REWEIGHTED BLOCK-BASED COMPRESSED SENSING USING SINGULAR VALUE DECOMPOSITION

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ABSTRACT

Existed inherent sparsity of natural images in some domains helps to reconstruct the signal with a lower number of measurements. To benefit from the sparsity, one should solve the *reweighted* ℓ_1-norm minimization algorithms. Although, the existed reweighted ℓ_1-norm minimization approaches work well for k-sparse signals, but, the performance of these methods for compressible signals are not competitive with unweighted one. Motivated by this challenge, in this paper, we propose a new reweighted ℓ_1-minimization algorithm based on singular value decomposition (SVD) of compressible signals like images. Moreover, we develop our proposed algorithm on the block-based compressed sensing (BCS) to make it applicable to large-size images. Simulation results also demonstrated the superiority of our proposed method over current state-of-the-art reweighted CS reconstruction algorithms for natural images.

Index Terms— Reweighted Compressed Sensing, Block-Based compressed sensing, Singular Value Decomposition.

1. INTRODUCTION

The signal reconstruction from a low number of sampling measurements is one of the popular aims of the signal processing. A conventional sampling method is Nyquist-Shannon theorem which states that if the signal's highest frequency is less than half of the sampling rate, the signal could be reconstructed perfectly. This sampling rate could be so high for most of the applications which makes the compression necessary after sampling process. In 2006, it is proved that the existence of some structure like sparsity as a side information could help to reduce the required number of samples for signal reconstruction which resulted in introducing the theory of Compressed Sensing (CS) [1].

There are several reconstruction algorithms for CS, such as the basis pursuit (BP) algorithm, total variation (TV) algorithm and the iterative thresholding (IT) algorithm. Recently, in order to achieve better performance, *reweighted CS* is developed in [2]. It is proved that replacing the ℓ_1-norm with a weighted ℓ_1-norm could often eventuate in a better recovery performance of the sparse signals. The weighted CS approach of [2] utilizes an iterative algorithm to determine weights as a function of the reconstructed signal in the previous iteration. Practically the ℓ_1-norm is replaced by logarithm function and then the popular Majorization-Minimization (MM) algorithm is used to solve the problem [2]. Although in [2] the performance of recovery for sparse signals has been enhanced, but as will be shown in this paper, reweighted ℓ_1-minimization approach still has not a better performance for compressible signals such as natural images and videos compared to unweighted CS.

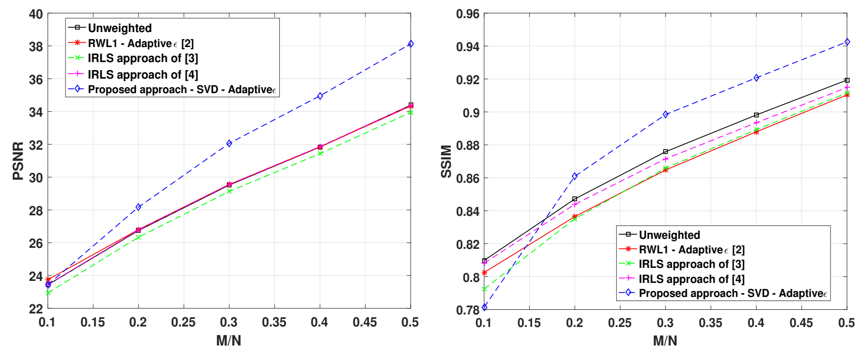
In this paper, unlike the conventional weighted CS approaches [2–4], we propose to design weights based on the SVD of natural images instead of signal values. Our proposed approach is also different from [5] in the sense that the prior information of activity of each entry of unknown signals is not used. But, since the SVD of the signal has all required information about the signal, we take advantage by using the SVD of unknown signal. Moreover, we develop our proposed algorithm on the block-based compressed sensing [6] to make it applicable to the large-size images.

2. PROPOSED APPROACH

Our aim is to propose a novel weighted compressed sensing method which can have suitable performance for compressible signals. To exhibit the relevancy between the previous algorithms and our proposed method, consider the following reweighted problem of [2]:

$$\begin{aligned} &\min_{\mathbf{s}\in\mathbb{R}^N} \sum_{i=1}^{N} \log\left(|s_i| + \varepsilon\right) \\ &\text{s.t. } \mathbf{y} = \underbrace{\mathbf{\Phi\Psi}}_{\boldsymbol{\theta}}\mathbf{s} \end{aligned} \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^M$ is a measurement vector and $\mathbf{s} \in \mathbb{R}^{N\times 1}$ is k-sparse signal which could be vectorized version of the some multidimensional signals ($\mathbf{x} \in \mathbb{R}^{N\times 1}$) like image and video in some basis $\mathbf{\Psi} \in \mathbb{R}^{N\times N}$, (i.e. $\mathbf{x} = \mathbf{\Psi s}$). Assume that the signal $\mathbf{x}$ is sampled by $M \times N$ linear measurement matrix $\mathbf{\Phi}\,(M < N)$. Since the synthetic exact k-sparse signal is assumed in [2], then $\mathbf{\Psi} = \mathbf{I}$ in (1). The trivial approach for extending the method of [2] for natural images is assuming $\mathbf{\Psi}$ as a DCT domain or wavelet domain or other ones. Our aim is to define some better transform domain in which the reweighted minimization problem work well. To explain our proposed approach, consider that $\mathbf{X} \in \mathbb{R}^{N_r\times N_c}$ represents the compressible two dimensional signal like image matrix. We partitioned the image into $B \times B$ blocks. Let $\mathbf{X}_j$ denotes j'th block of the input image $\mathbf{X}$ through raster scanning and vectorized representation of that as $\mathbf{x}_j \in \mathbb{R}^{B^2\times 1}$. To find the mentioned transform domain, we propose to use the vectorized representation of the SVD of $\mathbf{X}_j$. Note that the motivation behind the approach of [2] was that the larger coefficients of the signal are penalized more than smaller coefficients. Now, in this paper, we propose to penalize the magnitude of the singular values of the multidimensional signal in a similar way. To do this, consider that $\mathbf{X}_j = \mathbf{U}_j\mathbf{\Sigma}_j\mathbf{V}_j^T$, where $\mathbf{U}_j$ and $\mathbf{V}_j$ are real *unitary matrices*, and $\mathbf{\Sigma}_j$ is a *diagonal matrix* with non-negative real diagonal elements. The diagonal entries of $\mathbf{\Sigma}_j$ are known as the *singular values* of $\mathbf{X}_j$. By some simple linear algebra, the vectorized version of $\mathbf{X}_j$ can be written as



