Non-invasive Gearbox Fault Diagnosis Using Scattering Transform of Acoustic Emission

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Abstract—Monitoring acoustic emission from mechanical systems is an effective non-invasive way of diagnosing both system performance as well as short-term/long-term system failures. A difficulty however in fault detection in such systems is inter-class variability caused by non-uniform or unknown load conditions which decrease the classification accuracy. In this paper, a scattering transform is employed to diagnose gearbox faults using acoustic emission analysis. The results analysis and solution shows that a two layer scattering transform can diagnose four gearbox faults with an average accuracy of 97% even if the system is not exposed to data of all loads in the training phase.

Index Terms—Acoustic wave, Fault diagnosis, Discrete wavelet transforms, Scattering transform, Support vector machines

I. INTRODUCTION

A. Motivation

Gears, by changing the ratio of torque and speed, are important parts of many mechanical systems in industrial applications such as wind turbines, and electrical vehicles. An unpredicted fault in gearboxes may cause failure for the whole system resulting in a potential catastrophic disaster. Developing automatic fault diagnosis algorithms for these machines has received much attention from researchers during the last three decades [1, 2].

B. Prior Work

Fault diagnosis in mechanical systems can be done using vibration signal [3–5] or acoustic emission [6–8]. The majority of prior fault diagnosis algorithms are data-driven approaches. In this approach, the fault diagnosis is considered as a pattern recognition problem and signal processing techniques are applied to one these measurements to extract features. Next, the health state of the machine is diagnosed by classifying the extracted features. The key difference among prior data-driven methods is related to the feature extraction technique. Frequency domain features are proposed in [9, 10]. The authors in [11, 12] proposed wavelet transform for feature extraction. Empirical mode decomposition is also used in [13, 14].

In spite of the number of proposed methods, existing techniques suffer from low accuracy, limiting the number of faults that they can diagnose. A limiting factor is the variability caused by changes in the load condition. This becomes more challenging in machines with continuous load changes like coal mill, compressors or variable speed machines like pumps and conveyor belts or electric vehicles. The majority of prior works are developed to diagnose the steady state of vibration or acoustic signals in frequency domain which is not applicable to such non-stationary scenarios [15]. This topic has become a mainstream in research in this field recently. Authors in [16] used the Gabor wavelet to model the transient trends in an induction motor application. Complex wavelet analysis is exploited in [17] to extract features of vibration signal which are robust against load condition. A scale invariant method is proposed in [18] to capture non-stationary trends.

C. Key Contribution

In this paper, we propose an automatic fault diagnosis method for monitoring gearbox health status based on acoustic emission. We address the problem of variable load and nonstationarity in measurements from a feature extraction perspective. Specifically, we use the scattering transform for this purpose which is a robust representation against deformation caused by a change in load conditions. The proposed method has an affordable computational cost for real-time implementation and achieves high overall accuracy.

II. SIGNAL PROCESSING FRAMEWORK

We assume that the acoustic wave for various loads are a deformed version of each other by some time warp. In this section, the mathematical framework of our feature extraction method which is invariant against this deformation is briefly introduced.

A. Deformation and Notion of Stability

In the steady state analysis, the frequency domain is a useful tool to extract features from measured signals. Let $\hat{x}(\omega) = \int x(t)e^{-i\omega t}dt$ denote the Fourier integral of a signal $x(t) \in L^2_{\mathbb{R}}$. If $x_c(t) = x(t-c)$ is a deformed version of x(t) by translation, then $|\hat{x}_c(\omega)| = |\hat{x}(\omega)|$. So the modulus of the Fourier transform is a representation of signals which is invariant against translation. In practice, translation is not the only possible deformation which causes a variability. Time-warping is a more general class of deformation which is modeled as $x_\tau(t) = x(t - \tau(t))$ with $|\tau'(t)| < 1$. Time warp is a broad model for deformation used in several signal processing applications such as speech processing [19]. Here, the deformation can be quantified using $|\tau'(t)|$. If the derivative is zero the deformation simplifies to a pure translation.

