# HIGH PRECISION ROBUST MODELING OF LONG ROOM RESPONSES USING WAVELET TRANSFORM

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# ABSTRACT

Modeling of a room impulse response (RIR) is required in many audio processing applications; however, this is challenging since room responses are usually long and complex in practice and drastically vary as the source and microphone locations change. In this paper, a subband multichannel modeling method is proposed, which is computationally efficient, precise, and robust against RIR variations. A dual-tree complex wavelet packet transform is utilized to decompose a multichannel RIR into aliasing-free subband signals, and low order adaptive Kautz filters are designed to model subband signals using the poles common to the RIR channels. A least-squares algorithm is introduced to efficiently estimate the common poles at each subband. Experimental results indicate that the proposed method accurately models long room responses, while exhibiting significant robustness against room response variations caused by changing the source and microphone locations.

*Index Terms*— Wavelet transform, Kautz filter, least-squares approximation, room acoustics.

# 1. INTRODUCTION

A room impulse response (RIR) describes the sound propagation characteristics between a source and a microphone placed inside a room. Accurate modeling of an RIR is essential in many acoustic signal processing applications such as room acoustic virtualization [1] and acoustic feedback cancellation [2]. In practice, modeling of an RIR is challenging since (i) an RIR can be tens of thousands taps long, requiring high order filters for accurate representation, and (ii) an RIR drastically changes with slight variations in the source and microphone locations.

Several methods for modeling of an RIR have been proposed. All-zero modeling [3, 4] is the simplest method. For a room with a relatively long reverberation time, however, an all-zero model requires a large number of parameters. Since poles represent room resonances with fewer parameters than zeros, a pole-zero method [4] provides a more compact model of an RIR. However, pole-zero models may require nonlinear optimization, suffering from convergence to a local minima [5]. An alternative to conventional modeling methods is the Kautz filtering [6] which utilizes orthonormal basis functions to provide a more precise model with fewer parameters.

In practice, due to the complex time-frequency structure of a room response, a conventional modeling method or a kautz filter may not accurately model the full audio frequency range and temporal decay of a room response [6-9]. To alleviate this restriction, different subband modeling methods have been proposed. A multirate system [7], the frequency zooming ARMA model [8], and polyphase Kautz model [9] have been shown to provide gentler performance as compared to the fullband counterparts. These methods, however, do not address the robustness against RIR variations. Models less sensitive to RIR variations have been proposed in [10-12], where the common acoustical poles of the room are utilized to develop a pole-zero model [10, 11] or a model based on orthonormal basis functions [12]. These models, however, do not represent the room response over the full audio frequency range and are limited to low frequency components of an RIR.

In this paper, we introduce a method for modeling of long room responses. The proposed model exhibits high precision and significant robustness against the RIR variations for the full audio frequency range. Given a multichannel RIR, we decompose the RIR into subband equivalent signals and design a low order adaptive Kautz filter at each subband before a fullband signal is reconstructed. We utilize the dual-tree complex wavelet packet transform [13, 14] to produce aliasingfree subbands. We introduce a least-squares (LS) algorithm for the efficient approximation of the poles common to the RIR channels at each subband; common poles (CPs) enhance the model robustness against the RIR variations. Estimated poles are then fixed, and the model is validated with room responses measured at source and microphone locations not used for the approximation of the CPs. Experimental results indicate that the proposed model provides a high precision robust representation of long room responses by benefitting from time-frequency decomposition and efficient approximation of CPs. In Section 2, we propose the subband multichannel RIR model. In Section 3, we validate the model with experimental data. Conclusions are drawn in Section 4.

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#### 2. SUBBAND MULTICHANNEL RIR MODELING

A room response in the time domain can be divided into direct sound, early reflections, and late reverberations [15]. Alternatively, in the frequency domain, a room response can be characterized by discrete low frequency modes and diffuse overlapping modes [15, 16]. This complex time-frequency structure motivates the development of a subband multichannel RIR model. Since the wavelet oscillate locally, it requires an order of magnitude fewer coefficients than the Fourier basis to approximate within the same error [13]. We use the packet form of the discrete-time complex wavelet transform (DT- $\mathbb{C}WT$ ) [14] to produce aliasing-free subbands and perfect reconstruction. Aliasing-free subbands are essential since aliasing causes erroneous results at the band edges [13].

