# LISTENING-AREA-INFORMED SOUND FIELD REPRODUCTION BASED ON CIRCULAR HARMONIC EXPANSION

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## ABSTRACT

A sound field reproduction method that exploits prior information on listening areas is proposed. Most current methods are aimed at reproducing the sound field over the entire space or around listener locations. We formulate the objective function for this problem as the expectation minimization of the spatial squared error of the sound pressure inside the listening areas. The optimal driving signals are obtained by circular harmonic expansion. Comparing the proposed method with the mode-matching method, the advantage of the proposed method appears in the optimal weighting matrix of the circular harmonics, which depends on the location and range of the listening area. Numerical simulations indicated that high reproduction accuracy compared with other current methods can be maintained in the listening areas at high frequencies by using the proposed method.

*Index Terms*— Sound field reproduction, circular harmonics, mode-matching method, local reproduction, super-resolution

#### 1. INTRODUCTION

The physical reproduction of a sound field makes it possible to achieve high-fidelity audio systems. Most current methods are aimed at synthesizing the desired sound field over the entire space or around listener locations using an array of loudspeakers.

Wave field synthesis (WFS) [1, 2] is a method based on the Kirchhoff–Helmholtz or Rayleigh integrals. WFS is applied rigorously to a planar array of omnidirectional loudspeakers. Several approximation techniques have been derived to extend it to linear and other array geometries. When a planar array is used, the sound field in the half-space bounded by the loudspeaker array is reproduced. A method using a spherical or circular array is generally referred to as higher-order ambisonics (HOA) [3–6], which is based on sound field analysis and synthesis in a spherical or circular harmonic domain. HOA is basically aimed at reproducing the sound field around the center of the array; however, the reproduced region can be enlarged by increasing the maximum harmonic order. Since these methods are typically formulated under the assumption of a continuous secondary source (loudspeaker) distribution, we refer to them as *continuous distribution methods*.

Another type of sound field reproduction method is based on controlling sound pressures at discrete positions [7–9]. The inverse of a known transfer function matrix between loudspeakers and control points is numerically calculated. A similar but more sophisticated approach is the mode-matching method [4], which is aimed at controlling spherical or circular harmonics at listening positions so that they correspond to the desired ones. These methods can be inherently applied to arbitrary array geometries because a discrete arrangement of loudspeakers is assumed.

In several practical systems, it will be sufficient for only the specific sound field inside predefined listening areas to be reproduced with high accuracy. Additionally, it will be difficult to install a large number of loudspeakers in some situations. Therefore, in this paper, we propose a new efficient sound field reproduction method that exploits prior information on the location and range of the listening areas. An objective function is formulated as the expectation minimization of the spatial squared error of the sound pressure inside these listening areas. The solution is obtained by circular harmonic expansion of the sound field. By limiting the target reproduction areas, high reproduction accuracy can be achieved inside these areas at high frequencies even with a small number of loudspeakers. In addition, the proposed method can be applied to arbitrary array geometries owing to the assumption of a discrete arrangement of loudspeakers. The proposed method is closely related to the modematching method and has several advantages, which are discussed in Sec. 4 in detail.

Several methods for solving the problem of sound field reproduction in local regions have been proposed. Ahrens and Spors proposed a method based on the band limitation of spatial frequencies [10,11]. By controlling the spatial bandwidth of driving signals obtained by the continuous distribution method, the reproduction accuracy of the predefined directions or locations of the target sound field can be enhanced. In the mode-matching method, the reproduction accuracy in multiple local regions can be increased by using a translation technique [12, 13]. The translation operator enables multiple local harmonic coefficients to be associated with global harmonic coefficients. We conducted numerical simulations using circular and linear arrays to compare the proposed method with these state-of-the-art methods.

#### 2. PROBLEM STATEMENT

We assume that a desired sound field is synthesized by using L secondary sources located at  $r_l$   $(l \in \{1, \dots, L\})$ . The driving signal of the *l*th secondary source and its transfer function to position r are denoted as  $d_l(\omega)$  and  $G(r|r_l, \omega)$ , respectively, at temporal frequency  $\omega$ . The sound pressure at r synthesized by the secondary sources is represented as

$$p_{\rm syn}(\boldsymbol{r},\omega) = \sum_{l=1}^{L} d_l(\omega) G(\boldsymbol{r}|\boldsymbol{r}_l,\omega) = \boldsymbol{g}(\boldsymbol{r},\omega)^{\top} \boldsymbol{d}(\omega), \quad (1)$$

