COMPRESSED SENSING MRI USING DOUBLE SPARSITY WITH ADDITIONAL TRAINING IMAGES

Chenmin Tang[⋆] *Norihito Inamuro*[⋆] *Takashi Ijiri*[⋄] *Akira Hirabayashi*[⋄†]

[⋆] Ritsumeikan University
Graduate School of Info. Science & Eng.
Kusatsu, Shiga 525-8577, Japan

[⋄] Ritsumeikan University
Graduate School of Info. Science & Eng.
Kusatsu, Shiga 525-8577, Japan

ABSTRACT

The compressed sensing using dictionary learning has led to state-of-the-art results for magnetic resonance imaging (MRI) reconstruction from highly under-sampled measurements. Dictionary learning had been considered time-consuming especially when the patch size or the number of training patches is large. Recently, double sparsity model and online dictionary learning algorithm were proposed to obtain dictionaries with much less computational time. In this paper, we propose an efficient MRI reconstruction method by adopting the double sparsity model with the online dictionary learning method. Besides, for better reconstruction, we use separately prepared fully-sampled MRI images to train dictionaries. We compare results of the proposed technique to traditional offline methods with and without double sparsity model. Our simulation results show that the proposed technique is approximately twice faster than the traditional methods while maintaining the same reconstruction quality. Furthermore, our technique performed even better for lower sampling rate.

***Index Terms*—** MRI, compressed sensing, online dictionary learning, double sparsity model

1. INTRODUCTION

Magnetic resonance imaging (MRI) is a medical imaging technique based upon the nuclear magnetic resonance phenomenon [1]. Since MRI provides abundant and detailed anatomical information without any radiation exposure, it is widely used for diagnosis and treatment. In MRI, signals are sampled in k-space (the spatial Fourier transform domain) sequentially in time, thus time consuming. The long scanning time may cause motion artifacts or discomfort of patients. Therefore, reducing the amount of signals to speed up the acquisition under the promise of high imaging quality has become a hot research topic, since Lustig *et al.* addressed this problem in their milestone paper [2].

The compressed sensing (CS) theory enables to recover signals or images from far fewer samples when the signal has a sparse representation in some transform domains and the acquisition of signal is incoherent [3]. The CS theory is directly applicable to the problem of accelerating MRI scanning, because MRI images meet the sparsity condition and MRI acquisition can be designed to achieve incoherent undersampling [2].

In CS-based MRI recovery, wavelet transform and total variation are frequently employed [4], [5]. These transforms are, however, limited for not highly under-sampled signals because such fixed sparse transforms are not always optimal for individual signals. To find a more appropriate transform, dictionary learning has been recently applied to CSMRI. Ravishankar *et al.* proposed an outstanding MRI method named DLMRI using a patch-based dictionary learning algorithm [6] with K-SVD [7]. This is a typical dictionary learning method exploited in CS-MRI for a single image [8], [9] or image series [10]. Because K-SVD is time consuming, however, DLMRI is computationally expensive.

In this paper, to accelerate MRI recovery, we propose to use the double sparsity model [11] with the online dictionary learning method [12]. Since double sparsity model imposes a special structure of a product between a fixed base dictionary and an adaptive sparse dictionary, the reconstruction process can be accelerated. On the other hand, because of the structure, reconstruction quality can be degraded. To prevent it, we further propose to use separately prepared fully-sampled images to train dictionary. After training a dictionary, the orthogonal matching pursuit (OMP) algorithm computes sparse representations for all patches. Simulation results show that our technique saves more than half of computational time and recovers MRI images with the same high quality compared to the DLMRI method. It should be noted that our technique works even better for higher compression rate.

The rest of the present paper is organized as follows. Section 2 reviews the method of DLMRI. Section 3 shows our technique to apply the double sparsity to DLMRI and to use fully-sampled images to improve results. Section 4 shows that we apply the online dictionary learning method to further reduce the run time. Section 5 evaluates our technique by performing simulations using actual MRI images. Section 6 concludes the paper.