#### 2.1. Wavelet Decomposition

The DT-CWT consists of two wavelet transforms operating in parallel on a given signal [13]. We denote the wavelet associated with the first (second) wavelet filter bank (FB) as  $\psi(t)$  ( $\psi'(t)$ ). Wavelet  $\psi(t)$  is defined as

$$\psi(t) = \sqrt{2} \sum_{n} h_{hp}(n)\phi(2t-n), \qquad (1)$$

where  $\phi(t) = \sqrt{2} \sum_{n} h_{lp}(n) \phi(2t-n)$ . Here,  $h_{lp}(n)$  and  $h_{hp}(n)$  represent a discrete-time low-pass and a discrete-time high-pass filter, respectively. Wavelet  $\psi'(t)$  is defined similarly in terms of  $\{h'_{lp}(n), h'_{hp}(n)\}$ . For the ideal DT-CWT, wavelet  $\psi'(t)$  is the Hilbert transform of wavelet  $\psi(t)$ ,

$$\psi'(t) = \mathcal{H}\{\psi(t)\}.$$
(2)

As shown in [13], [14], if  $h'_{lp}(n)$  is the half-sample delayed version of  $h_{lp}(n)$ , wavelets produced by DT- $\mathbb{C}WT$  satisfy (2).

To construct the DT- $\mathbb{C}WPT$  (the packet form of the DT- $\mathbb{C}WT$ ) [14], each subband should be repeatedly decomposed using low-pass/high-pass perfect reconstruction filter banks (PR FBs). The PR FBs should be chosen so that the response of each branch of the second wavelet packet FB is the discrete Hilbert transform of the corresponding branch of the first wavelet packet FB. Hence, each subband of the DT- $\mathbb{C}WPT$  will be analytic. Detailed properties in the context of mathematical proofs are discussed in [13], [14]. The DT- $\mathbb{C}WPT$ , obtained by iterating PR FBs on both low-pass and high-pass outputs, provides a linear-band analysis, leading to a high resolution over the entire audio frequency range.

# 2.2. Subband Kautz Filters

Kautz filters are a special class of fixed-pole IIR filters, designed to produce orthonormal tap-output impulse responses [6]. The orthonormalization process provides control on individual resonances, enabling a Kautz filter to efficiently model an audio response. A Kautz filter is defined by a set of stable



Fig. 1. Kautz filter for N pairs of complex conjugate poles.

poles  $\mathbf{p} = \{p_m\}_{m=1}^N$  and a corresponding set of tap-output weights. A common assumption in modeling of a real room response is that the poles are real or complex conjugate [6]. For complex conjugate poles, a real Kautz filter formulation, shown in Fig. 1, prevents dealing with complex internal signals and filter weights. Normalization terms are given by [6]

$$\alpha_m = \sqrt{(1 - \rho_m)(1 + \rho_m - \gamma_m)/2}$$
 (3)

$$\beta_m = \sqrt{(1 - \rho_m)(1 + \rho_m + \gamma_m)/2},$$
 (4)

where  $\rho_m = |p_m|^2$  and  $\gamma_m = -2\Re\{p_m\}$ . As illustrated in Fig. 1, the filter output is expressed as

$$y(n, \mathbf{p}, \mathbf{w}) = \varphi^T(\mathbf{p}, n) \mathbf{w}(n), \tag{5}$$

with  $\mathbf{w}(n) = [w_1(n), \dots, w_{2N}(n)]^T$  and  $\varphi(\mathbf{p}, n) = [\varphi_1(n), \dots, \varphi_{2N}(n)]^T$ . Due to the short length of the subband signals, a low order Kautz filter at each subband is sufficient; furthermore, subbands are processed in parallel, enabling efficient computations. To enhance the robustness against room response variations, we use poles which are common to the RIRs measured at different source and microphone locations.

#### 2.3. Least-Squares Approximation of common poles

Brandenstein and Unbehauen proposed an LS method, known as the BU method, for the FIR-to-IIR filter conversion [17], which produces unconditionally stable and optimal pole sets for a desired IIR filter order [17]. Inspired by the BU method, we propose an iterative LS algorithm that produces CPs at low expense. We refer to this algorithm as the CPBU method.