Suppose $\Phi(x)$ is an arbitrary representation (feature) of x(t)and $\Phi(x_{\tau})$ is the same representation for $x_{\tau}(t)$, the deformed version of x(t); then, $\Phi(\cdot)$ is considered as a robust representation if the difference between $\Phi(x)$ and $\Phi(x_{\tau})$ caused by the deformation is small. This similarity can be quantified using the Euclidean norm as $d(x, x_{\tau}) = \|\Phi(x) - \Phi(x_{\tau})\|$. Basically, we are looking for a $\Phi(\cdot)$ which is invariant to this deformation. This invariance can be characterized using notion of stability. $\Phi(\cdot)$ is stable if a small change in x(t) does not lead to a big change in d. Mathematically, stability is defined as Lipschitz continuity condition respect to the norm, if there exists a constant C > 0 such that for all τ with $sup|\tau'(t)| < 1$:

$$\|\Phi(x) - \Phi(x_{\tau})\| \le C \sup_{t} |\tau'(t)| \, \|x\| \tag{1}$$

where *sup* denotes supremum of a set. The constant C gives a measure of stability. Indeed, if the Lipschitz condition holds true, a change in x(t) caused by time warp leads to a linear change in its representation. Since, time-warping is locally linearized by $\Phi(x(t))$ and $\Phi(x) - \Phi(x_{\tau})$ can be approximated by a linear operator if $sup|\tau'(t)|$ is small. In other words, in the feature space $\Phi(x)$ and $\Phi(x_{\tau})$ are on the same hyperplane and the corresponding features do not spread all over the space.

It can be shown that the Fourier modulus is not stable to time-warp deformation. In the Fourier representation, time warp changes each frequency component differently, (i.e., high frequency content are changed more than low frequency content, thus causes instability [19]). The notation of stability helps us to understand why feature extraction methods based on steady state analysis do not work for variable load condition. Technically, a deformation caused by a change in load spreads feature points corresponding to the deformed signals in the feature space in a random manner and increases the overlapping of classes. So, we are looking for a feature extraction method such that the feature point corresponding to a time warped signal lie on a hyperplane instead of spreading in the feature space.

B. Analytic Wavelet Transform Modulus

The analytic wavelet transform, which is robust against shift deformation, can be calculated using constant Q filterbanks [20]. A wavelet like $\psi(t)$ is a band pass filter where $\hat{\psi}(0) = 0$. In the analytical wavelet transform $\hat{\psi}(\omega) \simeq 0$ for $\omega < 0$ [21]. A dilated version of $\psi(t)$ with the central frequency of $\lambda > 0$ can be written as $\psi_{\lambda}(t) = \lambda \psi(\lambda t)$ or in frequency domain $\hat{\psi}_{\lambda}(\omega) = \hat{\psi}(\frac{\omega}{\lambda})$, where the central frequency of $\psi(\omega)$ is normalized to 1 and Q is chosen as the number of wavelets per octave, $\lambda = 2^{k/Q}$ for $k \in \mathbb{Z}$, which guarantees the bandwidth of $\hat{\psi}_{\lambda}$ to be in order of Q^{-1} and its central frequency is at λ . In this way, different $\hat{\psi}_{\lambda}$'s cover all frequency axis except DC which is covered using a low-pass filter ϕ . Let Λ denote the set of all values of λ , the wavelet transform of signal x(t) can be calculated by convolution of these filters:

$$Wx = (x(t) * \phi(t), x * \psi_{\lambda}(t)) \qquad t \in \mathbb{R}, \ \lambda \in \Lambda$$
 (2)

Here, t is not critically sampled as the wavelet bases, so this representation is redundant. The filters ϕ and ψ need to be designed such that the entire frequency axis is covered which requires:

$$A(\omega) = |\hat{\phi}(\omega)|^2 + \frac{1}{2} \sum_{\lambda \in \Lambda} (|\hat{\psi}_{\lambda}(\omega)|^2 + |\hat{\psi}_{\lambda}(-\omega)|^2) \quad (3)$$

for all $\omega \in \mathbb{R}$ satisfies [22]:

$$1 - \alpha \le A(\omega) \le 1$$
 for $\alpha \le 1$ (4)