[†] The author thanks for the Research Grant from the Kayamori Foundation of Informational Science Advancement.

2. DICTIONARY LEARNING MRI

Following [6], we also handle MR image reconstruction as well as dictionary learning using overlapping 2D image patches. The overlap stride r is defined to be the distance in pixels between corresponding pixel locations in adjacent image patches. For an image $\boldsymbol{x} \in \mathbb{C}^P$, $\boldsymbol{x}_{ij} \in \mathbb{C}^n$ denotes a square image patch of size $\sqrt{n} \times \sqrt{n}$ while (i, j) means the location of its top-left in the image. R_{ij} denotes the operator that extracts a patch from the image $\boldsymbol{x}$, meaning $\boldsymbol{x}_{ij} = R_{ij}\boldsymbol{x}$. Assuming patches wrap around edges of image, each pixel will be represented by n patches when the overlap stride $r = 1$. The process of reconstruction aims to solve the problem

$$\begin{aligned} \underset{\boldsymbol{x},D,C}{\operatorname{argmin}} \sum_{ij} \left\|R_{ij}\boldsymbol{x} - D\boldsymbol{c}_{ij}\right\|_2^2 + \nu \|F_u\boldsymbol{x} - \boldsymbol{y}\|_2^2 \\ s.t. \left\|\boldsymbol{c}_{ij}\right\|_0 \leqslant T_0 \quad \forall i, j, \end{aligned} \tag{1}$$

where F_u is the under-sampled Fourier transform and $\boldsymbol{x}$ is a vector representation of the reconstructed image from the measurements $\boldsymbol{y}$. D denotes the dictionary of size $n \times m$. $\boldsymbol{c}_{ij} \in \mathbb{C}^m$ denotes the sparse representation for $\boldsymbol{x}_{ij}$. C of size $m \times N$ denotes the set of representations $\boldsymbol{c}_{ij}$ for all patches. T_0 is the required sparsity level for $\boldsymbol{c}_{ij}$. The weight ν is a positive constant that makes the process of reconstruction more robust to noise. The first term controls the quality of sparse approximations of the image patches with respect to the dictionary D. The second term enforces the data consistency in k-space. However, the process of reconstruction is computationally expensive.

3. DOUBLE SPARSITY DLMRI WITH TRAINING DATA

To accelerate MRI recovery, we apply double sparsity model to DLMRI. This model represents the dictionary D by a product of a fixed base dictionary Φ of size $n \times L$ and an adaptive sparse dictionary A of size $L \times m$ as $D = \Phi A$. Then, the problem in (1) becomes

$$\begin{aligned} \underset{\boldsymbol{x},A,C}{\operatorname{argmin}} \sum_{ij} \left\|R_{ij}\boldsymbol{x} - \Phi A\boldsymbol{c}_{ij}\right\|_2^2 + \nu \|F_u\boldsymbol{x} - \boldsymbol{y}\|_2^2 \\ s.t. \begin{cases} \left\|\boldsymbol{c}_{ij}\right\|_0 \leqslant T_0 & \forall i, j, \\ \|\boldsymbol{a}_k\|_0 \leqslant T_1 & \forall k, \end{cases} \end{aligned} \tag{2}$$

where T_1 is the required sparse level for each column $\boldsymbol{a}_k$ in the sparse dictionary A. However, since double sparsity model imposes a special structure to dictionary, reconstruction quality can be degraded. To prevent it, we use fully-sampled data in each iteration to improve the quality. Thus, the problem in (3) becomes

$$\begin{aligned} \underset{\boldsymbol{x},A,C,C_z}{\operatorname{argmin}} \sum_{ij} \left\|R_{ij}\boldsymbol{x} - \Phi A\boldsymbol{c}_{ij}\right\|_2^2 + \nu \|F_u\boldsymbol{x} - \boldsymbol{y}\|_2^2 \\ + \mu \|Z - \Phi A C_z\|_F^2 \quad s.t. \begin{cases} \left\|\boldsymbol{c}_{ij}\right\|_0 \leqslant T_0 & \forall i, j, \\ \|\boldsymbol{a}_k\|_0 \leqslant T_1 & \forall k, \\ \left\|\boldsymbol{c}_l^z\right\|_0 \leqslant T_0 & \forall l, \end{cases} \end{aligned} \tag{3}$$