#### 2.3.1. Problem formulation

The *j*-th subband equivalent signal of the *i*-th channel of a multichannel RTF, denoted as  $H_{ij}(z)$  (i = 1, ..., M and j = 1, ..., J), is an FIR transfer function of length L. The IIR approximation of  $H_{ij}(z)$  in the least-squares sense is the IIR transfer function  $G_{ij}(z) = \frac{N_{ij}(z)}{D_{ij}(z)}$  of order N (N < L) such that

$$E_{ij} = ||\Delta_{ij}(z)||_2 = ||H_{ij}(z) - G_{ij}(z)||_2$$
(6)

is minimal, where  $|| \cdot ||_2$  denotes the  $l_2$ -norm. To estimate CPs, which are independent of the source and microphone

locations, we define  $D_{1,j}(z) = \cdots = D_{M,j}(z) = D_j(z)$ , where

$$D_{j}(z) = \sum_{n=0}^{N} d_{n} z^{-n} = 1 + z^{-1} \sum_{n=0}^{N-1} d_{n+1} z^{-n} = 1 + z^{-1} \bar{D}_{j}(z).$$
(7)

Hence, approximating CPs in the least-squares sense means that we need to determine N real coefficients  $d_n$  (n = 1, ..., N) such that the  $l_2$ -norm of the approximation error

$$\mathbf{E}_j = [E_{1j}, \dots, E_{Mj}]^T \tag{8}$$

is minimized. As the CPs must be stable, the rational transfer function  $G_{ij}(z)$  is required to be analytic in  $|z| \ge 1$ . Therefore, by applying the Walsh theorem [18], as discussed in [17], the difference function  $\Delta_{ij}(z)$  can be rewritten as

$$\Delta_{ij}(z) = z^{-1} A_j(z) R_{ij}(z), \ A_j(z) = \frac{z^{-N} D_j(z^{-1})}{D_j(z)}, \ (9)$$

where  $A_j(z)$  is an allpass filter, and  $R_{ij}(z)$  is an FIR transfer function with length L and real coefficients  $r_{ij,n}$   $(n = 0, \dots, L-1)$  which are computed through allpass filtering of  $X_{ij}(z) = z^{-L}H_{ij}(z^{-1})$  by  $A_j(z)$  as

$$U_{ij}(z) = \sum_{n=0}^{\infty} u_{ij}(n) z^{-n} = z^{-L} H_{ij}(z^{-1}) A_j(z), \quad (10)$$

$$r_{ij,L-1-n} = u_{ij}(n) \quad (n = 0, \cdots, L-1).$$
 (11)

Using equations (6), (9), and (11), we have

$$\mathbf{E}_{j}^{2} = \sum_{i=1}^{M} E_{ij}^{2} = \sum_{i=1}^{M} ||\Delta_{ij}||_{2}^{2} = \sum_{i=1}^{M} \sum_{n=0}^{L-1} r_{ij,n}^{2} = \sum_{i=1}^{M} \sum_{n=0}^{L-1} u_{ij}^{2}(n).$$
(12)

Hence, least-squares approximation of the CPs is formulated as approximating  $D_j(z)$  such that the energy of  $\mathbf{u}_j := [\mathbf{u}_{1j}, \ldots, \mathbf{u}_{Mj}]^T$  is minimal over the first L samples, where  $\mathbf{u}_{ij} = [u_{ij}(0), \ldots, u_{ij}(L-1)]$   $(i = 1, \ldots, M)$ .

#### 2.3.2. Approximation algorithm

We define the digital filter

$$A_j^{(k)}(z) = \frac{z^{-N} D_j^{(k)}(z^{-1})}{D_j^{(k-1)}(z)}$$
(13)

that approaches an allpass if  $||D_j^{(k)}(z) - D_j^{(k-1)}(z)||_2 \rightarrow 0$  for  $k \rightarrow \infty$ . Filtering  $X_{ij}(z) = z^{-L}H_{ij}(z^{-1})$  by  $A_j^{(k)}(z)$  leads to

$$U_{ij}^{(k)}(z) = A_j^{(k)}(z)X_{ij}(z) = z^{-N}D_j^{(k)}(z^{-1})X_{ij}^{(k)}(z),$$
(14)

