MULTIPLE-INPUT MULTIPLE-OUTPUT (MIMO) MRI: AN EFFICIENT PULSE DESIGN ALGORITHM TO COMBINE PARALLEL EXCITATION AND PARALLEL IMAGING

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ABSTRACT

Magnetic resonance imaging (MRI) plays a critical role in medicine today. Over the last few years, there has been an interest in techniques that use multiple coils on one side of the imaging system such as parallel excitation or parallel imaging to improve performance in high-field MRI. In this paper, we explore the use of multiple coils at *both* the transmit and receive sides for enhanced imaging. We term this a multiple-input multiple-output (MIMO) MRI system. We use tools from the well-developed MIMO communication models and propose a novel radio-frequency (RF) pulse design method for multiple coils. We show that our MIMO MRI techniques can provide an improved signal-to-noise ratio (SNR) in the reconstructed image.

Index Terms— MRI, MIMO, parallel excitation, parallel imaging, RF pulse design

1. INTRODUCTION

Magnetic resonance imaging (MRI) systems are now common place for their diagnostic and investigative benefits. To improve performance, multiple coil techniques have been developed and deployed for MRI. The most popular multiple coil techniques are currently used in MRI receivers to enable *parallel imaging* (e.g., [1], [2]). Parallel imaging is a robust method of accelerating the acquisition of MRI data and has impacted many possible applications of MR imaging. Parallel imaging techniques [3] work by using multiple receive coils to obtain a reduction in the amount of k-space data required for a fixed imaging quality. There is also work on improving imaging by using multiple independent coils at the transmitter under the framework of *parallel excitation* (e.g., [4], [5]).

Despite the work on using multiple coils at the transmit side or the receive side, there has been little research exploring techniques optimized for simultaneous multiple coil processing at both the transmitter and receiver. There has been some work on transceiver multi-coil design for both transmission and reception (e.g., [6], [7]), suggesting that the transceive coil design can be effectively used to enable transmit and/or receive-only parallel imaging techniques while maintaining high signal-to-noise ratio (SNR) sensitivity. Other work proposed a mean-squared error (MSE)-based excitation pattern design for parallel transmit and receive MR image reconstruction [8]. We show that an alternative novel method can be used to enhance performance of MRI systems using parallel excitation and imaging, which needs the multiple-input multiple-output (MIMO) techniques.

In this paper we address the problem of multi-coil radio-frequency (RF) pulse design for MIMO MRI. This RF pulse design method is proposed based on the idea of the spatial domain method of [9], [10]. Our method is designed specifically for the multiple transmit and receive coils scenario using an SNR cost function. The RF pulse design method optimizes the parallel excitation errors as well as the SNR of the parallel reconstructed image. This method has several advantages as it allows for region of interest (ROI) specification and gives control over the receive side of the MRI system. Furthermore, it allows for spatially-varying excitation error weighting and is adapted to the main magnetic field inhomogeneity. Our pulse design method is analogous to the transmit signal design techniques used for MIMO wireless communication systems. The present work outlines the theoretical framework of this MIMO MRI system and all derivations were made under the assumption of the validity of the reciprocity principle, using the small-tip-angle approximation. Computer simulations are employed to verify this theoretical study.

2. SYSTEM MODEL AND METHODS

We consider an MRI system, which has a transmitting coil array with N_t elements and a receiving coil array with N_r elements. Most prior work focuses on the case where $\min(N_t, N_r) = 1$. However, we are explicitly interested in systems optimized for the case when $\min(N_t, N_r) > 1$, but our work can also be applied when $\min(N_t, N_r) = 1$.