where Z is a matrix of size $n \times M$ containing M patches extracted from fully-sampled training MR images, C_z is a set of sparse representations for Z, and $\|\cdot\|_F$ is the Frobenius norm. We request the same sparse level T_0 for each column $\boldsymbol{c}_l^z$ of C_z. The third term provides prior information from the training images and μ controls the balance between the three terms.

4. MR IMAGE RECONSTRUCTION USING ONLINE DICTIONARY LEARNING

The problem proposed in (3) is NP-hard. Thus, we choose to divide it into a sparse dictionary learning step and a reconstruction update step and iteratively solve them with some variables fixed.

4.1. Dictionary Learning

In this stage, $\boldsymbol{x}$ is fixed while A, C, and C_z are free. In other words, we update the sparse dictionary and sparse representations in this step. The problem can be stated as

$$\begin{aligned} \underset{A,C,C_z}{\operatorname{argmin}} \sum_{ij} \left\|R_{ij}\boldsymbol{x} - \Phi A\boldsymbol{c}_{ij}\right\|_2^2 + \mu \|Z - \Phi A C_z\|_F^2 \\ s.t. \begin{cases} \left\|\boldsymbol{c}_{ij}\right\|_0 \leqslant T_0 & \forall i, j, \\ \|\boldsymbol{a}_k\|_0 \leqslant T_1 & \forall k, \\ \left\|\boldsymbol{c}_l^z\right\|_0 \leqslant T_0 & \forall l. \end{cases} \end{aligned} \tag{4}$$

In [11], the base dictionary is discrete Fourier transform while [12] chose the wavelet transform that performed better. In this paper, we also use the wavelet transform as the base dictionary. In particular, Daubechies 9 was used in the simulations below. Although this problem can be solved by K-SVD, we adopt an online method [12] to accelerate the procedure. In the online method, training patches will be divided into some mini-batches. The sparse dictionary A is updated by computing a gradient using one of the mini-batches. This update continues until all mini-batches are used. For better quality of dictionary, we can repeat the process for several times. To further reduce computational complexity, we only use a fraction of all patches of the reconstructing image and fully-sampled images as training data. To exploit various image characteristics as much as possible, we randomly choose the patches from the fully-sampled images, which is also changed at every iteration.

Once the sparse dictionary A has been learnt from the training data, we apply it to all patches of reconstructing image $\boldsymbol{x}$ and update the sparse representations by the OMP algorithm.

4.2. Image Reconstruction

There is only one free variable in the second sub-problem. The problem becomes

$$\underset{\boldsymbol{x}}{\operatorname{argmin}} \sum_{ij} \left\| R_{ij}\boldsymbol{x} - \Phi A \boldsymbol{c}_{ij} \right\|_2^2 + \nu \left\| F_u \boldsymbol{x} - \boldsymbol{y} \right\|_2^2. \tag{5}$$

This is a simple least squares problem admitting an analytical solution. The solution of this problem can be obtained similarly to the reconstruction update step in the DLMRI algorithm [6]. The update of $\boldsymbol{x}$ is essentially done in k-space (Fourier domain) by averaging the sparse solutions and the original acquisitions for k-space locations that were acquired. Then, the final expression for reconstructed image $\boldsymbol{x}$ at this iteration is

$$(F\boldsymbol{x})(k_x, k_y) = \begin{cases} \boldsymbol{s}(k_x, k_y), & (k_x, k_y) \notin \Omega, \\ \frac{\boldsymbol{s}(k_x,k_y) + \nu \boldsymbol{s}_0(k_x,k_y)}{1+\nu}, & (k_x, k_y) \in \Omega, \end{cases} \tag{6}$$

where $(F\boldsymbol{x})(k_x, k_y)$ represents the updated value of 2D Fourier transform at location (k_x, k_y), $\boldsymbol{s}_0 = FF_u^H\boldsymbol{y}$ represents the zero-filled k-space measurements, $\boldsymbol{s}(k_x, k_y)$ represents the updated k-space data value obtained by averaging contributions of patches, and Ω represents the subset of k-space data that has been sampled.