where  $d(\omega) = [d_1(\omega), \dots, d_L(\omega)]^\top$  is the vector of the driving signals and  $g(\mathbf{r}, \omega) = [G(\mathbf{r}|\mathbf{r}_1, \omega), \dots, G(\mathbf{r}|\mathbf{r}_L, \omega)]^\top$  is the vector of the transfer functions. Hereafter,  $\omega$  is omitted for notational simplicity. We here consider the case of a two-dimensional sound field for simplicity; however, the proposed method can be extended to the three-dimensional case with several modifications. Our objective is to synthesize a desired sound field inside a prioritized listening area with high reproduction accuracy. This can be formulated as the expectation minimization of the squared error of the sound pressure inside the area V as

minimize 
$$\mathcal{J} = \int_{\boldsymbol{r} \in V} |p_{\text{syn}}(\boldsymbol{r}) - p_{\text{des}}(\boldsymbol{r})|^2 \mathrm{d}\boldsymbol{r},$$
 (2)

where  $p_{\text{des}}(r)$  is the given desired sound pressure at r. We assume that V consists of single or multiple circular areas  $V_q$  ( $q \in \{1, \dots, Q\}$ ), where Q is the number of subareas. The center and radius of  $V_q$  are denoted as  $r_q$  and  $R_q$ , respectively. Using (1), the objective function  $\mathcal{J}$  can be rewritten as

$$\mathcal{J} = \sum_{q=1}^{Q} \int_{\boldsymbol{r} \in V_q} \left| \boldsymbol{g}(\boldsymbol{r})^{\top} \boldsymbol{d} - p_{\text{des}}(\boldsymbol{r}) \right|^2 \mathrm{d}\boldsymbol{r}.$$
 (3)

#### 3. DERIVATION OF OPTIMAL DRIVING SIGNALS

As in (3), the objective function  $\mathcal{J}$  is a quadratic form of the driving signals d; however, their coefficients include multiple integrals for which there is generally no analytical solution. We derive the driving signals d approximately by minimizing  $\mathcal{J}$  on the basis of the circular harmonic expansion. In polar coordinates  $\mathbf{r} = (r, \phi)$ , the solution of the Helmholtz equation can be expanded in terms of the circular harmonics as

$$p(r,\phi) = \sum_{m=-\infty}^{\infty} \alpha_m J_m(kr) e^{jm\phi},$$
(4)

where  $J_m(\cdot)$  represents the *m*th-order Bessel function of the first kind,  $k (= \omega/c)$  is the wave number, and *c* is the sound velocity. We define the basis function  $\varphi_m^{(q)}(\mathbf{r})$  as

$$\varphi_m^{(q)}(\mathbf{r}) = J_m(kr^{(q)})e^{jm\phi^{(q)}},$$
(5)

where  $(r^{(q)}, \phi^{(q)})$  denotes position r in polar coordinates whose origin is at  $r_q$ . In a similar manner, the transfer function and the desired sound pressure can be respectively represented as

$$G(\boldsymbol{r}|\boldsymbol{r}_l) = \sum_{m=-\infty}^{\infty} c_{m,l}^{(q)} \varphi_m^{(q)}(\boldsymbol{r}), \qquad (6)$$

$$p_{\rm des}(\boldsymbol{r}) = \sum_{m=-\infty}^{\infty} b_m^{(q)} \varphi_m^{(q)}(\boldsymbol{r}). \tag{7}$$

By truncating the maximum order of the circular harmonics to  $M_q$  in the summation of (6) and (7),  $\boldsymbol{g}(\boldsymbol{r})$  and  $p_{\rm des}(\boldsymbol{r})$  can be approximated as

$$\boldsymbol{g}(\boldsymbol{r})^{\top} \approx \boldsymbol{\varphi}^{(q)}(\boldsymbol{r})^{\top} \boldsymbol{C}^{(q)},$$
 (8)

$$p_{\rm des}(\boldsymbol{r}) \approx \boldsymbol{\varphi}^{(q)}(\boldsymbol{r})^{\top} \boldsymbol{b}^{(q)},$$
 (9)

where  $\boldsymbol{\varphi}^{(q)}(\boldsymbol{r}) \in \mathbb{C}^{2M_q+1}, \ \boldsymbol{C}^{(q)} \in \mathbb{C}^{(2M_q+1) \times L}$ , and  $\boldsymbol{b}^{(q)} \in \mathbb{C}^{2M_q+1}$  are defined as