2.1. RF Modeling for Parallel Excitation

Parallel excitation uses multiple parallel coils transmitting different RF excitations. The complex transverse magnetiza-

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tion $M(\boldsymbol{r})$ ($\boldsymbol{r} = (x, y, z)$) generated by a multidimensional spatially selective excitation pulse from a single, homogeneous transmit coil can be described by [9] [11]

$$M(\boldsymbol{r}) = i\gamma M_0 \int_0^T b_1(t) e^{i\gamma \Delta B_0(\boldsymbol{r})(T-t)} e^{i\boldsymbol{r}\cdot\boldsymbol{k}(t)} dt, \quad (1)$$

where $b_1(t)$ is the complex RF pulse (which is a one-dimensional temporal function), γ is the gyromagnetic ratio, T is the pulse length, and $\boldsymbol{k}(t)$ represents the 3-dimensional k-space trajectory $(k_x(t), k_y(t), k_z(t))$, $e^{i\gamma\Delta B_0(\boldsymbol{r})(T-t)}$ represents the phase accured due to the outer field deviation $\Delta B_0(\boldsymbol{r})$, and M_0 is the magnetization at the equilibrium state.

Therefore, when an array of N_t transmit coils with different sensitivity profiles are used, the resulting excitation pattern can be expressed as a linear, sensitivity-weighted combination of virtual single-element patterns giving

$$M_{res}(\boldsymbol{r}) = \sum_{n=1}^{N_t} S_n(\boldsymbol{r}) M_n(\boldsymbol{r}), \quad (2)$$

with

$$M_n(\boldsymbol{r}) = i\gamma M_0 \int_0^T b_{1,n}(t) e^{i\gamma \Delta B_0(\boldsymbol{r})(T-t)} e^{i\boldsymbol{r}\cdot\boldsymbol{k}(t)} dt. \quad (3)$$

The RF field $B_{1,n}(\boldsymbol{r})$ is induced by the n^{th} RF pulse $b_{1,n}(t)$, and it is defined as

$$B_{1,n}(\boldsymbol{r}) = S_n(\boldsymbol{r}) \int_0^T b_{1,n}(t) e^{i\gamma \Delta B_0(\boldsymbol{r})(T-t)} dt. \quad (4)$$

If we assume that B_0 field to be parallel to the $z-$axis, then the $z-$component of B_1 field does not take part in the excitation. The transmitting and receiving fields ($B^+_{1,n}(\boldsymbol{r})$, $B^-_{1,n}(\boldsymbol{r})$, respectively [12]) can be described in terms of the *x* and *y* components (B_{1,n_x} and B_{1,n_y}) of the B_1 field ('*' is the conjugate operator)

$$B^+_{1,n}(\boldsymbol{r}) = \frac{(B_{1,n_x}(\boldsymbol{r}) + iB_{1,n_y}(\boldsymbol{r}))}{2}, \quad (5)$$

$$B^-_{1,n}(\boldsymbol{r}) = \frac{(B_{1,n_x}(\boldsymbol{r}) + iB_{1,n_y}(\boldsymbol{r}))^*}{2}. \quad (6)$$

It is possible to construct a linear combination of the transmit coils to form an aggregate pattern [9]. By discretizing time and space, (2) can be written as

$$\boldsymbol{m} = \begin{bmatrix} \boldsymbol{D}_1\boldsymbol{A} & \ldots & \boldsymbol{D}_n\boldsymbol{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \vdots \\ \boldsymbol{b}_n \end{bmatrix} = \boldsymbol{A}_{full}\boldsymbol{b}_{full}, \quad (7)$$

where $\boldsymbol{m}$ is the vector of spatial samples of the aggregate excitation pattern, $\boldsymbol{D}_n = diag(S_n(\boldsymbol{r}))$ is the diagonal matrix containing samples of the sensitivity pattern of coil n, and $\boldsymbol{b}_n$ is a vector of RF pulse samples for coil n. The system matrix $\boldsymbol{A}$ has (j, k) entry

$$a_{jk} = i\gamma M_0 \Delta t e^{i\gamma\Delta B_0(\boldsymbol{r}_j)(T-t_k)} e^{i\boldsymbol{r}_j\cdot\boldsymbol{k}(t_k)}.$$

One approach to the design of the RF pulses is to minimize a distortion from some target excitation. This corresponds to the optimization [9]

$$\hat{\boldsymbol{b}}_{full} = \underset{\boldsymbol{b}_{full}}{\operatorname{argmin}}\{||\boldsymbol{A}_{full}\boldsymbol{b}_{full} - \boldsymbol{m}||^2\}. \quad (8)$$

This minimization problem can be solved using the pseudoinverse or conjugate gradient (CG) method to obtain the optimal RF pulses subject to one or more waveform generation constraints [10].