The proposed algorithm is shown in Algorithm 1. The algorithm is initialized with a zero-filled Fourier reconstruction.

Algorithm 1 proposed algorithm

Input: under-sampled k-space measurements, $\boldsymbol{y}$
Output: reconstructed MR image, $\boldsymbol{x}$

1: Initialize reconstructed image $\boldsymbol{x} = F_u^H\boldsymbol{y}$, number of reconstruction iteration *Iter*
2: **for** $t = 1$ to *Iter* **do**
3: Randomly extract patches from $\boldsymbol{x}$ as training data Y_t and choose fully-sampled patches as training data Z_t;
4: Update sparse dictionary A for Y_t and Z_t by the method mentioned in section 4;
5: Apply A to all patches and obtain sparse representations C by OMP;
6: Update each pixel of $\boldsymbol{x}$ by averaging the value of all patches;
7: Transform image $\boldsymbol{x}$ to k-space data $\boldsymbol{s}$;
8: Update k-space data $\boldsymbol{s}$ by the method mentioned in section 4.2;
9: Transform k-space data $\boldsymbol{s}$ to image $\boldsymbol{x}$;
10: **end for**
11: **return** $\boldsymbol{x}$

5. SIMULATIONS

Fully sampled axial brain scans were acquired from a healthy male of twenty years old for the analysis of the proposed method. The images in the dataset are in the Dicom format and their size is 256 × 256. We choose 10 images of ’MR0050’ through ’MR0059’ for the simulation. Fig.1(a) shows the image of ’MR0050’. We used images from ’MR0060’ to ’MR0109’ as training data.

Table 1. PSNR(dB) under different methods

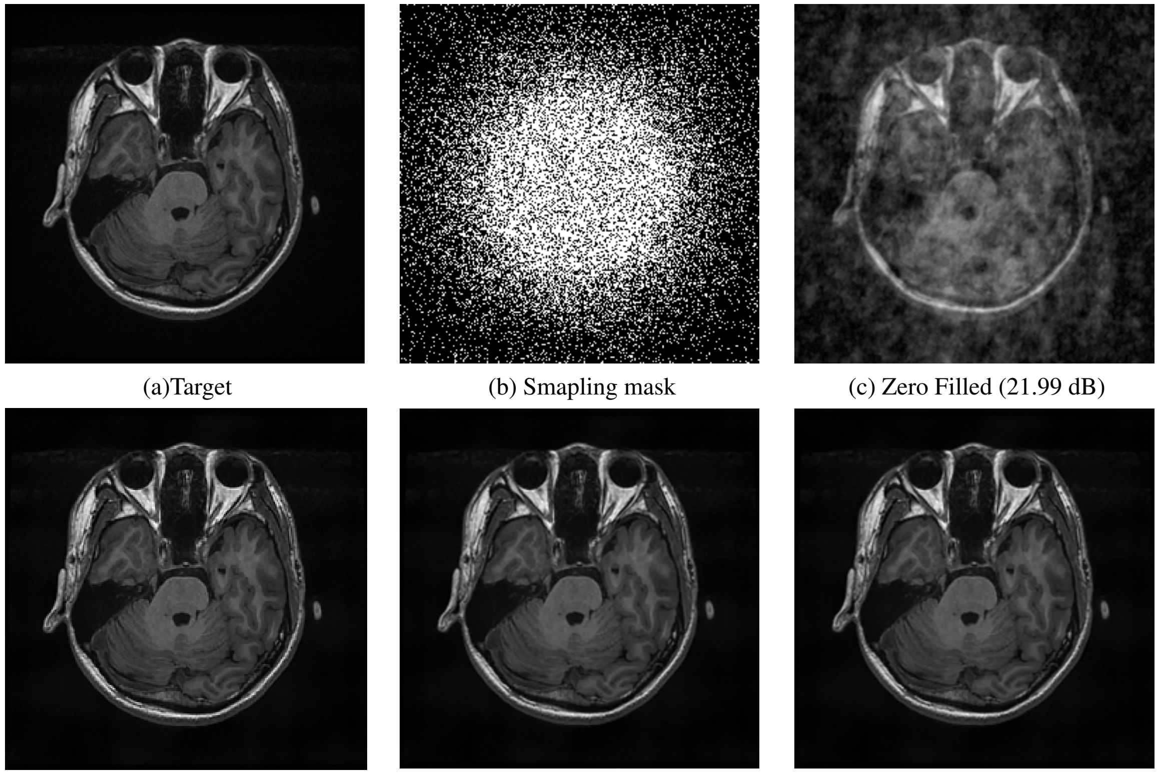
Sampling	1/3	1/5	1/8
Zero-Filled	20.80	20.79	16.56
DLMRI	40.73	38.84	32.06
DS+KSVD	40.64	38.75	32.27
DS+OL w/o Data	40.66	38.74	32.33
Proposed	40.79	38.87	32.30