where  $X_{ij}^{(k)}(z) = \frac{z^{-L}H_{ij}(z^{-1})}{D_j^{(k-1)}(z)}$ . Substituting (7) in (14) for i = 1, ..., M leads to

$$\begin{cases} X_{1,j}^{(k)}(z)z^{-(N-1)}\bar{D}_{j}^{(k)}(z^{-1}) = U_{1,j}^{(k)}(z) - z^{-N}X_{1,j}^{(k)}(z) \\ \vdots \\ X_{M,j}^{(k)}(z)z^{-(N-1)}\bar{D}_{j}^{(k)}(z^{-1}) = U_{M,j}^{(k)}(z) - z^{-N}X_{M,j}^{(k)}(z). \end{cases}$$
(15)

Equating the coefficients of  $z^0, z^{-1}, \ldots, z^{-(L-1)}$  on both sides of the equations in (15), we have

$$\mathbf{C}_{j}^{(k)}\mathbf{d}_{j}^{(k)} = \mathbf{u}_{j}^{(k)} + \mathbf{b}_{j}^{(k)}$$
(16)

where

$$\mathbf{C}_{j}^{(k)} = [\mathbf{C}_{1,j}^{(k)}, \dots, \mathbf{C}_{M,j}^{(k)}]^{T}, \quad \mathbf{d}_{j}^{(k)} = [d_{N,j}^{(k)}, \dots, d_{1,j}^{(k)}]^{T} \\
\mathbf{b}_{j}^{(k)} = [\mathbf{b}_{1,j}^{(k)}, \dots, \mathbf{b}_{M,j}^{(k)}]^{T}, \quad \mathbf{u}_{j}^{(k)} = [\mathbf{u}_{1,j}^{(k)}, \dots, \mathbf{u}_{M,j}^{(k)}]^{T} \\
\mathbf{b}_{ij}^{(k)} = -[0, \dots, 0, x_{ij}^{(k)}(0), \dots, x_{ij}^{(k)}(L-N-1)] \\
\mathbf{C}_{ij}^{(k)} = \begin{bmatrix} x_{ij}^{(k)}(0) & 0 & 0 & \dots & 0 \\ x_{ij}^{(k)}(1) & x_{ij}^{(k)}(0) & \dots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{ij}^{(k)}(L-1) & \dots & \dots & x_{ij}^{(k)}(L-N) \end{bmatrix}. \quad (17)$$

Solving  $\mathbf{C}_{j}^{(k)}\mathbf{d}_{j}^{(k)} = \mathbf{b}_{j}^{(k)}$  in the LS sense, a vector  $\mathbf{d}_{j}^{(k)}$  is obtained that minimizes the norm of  $\mathbf{u}_{j}^{(k)} = \mathbf{C}_{j}^{(k)}\mathbf{d}_{j}^{(k)} - \mathbf{b}_{j}^{(k)}$ . Hence, we have found  $D_{j}^{(k)}(z)$  that minimizes (12). We repeat the procedure as listed in Algorithm 1, creating a sequence of  $D_{j}^{(k)}(z)$  polynomials. Roots of the  $D_{j}^{(k)}(z)$  with the minimum error in sequence give the CPs.

Algorithm 1 CPBU Algorithm **Input:** Subband signals  $\{H_{ij}(z)\}_{i=1}^{M}$ , number of CPs (N) 1:  $D_j^{(0)} \leftarrow 1$ 2: for  $k = 1, 2, \cdots, K$  do for all i do 3: Update  $X_{ij}^{(k)}(z) = z^{-L} H_{ij}(z^{-1}) / D_j^{(k-1)}(z)$ 4: 5: Update  $\mathbf{C}_{j}^{(k)}$  and  $\mathbf{b}_{j}^{(k)}$  using  $X_{ij}^{(k)}(z)$  as in (17) Solve  $\mathbf{C}_{j}^{(k)}\mathbf{d}_{j}^{(k)} = \mathbf{b}_{j}^{(k)}$  in the least-squares sense Store  $\mathbf{d}_{j}^{(k)}$  and  $\mathbf{u}_{j}^{(k)}$ 6: 7: 8: 9: end for 10: Choose  $\mathbf{d}_j$  corresponding to the  $\mathbf{u}_j$  with the minimum norm 11: Create the  $D_j(z)$  polynomial utilizing elements of  $\mathbf{d}_j$  as in (7) 12: Compute roots  $p_{1,j}, \ldots, p_{N,j}$  of polynomial  $D_j(z)$ **Output:** The  $p_{1,i}, \ldots, p_{N,i}$  represent the CPs