2.2. Parallel Imaging Overview

Parallel imaging, which uses multiple receive coils for parallel reception, enables faster imaging through k-space (i.e., frequency domain) undersampling. Most popular parallel imaging systems, e.g., SiMultaneous Acquisition of Spatial Harmonic (SMASH) [1], SENSitivity-Encoding (SENSE) [2], and Generalized Auto-calibrating Partially Parallel Acquisition (GRAPPA) [13], use under-sampled k-space data from each of the N_r receive coils.

We assume the sensitivity profiles are not uniform for each receiving coil, and thus we will use a SENSE method to reconstruct the image based on the receiving sensitivity profiles, the B_1^- field. The ideal image combining procedure in the receive array of coils is shown in Fig. 1.

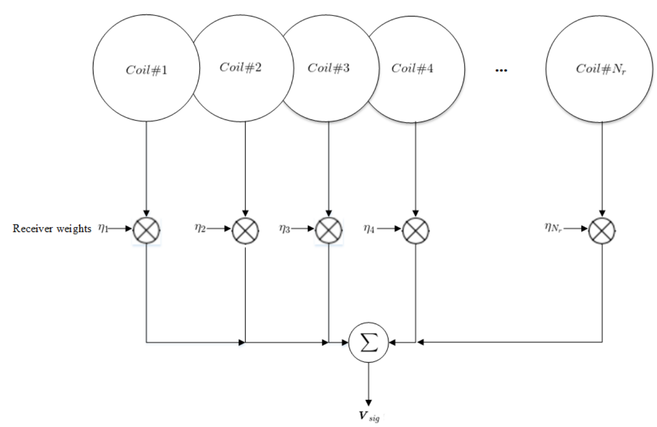

Fig. 1. N_r receive coils with outputs phase shifted and summed through a set of transformers. η_j is the receiver weight of j^{th} receiving coil.

The SNR is defined as the ratio of the nuclear magnetic resonance (NMR) signal to the root-mean-square (RMS) noise voltage. Therefore, the general definition of SNR in MRI is commonly defined as [14]

$$SNR = \frac{|\boldsymbol{V}_{sig}|}{\sqrt{4k_B T \Delta f R_{total}}}, \quad (9)$$

where Δf is the receiver bandwidth, T is the absolute temperature in kelvin, and k_B is Boltzmann's constant in Joules per Kelvin. $\boldsymbol{V}_{sig}$ is the complex electromotive force (emf) of the NMR voltage output, and it is a $N_r \times N_s$ matrix, where N_r is the number of receive coils and N_s is the total number of voxels in the selected ROI. For each voxel voltage of each receiving coil, its NMR signal voltage is defined as the emf induced at that specific location, which can be further regarded as a certain function of the expectation of all the instantaneous NMR signal voltages.

The total noise R_{total} at each location $\boldsymbol{r}$ can be defined as

$$R_{total}(\boldsymbol{r}) = \sum_{j=1}^{N_r}\sum_{k=1}^{N_r} \eta_j(\boldsymbol{r})^* \eta_k(\boldsymbol{r}) R_{jk} = \boldsymbol{\eta}(\boldsymbol{r})^H \boldsymbol{R}\boldsymbol{\eta}(\boldsymbol{r}), \quad (10)$$

where η_j is the receiver weight of the j^{th} receiving coil and a superscript 'H' denotes conjugate transposition. R_{jk} represents the correlation between the noise of the j^{th} and k^{th} coils.. In reality, R_{jk} depends on the electric field induced in j^{th} and k^{th} receiving coils. However in the simulation, we assume that the thermal noises are well modeled as a complex Gaussian vector, satisfying $E[\mathfrak{n}] = 0$; $E[\mathfrak{n}\mathfrak{n}^{\mathrm{H}}] = \boldsymbol{R}$. We use the covariance matrix $\boldsymbol{R}$ as the noise resistance matrix in (10).