Table 2. Run time(s) under different methods

Sampling	1/3	1/5	1/8
DLMRI	944.44	874.33	877.09
DS+KSVD	668.02	643.79	645.92
DS+OL w/o Data	395.84	372.51	374.05
Proposed	398.79	374.36	376.37

The parameters for the proposed algorithm were set as patch size $\sqrt{n} = 4$, number of sparse dictionary atoms $L = 64$, sparsity level of sparse representations $T_0 = 0.15 \times 16$, sparsity level of sparse dictionary $T_1 = 40$, the error target in the process of sparse coding $\epsilon = 0.023$, weight $\nu = 140$, weight $\mu = 1$. We used $N + M = 20,000$ patches for dictionary learning and divide them into 5 batches. In the step 4 of Algorithm 1, each of these 5 batches is used 3 times to update the sparse dictionary A. These parameters were chosen based on empirical tradeoffs between performance and efficiency. We apply random 2D discrete Fourier transform to all fully sampled data, and the compression rates we used are 1/3, 1/5, 1/8. Fig.1(b) shows the 3 fold under-sampling mask we used. The noise was not introduced manually because the data we used here are real observed ones.

The proposed method was compared to the DLMRI, the double sparsity model implemented with KSVD (DS+KSVD), and the double sparsity model implemented with online dictionary learning without any additional fully-sampled data in the process of reconstruction (DS+OL w/o Data). For these methods, we used the same parameters as the proposed method. The implementation of DLMRI is publicly available in [13]. Since the step of sparse coding in this code is written by MATLAB and the OMP algorithm in this code is not well implemented, the quality of the reconstructed images was not good. For better comparison, we replaced the OMP algorithm in this program by another implementation written by C++ [14]. All programs were run using Matlab R2015b on Macbook with 2.4GHz Intel Core i5 processor and 8 GB memory. The quality of the reconstruction is measured using PSNR (in dB) which is computed as the ratio of the peak intensity value of the reference image to the root mean square



(a)Target (b) Smapling mask (c) Zero Filled (21.99 dB)

(d)DLMRI (39.74 dB) (e)DS+KSVD (39.65 dB) (f) Proposed Method (39.74 dB)

Fig. 1. Simulation Results.

reconstruction error relative to the reference (error computed between image magnitudes).

Fig.1 presents results for the axial brain data with 2D 3 fold random under-sampling. The PSNR of the proposed method (39.74dB) is same as that of the DLMRI (39.74dB), indicating that our method can obtain the same high quality image as DLMRI. Table 1 presents the average of the PSNRs for the 10 images under different methods and three different under-sampling rates. We can see that double sparsity model degraded image quality than DLMRI as expected. The use of the online dictionary learning and the fully-sampled data recovered the image quality. Note that finally, the quality obtained by the proposed method is slightly better than DLMRI. Table 2 presents the run time of three methods under three different under-sampling rates. The run time of proposed method is less than half of that for DLMRI. From the results, we can see that both the double sparsity model and online algorithm helped to reduce the run time and that the use of training images helped to improve the image quality. Note that the results have converged when the compression rate is 1/3 and 1/5 while the results did not yet converge when compression rate is 1/8.

