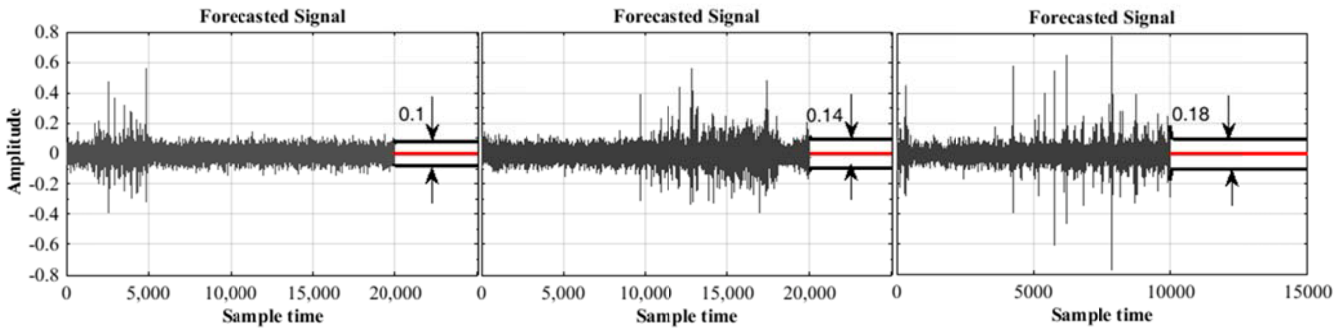

Fig. 2: 5000-period future horizon; figures from left, are respectively for 40 min, 25 min and 10 min before seizure onset

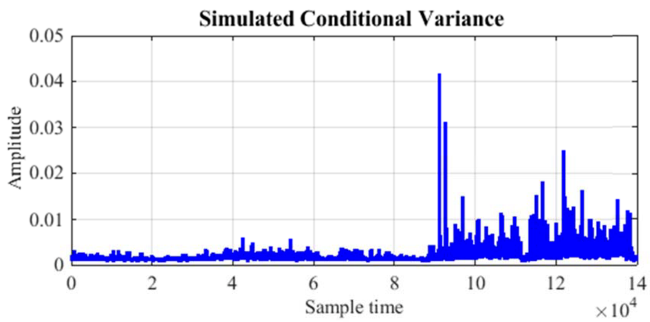

Fig. 3: Simulated conditional variance; pre-seizure variations can be observed and utilized for prediction.

present and past values of EEG, but, in this paper, based on ARIMA-GARCH modeling, we introduce a new method which is based on the present and past values of EEG to simulate(predict) the future values according to 5000-period future horizon, for the typical EEG scenario in Fig 2 and Fig 3 ARIMA(2,1,2)-GARCH(1,1) model used in the 5000-period future horizon follows below

$$\begin{aligned} h_t + X_t - X_{t-1} \quad = \quad & (-0.4360)(X_{t-1} - X_{t-2}) \\ & +(0.3612)(X_{t-2} - X_{t-3}) \\ & +(-0.1294)\epsilon_{t-1} + (-0.8702)\epsilon_{t-2} \\ & +(1.40606e - 05) \\ & +(0.9548)\epsilon_{t-1}^2 + (0.0375)h_{t-1} \end{aligned}$$

ARIMA(2,1,2) model focuses on EEG time series linearly. ARIMA(2,1,2) possess unconditional variance which is constant. GARCH(1,1) is to measure the volatility of the series which is tantamount to model the noise term of ARIMA(2,1,2); i.e, GARCH(1,1) incorporates the noise term of ARIMA(2,1,2) to forecast the future EEG values of EEG based on 5000-period future horizon.

5. CONCLUSION

In this paper, with an emphasis on the heteroskedasticity of EEG signal and ARIMA-GARCH modeling provides a mathematical modeling of the EEG signal fluctuations corresponding to epileptic seizures. Accordingly, first ARIMA, as a linear modeling is performed on the signal and then ARCH or GARCH as a non-linear modeling, is addressed to model and present the residuals of the ARIMA model. Finally using the formulation of model, simulated conditional variance , is depicted as used to predict the seizure. A new methodology termed as " 5000-period future horizon" is also proposed to predict the future EEG signal values for seizure prediction.

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