## 2.4. Filter Weights Adaptation

Once the CPs are found from the measured data, the filter weights  $\mathbf{w}(n)$  can be adaptively computed by a standard algorithm such as Kalman filter, recursive least squares, and normalized least mean squares (NLMS) algorithm [19], [20]. For simplicity, we choose the NLMS with the adaptation rule

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \frac{\mu}{||\varphi(n,.)||^2} \varphi(n,.)(y(n) - \varphi^T(n,.)\hat{\mathbf{w}}(n))$$
(18)

where  $\mu \in (0, 2)$  is a gain. Linearity in filter weights leads to global convergence under the same conditions as for an FIR filter with the same number of parameters, and orthonormality assures faster convergence of the adaptation algorithm [20].



**Fig. 2**. Error of the SBCP-Kautz and CP-Kautz methods at different locations of the source and microphone.

#### **3. EXPERIMENTAL RESULTS**

Experimental results are presented to demonstrate the performance of the proposed subband multichannel Kautz method developed using the CPs (SBCP-Kautz). Experiments aim to evaluate the precision and robustness of the SBCP-Kautz method in both time and frequency domains in comparison to the fullband counterpart (CP-Kautz). To study the performance of the SBCP-Kautz method in modeling of the *i*-th channel of a multichannel RIR, the normalized mean square error (NMSE) is defined as

NMSE<sub>i</sub> = 20 log 10 
$$\frac{||\mathbf{h}_i - \hat{\mathbf{h}}_i||_2}{||\mathbf{h}_i||_2}$$
 (dB), (19)

where  $h_i$  and  $h_i$  denote the measured and modeled RIRs.

Experiments are performed using the MARDY database [21]. The RIRs were measured from three sources (placed at left (L), center (C), and right (R)) to a linear array of microphones placed at three different locations in a room with reflective panels. The source-to-microphone distances varied between 1 m and 3 m (1 m increments). The  $T_{60}$  is 447 ms. Each RIR has a length of 65536 samples with sample rate  $f_s = 48$  KHz. The SBCP-Kautz with 64 subbands is utilized, where each subband is modeled with 16 CP pairs (32-nd order filter). The CPBU algorithm (10 iterations) is used to estimate CPs from training data comprised of 7 RIRs measured from the source. The CPs are then fixed, and the SBCP-Kautz is applied to model room responses not used in the training step.

As illustrated in Fig. 2, the SBCP-Kautz method achieves nearly perfect modeling (NMSE  $\approx -80$  dB), independent of the source and microphone locations. To provide a comparison, the CP-Kautz with a 64-th order filter was utilized to model the same room responses. The CP-Kutz is computationally expensive and gives much lower accuracy (NMSE  $\approx$ -11 to -17 dB) which, depending on the source and microphone locations, varies by more than 50%. Higher orders of



**Fig. 3**. Frequency domain performance of the (a) SBCP-Kautz method and (b) CP-Kautz method.

a filter slightly enhance the accuracy of the CP-Kautz at the expense of highly increased computational load.

To evaluate the performance of the SBCP-Kautz method in the frequency domain, an arbitrary channel from the described databased is considered. Fig. 3 shows the channel frequency response and corresponding modeled response, followed by the error signal. The SBCP-Kautz models the room response over the full audio frequency range with almost no degradation, while benefiting from the low order filters.

#### 4. CONCLUAIONS

A method for modeling of long room responses has been introduced, where the RIR is decomposed into aliasing-free subband signals and a low order adaptive Kautz filter is designed to model each subband using CPs. A least-squares algorithm is introduced to efficiently estimate the CPs. Experimental results indicate that the proposed method (i) precisely models RIRs over the entire audio frequency range, and (ii) is robust against RIR variations. Computational efficiencies afforded by employing the subband method make it practical to implement the proposed method on modest signal processing platforms.

## 5. REFERENCES

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