$$\boldsymbol{\varphi}^{(q)}(\boldsymbol{r}) = \begin{bmatrix} \varphi_{-M_q}^{(q)}(\boldsymbol{r}) \\ \vdots \\ \varphi_{M_q}^{(q)}(\boldsymbol{r}) \end{bmatrix}, \qquad (10)$$

$$C^{(q)} = \begin{bmatrix} c_{-M_{q},1}^{(q)} & \dots & c_{-M_{q},L}^{(q)} \\ \vdots & \ddots & \vdots \\ c_{M_{q},1}^{(q)} & \dots & c_{M_{q},L}^{(q)} \end{bmatrix}, \quad (11)$$
$$\boldsymbol{b}^{(q)} = \begin{bmatrix} b_{-M_{q}}^{(q)} \\ \vdots \\ b_{M_{q}}^{(q)} \end{bmatrix}. \quad (12)$$

By substituting (8) and (9) into (3),  $\mathcal{J}$  is approximated as

$$\mathcal{J} \approx \sum_{q=1}^{Q} \int_{\boldsymbol{r} \in V_{q}} \left| \boldsymbol{\varphi}^{(q)}(\boldsymbol{r})^{\top} \left( \boldsymbol{C}^{(q)} \boldsymbol{d} - \boldsymbol{b}^{(q)} \right) \right|^{2} d\boldsymbol{r}$$

$$= \sum_{q=1}^{Q} \left\{ \left( \boldsymbol{C}^{(q)} \boldsymbol{d} - \boldsymbol{b}^{(q)} \right)^{\mathrm{H}} \cdot \int_{\boldsymbol{r} \in V_{q}} \boldsymbol{\varphi}^{(q)}(\boldsymbol{r})^{*} \boldsymbol{\varphi}^{(q)}(\boldsymbol{r})^{\top} d\boldsymbol{r} \left( \boldsymbol{C}^{(q)} \boldsymbol{d} - \boldsymbol{b}^{(q)} \right) \right\}$$

$$= \sum_{q=1}^{Q} \left( \boldsymbol{C}^{(q)} \boldsymbol{d} - \boldsymbol{b}^{(q)} \right)^{\mathrm{H}} \boldsymbol{W}^{(q)} \left( \boldsymbol{C}^{(q)} \boldsymbol{d} - \boldsymbol{b}^{(q)} \right), \quad (13)$$

where  $\boldsymbol{W}^{(q)} \in \mathbb{C}^{(2M_q+1) \times (2M_q+1)}$  and its elements are represented as

$$\boldsymbol{W}^{(q)} = \begin{bmatrix} w_{-M_q,-M_q}^{(q)} & \dots & w_{-M_q,M_q}^{(q)} \\ \vdots & \ddots & \vdots \\ w_{M_q,-M_q}^{(q)} & \dots & w_{M_q,M_q}^{(q)} \end{bmatrix}, \quad (14)$$

$$w_{m,n}^{(q)} = \int_{\boldsymbol{r} \in V_q} \varphi_m^{(q)}(\boldsymbol{r})^* \varphi_n^{(q)}(\boldsymbol{r}) d\boldsymbol{r}$$
  
=  $\int_0^{R_q} \int_0^{2\pi} J_m(kr) J_n(kr) e^{j(n-m)\phi} r d\phi dr$   
=  $2\pi \delta_{m,n} \int_0^{R_q} J_m(kr)^2 r dr$   
=  $\pi R_q^2 \delta_{m,n} \left\{ J_m(kR_q)^2 - J_{m-1}(kR_q) J_{m+1}(kR_q) \right\}.$  (15)

Here,  $\delta_{n,m}$  is the Kronecker delta, and this equation indicates that  $W^{(q)}$  becomes a diagonal matrix with positive values. A similar equation to the third line of (15) was derived by Betlehem and Abhayapala in the context of sound field reproduction in a reverberant environment [14]; however, we newly derived the fourth line of (15) as a closed-form analytical solution without numerical integration, referring to the mathematical formula [15] (Chapter 7, Sec. 14.1, (10)). Finally, the optimal driving signals  $\hat{d}$  that approximately minimize  $\mathcal{J}$  are given by

$$\hat{d} = \left(\sum_{q=1}^{Q} C^{(q)^{H}} W^{(q)} C^{(q)} + \lambda I\right)^{-1} \sum_{q=1}^{Q} C^{(q)^{H}} W^{(q)} b^{(q)},$$
(16)



**Fig. 1.** Comparison of weighting on circular harmonics for k = 36.93 rad/m and R = 0.4 m.

where  $\lambda$  is a regularization parameter and  $\boldsymbol{I}$  is the identity matrix.