3. MIMO MRI TRANSMIT AND RECEIVE OPTIMIZATION

Mathematically, by ignoring noises, the NMR voltage output at the receive coils can be described as [14]

$$\boldsymbol{V}_{sig}(\boldsymbol{r}) \propto M_{res}(\boldsymbol{r}) \sum_{n=1}^{N_r} \eta_n^*(\boldsymbol{r}) B_{1,n}^-(\boldsymbol{r}), \quad (11)$$

where $M_{res}(\boldsymbol{r})$ is the induced magnetization at position $\boldsymbol{r}$, and $B_{1,n}^-(\boldsymbol{r})$ is the receiving field of the n^{th} coil.

For simplicity, we ignore the constants which remain unchanged from voxel to voxel in the SNR, giving a *coil signal-to-noise ratio* SNR_c, defining as

$$SNR_c(\boldsymbol{r}) = \frac{\left| M_{res}(\boldsymbol{r}) \sum_{n=1}^{N_r} \eta_n^*(\boldsymbol{r}) B_{1,n}^-(\boldsymbol{r}) \right|}{\sqrt{\sum_{j=1}^{N_r}\sum_{k=1}^{N_r} \eta_j^*(\boldsymbol{r})\eta_k(\boldsymbol{r}) R_{jk}}}, \quad (12)$$

where $B_{1,n}^-(\boldsymbol{r})$ is defined in (6).

In this paper, our goal is to maximize the SNR in (12) for enhanced reconstruction subject to maintaining a transverse magnetic field that is close to a target field distribution. SNR_c is a useful metric for RF pulse design algorithms because it preserves the image quality of the reconstructed MR images, and it further gives the SNR of each voxel in the image. Since it is not convenient to apply the SNR_c matrix within the optimization, we further define a global SNR (denoted as $gSNR$) as the mean value of SNR_c of the specific ROI so that it can be used within the optimization. The definition will be given later in this section.

An efficient RF pulse design algorithm is produced by optimizing the parallel excitation errors and the $gSNR$ term simultaneously. In the optimization, we first take the square of the SNR_c, then (12) will become

$$SNR_c^2(\boldsymbol{r}) = \frac{|M_{res}(\boldsymbol{r})|^2 \sum_{j=1}^{N_r}\sum_{k=1}^{N_r} \eta_j^*(\boldsymbol{r})\eta_k(\boldsymbol{r}) B_{1,j}^-(\boldsymbol{r}) B_{1,k}^+(\boldsymbol{r})}{\sum_{j=1}^{N_r}\sum_{k=1}^{N_r} \eta_j^*(\boldsymbol{r})\eta_k(\boldsymbol{r}) R_{jk}}. \quad (13)$$

Next, we discretize the term SNR_c^2 to an $N_s \times 1$ vector. For each voxel in the selected ROI, its corresponding SNR is given as

$$||\boldsymbol{SNR}_c(\boldsymbol{r})||^2 = \frac{|\boldsymbol{m}(\boldsymbol{r})|^2 \, \boldsymbol{\eta}(\boldsymbol{r})^{\mathrm{H}} \boldsymbol{B}_1^-(\boldsymbol{r}) \boldsymbol{B}_1^-(\boldsymbol{r})^{\mathrm{H}} \boldsymbol{\eta}(\boldsymbol{r})}{\boldsymbol{\eta}(\boldsymbol{r})^{\mathrm{H}} \boldsymbol{R} \boldsymbol{\eta}(\boldsymbol{r})}, \quad (14)$$