By multiplying both sides of this inequality by $|\hat{x}(\omega)|^2$ and apply Plancherel theorem one can obtain [23]:

$$(1 - \alpha) \|x\|^2 \le \|Wx\|^2 \le \|x\|^2 \tag{5}$$

where $||Wx||^2 = \int |x * \phi|^2 + \sum_{\lambda \in \Lambda} \int |x * \psi|^2$ is the squared norm of wavelet representation and $||x||^2 = \int |x(t)|^2 dt$ is the norm of signal. In Eq. 5, the lower bound guarantees a stable inverse while the upper bound shows that wavelet is a contractive operator [19]. If $\alpha = 0$, then W becomes a tight frame and x(t) can be reconstructed as $x(t) = (x * \phi(t)) * \phi(-t) + \sum_{\lambda \in \Lambda} Real\{(x * \psi(t)) * \psi(-t)\}$ [22].

In the scattering transform, the wavelet modulus is used for feature extraction. In spite of Fourier transform, which is not possible to reconstruct the signal just using its Fourier modulus, it is possible to reconstruct the signal using just modulus of complex wavelet [24]. This is due to the redundant representation in Eq. 2. In addition, since the complex modulus is contractive, ||a| - |b|| for any $(a, b) \in \mathbb{C}$, the wavelet modulus operator, |W| is contractive:

$$|||W|x - |W|x'||^2 \le ||Wx - Wx'||^2 \le ||x - x'||^2$$
 (6)

C. Wavelet Scattering Transform

The main idea behind the scattering transform is to analyze the signal using analytical wavelet and then average the wavelet coefficients over time to extract features. The intuition behind the averaging coefficient is to reduce the variability in features and is similar to averaging the Fourier coefficient over Mel frequency intervals to extract Mel frequency cepstral coefficients (MFCC) in speech processing [25]. However, MFCC loses information by averaging, while scattering transform preserves the reconstruction information [26].

In the scattering transform, a locally translation invariant descriptor is obtained by a time average $S_0x(t) = x * \phi(t)$ which removes the high frequency contents. However, these high frequency content are recovered by a wavelet modulus transform as $|W|_1 x = (x * \phi, |x * \psi_{\lambda_1}|)$. The first order of the scattering coefficients can be obtained as $S_1x(t, \lambda_1) = |x * \psi_{\lambda_1}| * \phi$. These coefficients measure the average signal amplitude in the frequency interval covered by ψ_{λ_1} with bandwidth corresponding to Q_1 . In essence, they are calculated by a second wavelet modulus operator as $|W|_2 |x * \psi_{\lambda_1}| = (|x * \psi_{\lambda_1}| * \phi, ||x * \psi_{\lambda_1}| * \psi_{\lambda_2}|)$. So, the second order scattering coefficients are $S_2 x = ||x * \psi_{\lambda_1}| * \psi_{\lambda_2}|$ which are computed by a ψ_{λ_2} with a bandwidth corresponding to Q_2 . Iterating this process defines the scattering coefficients at any desired order.

For any $m \geq 1$, iterated wavelet modulus convolutions are written as $U_m(t, \lambda_1, \dots, \lambda_m)x = |||x * \psi_{\lambda_1}| * \dots | * \psi_{\lambda_m}|$ where the m^{th} order wavelet have an octave resolution of Q_m and they satisfy condition in Eq. 4. Next; the m^{th} order scattering coefficients are obtained by averaging $U_m x$ with ϕ as $S_m x(t, \lambda_1, \dots, \lambda_m) = U_m x(t, \lambda_1, \dots, \lambda_m) * \phi$. So, the scattering decomposition of a signal with the maximum order of l is an iterative operation by applying $|W|_{m+1}$ on $U_m x$ to obtain $S_m x$ and $U_{m+1} x$ for $0 \leq m \leq l$ where $U_0 x = x$. The scattering transform is the collection of all coefficients from each order $Sx = \{S_m x | 0 \le m \le l\}$. Fig. 1 shows the the decomposition graph.