There are two approaches to obtaining the circular harmonic coefficients of the transfer functions  $C^{(q)}$  and those of the desired sound field  $b^{(q)}$ . One is to analytically derive them, which is possible when  $G(\mathbf{r}|\mathbf{r}_l)$  and  $p_{des}(\mathbf{r})$  are analytically represented, such as in the case of point sources and plane waves [4–6]. Although these representations are generally used to synthesize a given desired sound field, they are also useful when  $p_{des}(\mathbf{r})$  is obtained by using a sparse sound field decomposition method [16, 17]. The other approach is to directly measure the circular harmonic coefficients in a recording area, for instance, by a circular microphone array [18, 19].

## 4. RELATIONSHIP WITH MODE-MATCHING METHOD

We discuss the relationship between the proposed method and the mode-matching method [4] because they are closely related. Since we consider the case of a single listening area, i.e., Q = 1, to simplify the comparison, the superscript (q) is omitted here. The mode-matching method based on the equalization of the circular harmonics at a single listening position, expressed as

$$Cd = b. \tag{17}$$

The driving signals are obtained by the least-squares solution of (17) as

$$\hat{\boldsymbol{d}} = \left(\boldsymbol{C}^{\mathrm{H}}\boldsymbol{C} + \lambda \boldsymbol{I}\right)^{-1} \boldsymbol{C}^{\mathrm{H}}\boldsymbol{b}.$$
 (18)

Although the mode-matching method is aimed at minimizing the modal squared error at the single listening position, it does not directly minimize the spatial error inside the region. On the other hand, the proposed method is obtained by formulating a minimization of the spatial squared error, which depends on the location and range of the listening area. By comparing (16) and (18), one can find that a difference between the two methods appears in the diagonal matrix W. An appropriate order truncation of the circular harmonics, i.e., the adjustment of M and the sizes of C and b, is required in the mode-matching method. Truncating as  $M = \lceil kR \rceil$  is empirically known to give a high performance [13]. In the proposed method, however, the optimal weighting on each order is automatically determined by calculating the weighting matrix W, even for an overdetermined case. Therefore, by setting a larger M, higher reproduction accuracy can be achieved. We plotted the diagonal elements of W (15) for k = 36.93 rad/m and R = 0.4 m in Fig. 1. For comparison, the lines of  $M = \lceil kR \rceil$  are also shown.

To apply the mode-matching method to the multiple-listeningarea scenario, a translation of the circular harmonics is generally used [12, 13]. Specifically, multiple local circular harmonics are obtained from global circular harmonics as a result of the translation. In contrast, the proposed method simultaneously minimizes the spatial squared error in the multiple listening areas.



**Fig. 2.** Relationship between frequency and SDR for circular array geometry.



Fig. 3. Relationship between frequency and SDR for linear array geometry.

### 5. EXPERIMENTS

Numerical simulations were conducted under the two-dimensional free-field assumption to evaluate the proposed method. We compared the proposed method (Proposed) with the mode-matching method (MM) [12, 13], the continuous distribution method (CD) [2, 4], and the continuous distribution method with band limitation (CD w/ BL) [10, 11]. Two array geometries, circular and linear, were investigated for the synthesis of a desired sound field; therefore, CD corresponds to HOA and WFS in the circular and linear array cases, respectively. In the circular array case, 64 loudspeakers were equiangularly aligned on a circle of radius 2.0 m centered at the origin in Cartesian coordinates. In the linear array case, 25 loudspeakers were equidistantly aligned at intervals of 0.16 m on the line y = -2.0 m. In both cases, the loudspeakers were assumed to be point sources. A single point source (cylindrical wave) was located at (0.0, -4.0) m as a desired sound field. We evaluated the reproduction accuracy of the four methods when the number of listening areas was set to Q = 1 and 2. For Q = 1, the listening area was a circular region of 0.8 m radius centered at the origin. For Q = 2, the listening areas were two circular regions centered at (-0.8, 0.8) m and (0.8, 0.8) m and their radii were 0.4 m.