(a) PSNR (dB)

(b) SSIM

Fig. 1: PSNR and SSIM values vs sampling ratio for state-of-the-art algorithms and proposed algorithm with adaptive ε for Mondrian image.

$\mathbf{x}_j = vec\left(\mathbf{X}_j\right) = \left(\mathbf{V}_j \otimes \mathbf{U}_j\right)\mathbf{z}_j$. Therefore, we transformed the $\mathbf{x}_j$ into singular value domain using $\mathbf{\Psi}_j = \left(\mathbf{V}_j \otimes \mathbf{U}_j\right)$. It is noteworthy that $\mathbf{\Psi}_j$ depends on the signal to be recovered. Hence, we use a solution of unweighted ℓ_1-minimization algorithm to compute $\mathbf{\Psi}_j$. Consequently, similar to (1), we propose to reconstruct the image using the following reweighted optimization problem:

$$\begin{aligned} &\min_{\mathbf{z}_j \in \mathbb{R}^{B^2 \times 1}} \sum_{i=1}^{B^2} \log\left(|z_{j,i}| + \varepsilon\right) \\ &\text{s.t. } \mathbf{y}_j = \mathbf{\Phi}_B\left(\mathbf{V}_j \otimes \mathbf{U}_j\right)\mathbf{z}_j \end{aligned} \tag{2}$$

where $\mathbf{y}_j \in \mathbb{R}^{M_B \times 1}$ and $\mathbf{\Phi}_B \in \mathbb{R}^{M_B \times B^2}$ are given measurements vector and measurement matrix and ε is also a positive parameter in order to provide stability. Similar to [2], we use the popular and simple MM algorithm to solve the optimization problem of (2). Hence, using some simple linear algebra, the optimization problem of (2) becomes as follows:

$$\begin{aligned} &\mathbf{z}_j^{(l+1)} = \arg\min \left\|\mathbf{W}_j^{(l+1)}\mathbf{z}_j\right\|_{\ell_1} \\ &\text{s.t. } \mathbf{y}_j = \mathbf{\Phi}_B\left(\mathbf{V}_j \otimes \mathbf{U}_j\right)\mathbf{z}_j \end{aligned} \tag{3}$$

where $\mathbf{W}_j^{(l+1)}$ refers to a diagonal matrix with $\left\{w_{j,i}^{(l+1)}\right\}_{i=1}^{B^2}$ as a diagonal elements. Hence, for each i these weights are equal to $w_{j,i}^{(l+1)} = \frac{1}{\left|z_{j,i}^{(l)}\right| + \varepsilon}$.

3. EXPERIMENTS

Some experimental results are presented to illustrate the performance of the proposed method. Also, the SparseLab[1] software is used for solving the ℓ_1-minimization problems. We employ blocks of size $B = 16$ for our simulations. We also assume that the maximum number of iterations is equal to $l_{max} = 3$. To investigate the effectiveness of our proposed approach for reconstruction of multidimensional signal, it is compared with four state-of-the-art methods [2–4]. Since the parameter ε is effective for stability incredibly, this parameter should be adaptively selected based on the singular value of the reconstructed signal in each iteration. By this way there is no need to know the ε already. We should also note that this parameter is chosen in a decreasing sequence in each iteration.

[1] http://sparselab. stanford. edu

Fig. 1 demonstrates the performance of the proposed algorithm with adaptive ε for the Mondrian test image. As it is obvious, our proposed SVD-based algorithm outperforms the other state-of-the-art algorithms. As also expected, performance of the proposed method is improved when the available number of measurements increase. More simulations and discussions can be find in [7].

4. DISCUSSION AND CONCLUSIONS

In this paper, we proposed a novel block-based weighted ℓ_1-minimization algorithm based on SVD for reconstruction of the compressible signals like natural images. In our algorithm weights are updated based on the singular values and not based on the signal values. Besides, we develop our proposed algorithm on the block-based compressed sensing to make it applicable to large-size images. Finally, simulation results demonstrated the superiority of our proposed algorithm for images. Since the eigenvectors of the unweighted CS reconstructed signal is used to construct a sparsifying basis in this paper, as a future work, one could propose better way to approximate these eigenvectors.

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