6. CONCLUSION

In this work, we proposed an adaptive sparse dictionary learning based algorithm for MR image reconstruction from highly under-sampled k-space data using the double sparsity model with the online dictionary learning method. The proposed algorithm alternates between a sparse dictionary learning step, and a reconstruction update step. For each iteration, we used fully-sampled data to improve the results. The proposed algorithm, due to the efficiency of double sparsity model and online dictionary algorithm, provided comparable, slightly even better reconstructions to the previous methods. Most importantly, our approach is significantly faster than the previous DLMRI approach [6]. In the future, we expect to provide even greater speed-ups over DLMRI for larger patch sizes and higher image quality. To analyze the convergence of the proposed method theoretically is also our future work.

REFERENCES

[1] R. R. Edelman and S.Warach, "Magnetic resonance imaging," *New England Journal of Medicine*, vol. 328, no. 10, pp. 708–716, 1993.

[2] M. Lustig, D. L. Donoho, J. M. Santos, and J. M. Pauly, "Compressed sensing MRI," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 72–82, March 2008.

[3] D. L. Donoho, "Compressed sensing," *IEEE Trans. Information Theory*, vol. 52, no. 4, pp. 1289–1306, April 2006.

[4] S. Ma, W. Yin, Y. Zhang, and A. Chakraborty, "An efficient algorithm for compressed MR imaging using total variation and wavelets," in *IEEE Conf. Computer Vision and Pattern Recognition, 2008. CVPR 2008*, June 2008, pp. 1–8.

[5] Z. Zhang, Y. Shi, and B. Yin, "MR images reconstruction based on TV-group sparse model," in *2013 IEEE International Conference on Multimedia and Expo (ICME)*, July 2013, pp. 1–6.

[6] S. Ravishankar and Y. Bresler, "MR image reconstruction from highly undersampled k-space data by dictionary learning," *IEEE Trans. Medical Imaging*, vol. 30, no. 5, pp. 1028–1041, May 2011.

[7] M. Aharon, M. Elad, and A. Bruckstein, "K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation," *IEEE Trans. Signal Processing*, vol. 54, no. 11, pp. 4311–4322, Nov 2006.

[8] Y. Song, Z. Zhu, Y. Lu, Q. Liu, and J. Zhao, "Reconstruction of magnetic resonance imaging by three-dimensional dual-dictionary learning," *Magnetic resonance in medicine*, vol. 71, no. 3, pp. 1285–1298, 2014.

[9] Q. Liu, S. Wang, K. Yang, J. Luo, Y. Zhu, and D. Liang, "Highly undersampled magnetic resonance image reconstruction using two-level bregman method with dictionary updating," *IEEE Trans. Medical Imaging*, vol. 32, no. 7, pp. 1290–1301, 2013.

[10] J. Caballero, A. Price, D. Rueckert, and J. Hajnal, "Dictionary learning and time sparsity for dynamic MR data reconstruction," *IEEE Trans. medical imaging*, vol. 33, no. 4, pp. 979–994, 2014.

[11] R. Rubinstein, M. Zibulevsky, and M. Elad, "Double sparsity: Learning sparse dictionaries for sparse signal approximation," *IEEE Trans. Signal Processing*, vol. 58, no. 3, pp. 1553–1564, March 2010.

[12] J. Sulam, B. Ophir, M. Zibulevsky, and M. Elad, "Trainlets: Dictionary learning in high dimensions," *IEEE Trans. Signal Processing*, vol. 64, no. 12, pp. 3180–3193, June 2016.

[13] "http://www.ifp.illinois.edu/ yoram/dlmri-lab/dlmri.html," .

[14] "http://www.cs.technion.ac.il/ ronrubin/software.html," .