In **MM**, the translation operator was used for reproduction in multiple listening areas. We determined the truncation order in **MM** using  $M_q = \lceil kR_q \rceil$ . In the proposed method, the truncation order was 150. Note that too large  $M_q$  deteriorates the reproduction accuracy in **MM**, as opposed to **Proposed**, whose order is truncated with  $W^{(q)}$ . In **CD**/w **BL** for the circular array case, the local circular harmonics were band-limited as  $M_q = \lceil kR_q \rceil$ , then the desired circular harmonics [10]. For the linear array case, the band limitation of the spatial frequency was determined by the angle between the



Fig. 5. Normalized error distribution.

source location and the listening area [11], i.e., from 1.37 to 1.77 rad for Q = 1 and from 1.32 to 1.49 rad and from 1.65 to 1.82 rad for Q = 2. In **Proposed** and **MM**, the regularization parameter  $\lambda$  was determined as  $\sigma_{\max}(\sum_{q=1}^{Q} \boldsymbol{C}^{(q)^{H}} \boldsymbol{W}^{(q)} \boldsymbol{C}^{(q)}) \times 10^{-6}$  and  $\sigma_{\max}(\sum_{q=1}^{Q} \boldsymbol{C}^{(q)^{H}} \boldsymbol{C}^{(q)}) \times 10^{-6}$ , respectively, where  $\sigma_{\max}(\cdot)$  obtains the maximum eigenvalue.

To evaluate the general reproduction accuracy, we used the signal-to-distortion ratio (SDR) defined as

$$SDR(\omega) = 10 \log_{10} \frac{\int_{\boldsymbol{r} \in V} |p_{des}(\boldsymbol{r}, \omega)|^2 d\boldsymbol{r}}{\int_{\boldsymbol{r} \in V} |p_{syn}(\boldsymbol{r}, \omega) - p_{des}(\boldsymbol{r}, \omega)|^2 d\boldsymbol{r}}.$$
 (19)

SDRs were calculated in the frequency domain to compare the theoretical reproduction performances. The spatial integrals in (19) were discretized with an interval of 0.01 m in the target areas.

Figures 2 and 3 show the relationship between the frequency of the source signal and the SDR. The left figures show the results for the case of Q = 1 and the right figures show those for the case of Q = 2. **Proposed** outperformed the other three methods except at low frequencies in the circular array geometry. In the circular array case, the SDR of **CD** was very high at low frequencies because **CD** does not include numerical computation of the inverse matrix; however, its reproduction accuracy sharply decreased above 1.0 kHz owing to spatial aliasing artifacts. Since the other three methods are aimed at maintaining the reproduction accuracy in the listening areas, their SDRs were higher than those of **CD**, particularly at high frequencies. Although the SDRs of **Proposed** and **MM** were almost the same at very high frequencies, for instance, above 2.2 kHz, in the circular array case, **Proposed** achieved the highest SDRs in many cases.

As an example, the reproduced sound pressure distribution for

the case of the circular array and Q = 2 is shown in Fig. 4. The black dots and dashed lines indicate the loudspeaker positions and listening areas, respectively. The frequency of the source signal was 2.0 kHz. The normalized error distribution is also shown in Fig. 5. Because of the high frequency, in **CD**, unwanted sound waves were generated as a result of spatial aliasing. The reproduction accuracy of **CD**/w **BL** in the listening areas was lower than those of **Proposed** and **MM**. The region with high reproduction accuracy was larger for **Proposed** than for **MM**. This is owing to the optimal weighting of the circular harmonics using  $W^{(q)}$ .

## 6. CONCLUSION

We proposed a sound field reproduction method that exploits prior information on listening areas. The objective function is formulated as the expectation minimization of the spatial squared error of the sound pressure inside these listening areas, and the optimal driving signals are obtained by circular harmonic expansion. By defining the location and range of the listening areas, an optimal weighting matrix of the circular harmonics,  $W^{(q)}$  in (16), is automatically determined. Numerical simulations indicated that high SDRs compared with other current methods can be maintained at high frequencies by using the proposed method. Although we here only considered synthesizing a common sound field inside single or multiple listening areas, the proposed method can be applied to the multizone sound field reproduction problem [20], which remains as a future work.

## 7. ACKNOWLEDGEMENTS

This work was supported by JSPS KAKENHI Grant Numbers JP15H05312 and JP16H01735 and SECOM Science and Technology Foundation.