Fig. 1. Scattering transform of signal x by iterating the $|W|_m$ operator. The black dots show the output nodes.

One can prove that the scattering transform has the following properties [23]:

- Time warp deformation stability: it satisfies the Lipschitz condition, (i.e., there exist a constant C for any x such that $||Sx Sx_{\tau}|| \le C \sup |\tau'(t)| ||x||$).
- Contraction: since scattering is calculated by wavelet modulus, it is a contractive transform, (i.e., $||Sx Sx'|| \le ||x x'||$). As a result of this property, the scattering transform is robust against the additive noise.
- Energy conservation: if the chosen wavelet is a tight frame, then the scattering transform preserves the norm, (i.e., ||x||² = ||Sx||² + ||U_{l+1}x||²). As a result, ||U_{l+1}x||² vanishes as l → ∞. In practice, the coefficients become very small after a few iterations.

These properties make the scattering transform a suitable representation for signal classification tasks. In the next section, we propose a robust fault diagnosis algorithm based on this transform.

III. FAULT DIAGNOSIS METHODOLOGY

A. Signal Processing Pipeline

We propose a monitoring method for diagnosing gearboxes using the acoustic emission signal picked up by an free field microphone. Fig. 2 shows the signal processing pipeline of the proposed method.



Fig. 2. The proposed four-stage fault diagnosis algorithm based on acoustic emission of a gearbox picked up by an open field microphone.

In the first step, the acoustic signal is analyzed with the scattering transform. We use a two layer (l = 2) scattering network with the normalized Morlet wavelet defined as $\psi(t) = e^{-i\omega_0 t}\theta(t) = e^{-it}e^{-t^2/2\sigma^2}$, which is simply a modulated Gaussian function. In the frequency domain, $\hat{\psi}(\omega) = \hat{\theta}(\omega - 1)$ is a low-pass filter with a Gaussian shape with its central frequency at the normalized frequency of 1. The Morlet wavelet is almost an analytical function since $|\hat{\psi}(\omega)|$ is small for $\omega < 0$ but not zero. However, strictly speaking, Morlet is not an admissible wavelet [19]. For satisfying the

admissibility condition we use $\hat{\psi} = \hat{\theta}(\omega-1) - \hat{\theta}(\omega)\hat{\theta}(-1)/\hat{\theta}(0)$ which guarantees $\hat{\psi}(0) = 0$. The parameter σ^2 determines the bandwidth of the wavelet which is assigned based on the choice of Q in scattering network. Fig. 3 shows Morlet wavelets for $Q_1 = 8$ and $Q_2 = 1$ in our scattering network.



Fig. 3. Frequency response of Morlet wavelet used in scattering network. (a) First layer with $Q_1 = 8$, (b) Second layer with $Q_2 = 1$.

In the second stage, the scattering coefficients are postprocessed to get a better classification rate. This post processing has two steps: first, the scattering coefficients are normalized to be invariant against a change in the amplitude of input signal by dividing the coefficients in each layer by corresponding coefficients in the predecessor layer where the first layer coefficients are normalized by S_0x . After normalization, the log function is applied to the normalized coefficients as a range compressor. This leads to a better classification accuracy and also a better visualization contrast.

In the third stage, the dimensions of the feature space is reduced using linear discriminant analysis (LDA). Basically, LDA provides a trade-off between reducing the dimensionality of feature space and maximizing the Rayleigh criterion [27].

In the last stage, a multi-class support vector machine (SVM) classifier with the radial basis function (RBF) kernel diagnoses the health state of machine using the extracted feature. The multi-class classifier is built based on the error correcting output code scheme.