where the total noise resistance matrix $\boldsymbol{R}$ has (j,k) entries R_{jk}. Additionally, $\boldsymbol{B}_1^-(\boldsymbol{r})$ is defined as an $N_r \times 1$ vector that contains the B_1^- field at a specific position $\boldsymbol{r}$ from all receiving coils, which is given by

$$\boldsymbol{B}_1^-(\boldsymbol{r}) = (\boldsymbol{A}'(\boldsymbol{r}) \boldsymbol{b}_{full}^{\mathrm{H}})^\top. \quad (15)$$

where the $N_s \times N_p$ (N_p is the total number of RF pulse samples) system matrix $\boldsymbol{A}'$ has (j,k) entries

$$a'_{jk} = \frac{1}{2} S_n^*(\boldsymbol{r}_j) e^{-i\gamma \Delta B_0(\boldsymbol{r}_j)(T - t_k)}. \quad (16)$$

Therefore, $\boldsymbol{A}'(\boldsymbol{r})$ represents the total number of RF pulse samples contributes to a specific location $\boldsymbol{r}$. The $\boldsymbol{b}_n^*$ is the vector that contains the conjugate of the RF pulse samples in n^{th} receiving coil.

We can find the optimal SNR by finding when the gradient of $||\boldsymbol{SNR}_c||^2$ with respect to $\boldsymbol{\eta}(\boldsymbol{r})$ is zero. Solving for the $\boldsymbol{\eta}$'s yield the optimum weights for combining magitude images and can be easily controlled by the user. Using the optimum receive weights in [15],[16], we can substitute the optimal $\boldsymbol{\eta}$ into (14), resulting in

$$||\boldsymbol{SNR}_c(\boldsymbol{r})||^2 = |\boldsymbol{m}(\boldsymbol{r})|^2 (\boldsymbol{A}'(\boldsymbol{r}) \boldsymbol{b}_{full}^{\mathrm{H}})^* \boldsymbol{R}^{-1} (\boldsymbol{A}'(\boldsymbol{r}) \boldsymbol{b}_{full}^{\mathrm{H}})^\top. \quad (17)$$

Since $gSNR := (1/N_s) \cdot \sum_{\boldsymbol{r}} \boldsymbol{SNR}_c(\boldsymbol{r})$, if we combine the $gSNR$ maximization for parallel imaging and the RF modeling method for parallel excitation, we obtain the following model for multi-coil RF pulse designing on both MR transmitting and receiving sides:

$$\begin{aligned} &\text{maximize } \frac{1}{N_s} \cdot \sum_{\boldsymbol{r}} ||\boldsymbol{SNR}_c(\boldsymbol{r})||, \\ &\text{subject to } : ||\boldsymbol{A}_{full} \boldsymbol{b}_{full} - \boldsymbol{m}||^2 \le \epsilon_{max}. \end{aligned}$$

4. SIMULATIONS AND RESULTS

We performed Bloch equation simulations to test the RF pulse design methods. For the excitation, simulations were performed over a 64×64 transverse grid covering a $24cm \times 24cm$ region, assuming that all simulations were carried out by an eight element transceive array (e.g., [4], [17]). Transceive sensitivity patterns were obtained using finite-difference time-domain simulation [10] of the array at 3.0T, and the magnitude of sensitivity profiles of eight-channel parallel excitation were obtained from [17]. The desired excitation pattern, shown in Fig. 2, is a smoothed $radius \approx 4.5cm$ circle block, whose peak was scaled to $\frac{1}{18}\pi$, corresponding to a $10°$ tip angle. The field correction was applied so as to minimize the side effects of the nonuniform B_0 magnetic field.