#### 8. REFERENCES

- A. J. Berkhout, D. de Vries, and P. Vogel, "Acoustic control by wave field synthesis," *J. Acoust. Soc. Am.*, vol. 93, no. 5, pp. 2764–2778, 1993.
- [2] S. Spors, R. Rabenstein, and J. Ahrens, "The theory of wave field synthesis revisited," in *Proc. 124th AES Conv.*, Amsterdam, Oct. 2008.
- [3] J. Daniel, "Spatial sound encoding including near field effect: Introducing distance coding filters and a viable, new ambisonics format," in *Proc. 23rd AES Int. Conf.*, Copenhagen, May 2003.
- [4] M. A. Poletti, "Three-dimensional surround sound systems based on spherical harmonics," J. Audio Eng. Soc., vol. 53, no. 11, pp. 1004–1025, 2005.
- [5] J. Ahrens and S. Spors, "An analytical approach to sound field reproduction using circular and spherical loudspeaker distributions," *Acta Acustica United with Acustica*, vol. 94, pp. 988– 999, 2008.
- [6] Y. J. Wu and T. D. Abhayapala, "Theory and design of soundfield reproduction using continuous loudspeaker concept," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 17, no. 1, pp. 107–116, 2009.
- [7] O. Kirkeby and P. A. Nelson, "Reproduction of plane wave sound fields," J. Acoust. Soc. Am., vol. 94, no. 5, pp. 2992– 3000, 1993.
- [8] P. A. Nelson, "Active control of acoustic fields and the reproduction of sound," *J. Sound Vib.*, vol. 177, no. 4, pp. 447–477, 1993.
- [9] P.-A. Gauthier and A. Berry, "Sound-field reproduction inroom using optimal control techniques: Simulations in the frequency domain," *J. Acoust. Soc. Am.*, vol. 117, no. 2, pp. 662– 678, 2005.
- [10] J. Ahrens and S. Spors, "An analytical approach to sound field reproduction with a movable sweet spot using circular distributions of loudspeakers," in *Proc. IEEE Int. Conf. Acoust.*, *Speech, Signal Process. (ICASSP)*, Taipei, April 2009, pp. 273–276.
- [11] J. Ahrens and S. Spors, "An analytical approach to local sound field synthesis using linear arrays of loudspeakers," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Prague, May 2011, pp. 65–68.
- [12] H. Chen, T. D. Abhayapala, and W. Zhang, "Enhanced sound field reproduction within prioritized control region," in *Proc. INTER-NOISE*, Melbourne, Nov. 2014, pp. 4055–4064.
- [13] M. A. Poletti and T. Betlehem, "Creation of a single sound field for multiple listeners," in *Proc. INTER-NOISE*, Melbourne, Nov. 2014, pp. 2033–2040.
- [14] T. Betlehem and T. D. Abhayapala, "Theory and design of sound field reproduction in reverberant rooms," J. Acoust. Soc. Am., vol. 117, no. 4, pp. 2100–2111, 2005.
- [15] H. Bateman, *Higher transcendental functions*, vol. 2, McGraw-Hill, New York, 1955.
- [16] S. Koyama, S. Shimauchi, and H. Ohmuro, "Sparse sound field representation in recording and reproduction for reducing spatial aliasing artifacts," in *Proc. IEEE Int. Conf. Acoust.*, *Speech, Signal Process. (ICASSP)*, Florence, May 2014, pp. 4443–4447.

- [17] S. Koyama, N. Murata, and H. Saruwatari, "Structured sparse signal models and decomposition algorithm for superresolution in sound field recording and reproduction," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Brisbane, Apr. 2015, pp. 619–623.
- [18] J. Meyer and G. Elko, "A highly scalable spherical microphone array based on an orthogonal decomposition of the soundfield," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, Orlando, May 2002, pp. II–1781–1784.
- [19] S. Koyama, K. Furuya, K. Wakayama, S. Shimauchi, and H. Saruwatari, "Analytical approach to transforming filter design for sound field recording and reproduction using circular arrays with a spherical baffle," *J. Acoust. Soc. Am.*, vol. 139, no. 3, pp. 1024–1036, 2016.
- [20] Y. J. Wu and T. D. Abhayapala, "Spatial multizone soundfield reproduction: Theory and design," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 19, no. 6, pp. 1711–1720, 2011.