B. Test Setup

For testing the proposed fault diagnosis method, a pinionwheel gearbox drive by an electric motor is used [28]. A defect in one tooth of pinion, wheel and both pinion and wheel simultaneously are studied. The acoustic emission of gearbox is recorded using an open field microphone at the rate 5KHz for 5 loads conditions (20%, 40%, 60%, 80% and 100%) and four classes f_0 corresponding to fault-free and f_1 , f_2 , and f_3 which are pinion, wheel and simultaneous faults, respectively. Each recoding has a duration of 60 seconds and is repeated 5 times. Fig. 4 shows the corresponding acoustic waves for each fault.

IV. EXPERIMENTAL RESULTS

The collected dataset is processed by the aforementioned scattering network and 7,300 features from each class were extracted. The dimensionality of the feature space is reduced from 354 to 11 using LDA. The dimension of new feature



classes: Time domain (left) and periodogram (right).

Fig. 5. Scatter plot of features in 3D space.



space is chosen based on the proportional cumulative variance of eigenvalues of LDA [27]. Fig. 5 shows the projected features to the subspace spanned by the first three largest eigenvectors of LDA.

Accuracy of the proposed algorithm is measured twice using 5-fold cross validation. In the first test (Test 1 in Table I), the dataset is shuffled and four folds of data are used for tuning the LDA and the SVM classifier while system's accuracy is tested with the fifth fold. In this experiment, the system was exposed to data from all loads in the training phase. The average accuracy of 98.25% is obtained. For investigating the performance of the system to a load which is not exposed to the system in training phase, we arranged the second test (Test 2 in Table I). Here, we used data from 4 randomly chosen loads for training and we measured accuracy using data from the fifth load. The average accuracy of 97.61% is obtained. The confusion matrix of both tests are tabulated in Table I. The average accuracy of the system in both tests are almost equal.

TABLE I CONFUSION MATRIX OF CLASSIFIERS (%)

	Classified (Test 1)				Classified (Test 2)			
Target	f_0	f_1	f_2	f_3	f_0	f_1	f_2	f_3
f_0	98.40	0.50	0	1.10	97.15	1.05	0	1.80
f_1	1.70	98.30	0	0	2.35	97.25	0	0.40
f_2	0.40	0	98.50	1.10	0.60	0.08	98.14	1.18
f_3	1.07	0	1.13	97.80	1.17	0	0.93	97.90
Accuracy	98.25				97.61			
AUROC	100	99.0	99.5	99.1	98.2	98.0	99.3	99.7

A better way of comparing the two classifiers, is their receiver operating curves (ROC). Fig. 6 shows the ROC of classifiers in both tests. An ideal ROC is unit step between zero and one. This quality can be quantified by measuring the area under the ROC (AUROC) which is equal 1 (or 100%) in an ideal case. The last row of Table I shows the AUROC for each classifier. Comparing ROC curves also confirms that the performance of both classifiers are almost equal. However, in the second test, the system was not exposed to different loads for training and testing. This shows that scattering transform extracts features which are robust against variations in the load. Fig. 7 shows the scattering transform versus time of the acoustic emission of different classes and loads. Comparing the scattering transform transform t

(c) Wheel Fault (f_2) (c) Wheel Fault (f_3)

Fig. 7. Scattergram of the acoustic emission for four classes for different loads (L = 20%, 40%, 60%, 80% and 100%)

captures fine differences between different classes, it gives a similar representations for different loads.

We implemented the entire signal processing pipeline depicted in Fig. 2 in MATLAB on a PC running Ubuntu Linux 14.04 at a clock rate of 3.4 GHz. The average runtime (wall-clock) of processing a window of acoustic wave with a duration of 10 seconds is almost equal to 0.9 sec which shows the possibility of real-time implementation of the proposed method on this machine.

V. CONCLUSION

In this study, a novel method for fault diagnosis of gearboxes using the acoustic emission was proposed. This method was based on a wavelet scattering transform which provides a robust feature extraction against deformations caused by load changes in this signal (i.e. unsupervised/open set). The proposed method has an affordable computational cost for realtime implementation. Exploring the application of this new tool in other systems with different measurement modalities is our research plan for future work.

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