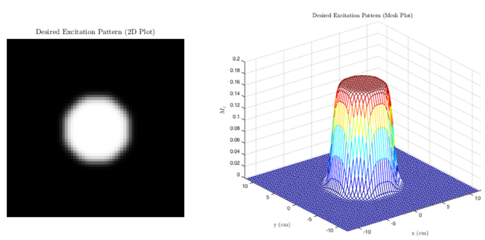

Fig. 2. Desired excitation pattern used in the simulations, which was a smoothed $radius \approx 4.5cm$ circle block, whose peak was scaled to $\frac{1}{18}\pi$, correponding to $10°$ tip angle.

We designed the RF pulses using echo-planar (EP) excitation k-space trajectories. EP trajectories were designed using $4G/cm$ maximum gradient amplitude and $15G/cm/ms$ maximum slew rate [18]. For these trajectories, acceleration was achieved via undersampling in the phase-encoded dimension. The sampling period for all RF pulses was $4\mu s$.

In the SNR-maximization process, we designed the RF pulses to excite the desired excitation pattern. The algorithm alternates between the small-tip-angle approximations to update the $gSNR$ and CG iterations to design the $\boldsymbol{b}_{full}$. The finalized RF pulses were designed not only to reduce the erroneous excitations, but also to maximize the SNR of the final reconstructed image.

The priority of the RF pulse design problem is to better simulate the transverse magnetization pattern, therefore improving the excitation accuracy. The desired cylinder in Fig. 2 indicates the target region and its excitation shape. With the self-designed RF pulses, we can produce a transverse magnetization pattern that is really close to the target excitation pattern, suggesting the high excitation accuracy (see Fig. 3). If we use the designed RF pulses to excite a test image of an analytical brain phantom [19] (see Fig. 4(a)), then the reconstructed image is analogous to the simulated result (see Fig. 4(b)). The reconstructed image is a bit blurry because of the low image resolution, however it states that the SNR of the final reconstructed image can be estimated and optimized using this RF design method. Note that in this experiment, full k-space sampling was applied in the transmit side, and 8 tranceive coils were used in the simulated MR system.

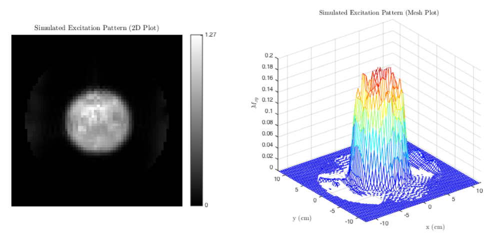

Fig. 3. Simulated voltage output $\boldsymbol{V}_{sig}$ produced by the designed RF pulses with the small-tip-angle regime, the resulting pattern can be regarded as the M_{xy} excitation pattern under the MIMO MRI system. *Left* shows the 2D plot while the *right* shows the mesh plot of the voltage output.

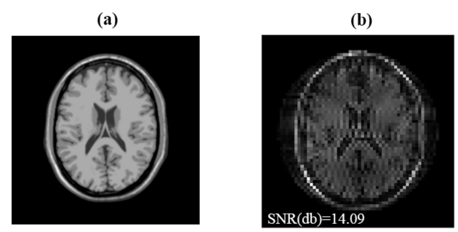

Fig. 4. (a): The test analytical brain phantom. (b): Reconstructed MR images after the obtained $gSNR$ converges.

5. CONCLUSION

In this work we have proposed a novel theoretical framework for the MIMO MRI model, developed an RF pulse designing algorithm, and verified it with computer simulations. This novel RF pulse design method is formulated as an optimization problem in which the resulting RF pulse is the optimizer of several quadratic cost functions comprising SNR, weighted error terms, and regularization terms.

The MIMO MRI framework was built based on the ideas of the spatial domain method [9] and the transmit SENSE method [20]. This method allows for a spatially varying excitation error weighting and leads to a good excitation accuracy even at high acceleration factors. Traditional RF pulse design methods target at the ROI specification and the B_1 field inhomogeniety compensation. This method maintains these two advantages in the MR transmit side, and allows the conventional B_1^+, B_1^- field design while maintaining a high SNR sensitivity in the reconstructed image, if more than one coil is used in both MR transmit and receive sides.

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