Multilinear Parameter Estimation and Applications Based on Tensor Decomposition

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I. Introduction

Parameter estimation has always played a key role in signal processing. The conventional methods are based on vector or matrix modeling since the completeness of matrix theory. However, the real-world signal often has the higher-order tensor structure. For instance, the color image can be described as a third-order tensor data including RGB color and coordinate information; similarly, a MIMO channel transfer function can be formulated as a higher-order harmonic signal. Such tensor signal maintains certain structural information among each mode, which would inevitably lost by dimension reduction process. In order to utilize the structural information, tensor decomposition framework has been introduced into signal processing in recent years [1]. Tensor modeling possesses many benefits, e.g., Tucker decomposition (TKD) can achieve a high efficiency in data compression [2]; the uniqueness of canonical polyadic decomposition (CPD) will facilitate the signal identification [3]. In this research, two kinds of parameter estimation problems have been investigated: 1. Arrival angles estimation in array signal processing; 2. SAR imaging for block sparse scattering targets.

II. Arrival Angles Estimation by CPD Modeling

This research focuses on the two-dimensional angle-of-arrival (2D-AOA) estimation based on the multiple invariance feature and uniqueness of CPD, and exploits the cross correlation matrix (CCM) information of L-shaped array manifold to achieve pair-matching for estimated angles [4]. In consideration of the Vandermonde structure, the received signal of each axis can be segmented into P parts,

$$\boldsymbol{X}_p = \boldsymbol{A} D_p(\boldsymbol{\Psi}) \boldsymbol{S}^T, \quad p = 1, 2, ..., P, \tag{1}$$

where $\boldsymbol{S} \in \mathbb{C}^{N \times K}$ is the source signal, $\boldsymbol{A} \in \mathbb{C}^{M' \times K}$ is the sub-steering matrix and $D_p(\boldsymbol{\Psi})$ denotes a diagonal matrix consisted by the pth row of factor matrix $\boldsymbol{\Psi} \in \mathbb{C}^{P \times K}$. It is found that (1) matches the CPD model [1] and $\boldsymbol{X}_p$ can be considered as the pth slice matrix of received signal tensor $\mathcal{X} \in \mathbb{C}^{M' \times N \times P}$. If combined all slices into a rectangular matrix, (1) can be written as $\boldsymbol{X}_{(1)} = (\boldsymbol{\Psi} \odot \boldsymbol{S}) \boldsymbol{A}^T$, where $\odot$ denotes the Khatri-Rao product [1]. Hence it is possible to estimate the spacial frequency by using tenor decomposition. A typical algorithm to handle such problem is alternating least squares (ALS), which updates one factor with the others fixed by LS approach until the convergence condition is met. By introducing the Vandermonde feature as constraint, the sub-steering matrix estimation problem can be formulated as

$$\min \left\| \boldsymbol{X}_{(1)} - (\boldsymbol{\Psi} \odot \boldsymbol{S}) \boldsymbol{A}^T \right\|_F^2, \quad s.t. \quad \boldsymbol{A}_2 = \boldsymbol{A}_1 D_2(\boldsymbol{\Psi}), \tag{2}$$

where $\boldsymbol{A}_1, \boldsymbol{A}_2$ consist of the first and last $(M' - 1)$ rows of $\boldsymbol{A}$, respectively. Then the estimate of sub-steering matrix $\hat{\boldsymbol{A}}$ can be obtained. As a result, the spacial frequency can be calculated by typical subspace methods [5]. The CCM information is introduced to accomplish the pair-matching between the estimates of azimuth and pitch angles [6]. It is deduced that the uniqueness condition of this scenario is $\min(M', K) + \min(P - 1, K) \geqslant K + 1$, indicating that the uniqueness will be guaranteed even if the number of rows in steering matrix is less than the number of sources, which makes CPD-based method more flexible than classic subspace methods on applicability. For the coherent situation, the subspace of $\boldsymbol{A}$ can still be obtained by TKD modeling.

The RMSE curves of the proposed algorithm and reference methods are shown in Fig. 1. It is demonstrated that the proposed method is superior to the other methods. Note that both tensor-based methods achieve lower RMSE than the classic subspace methods. Besides, compared with the standard CPD algorithm, the proposed method performs better even in severe noise situation.

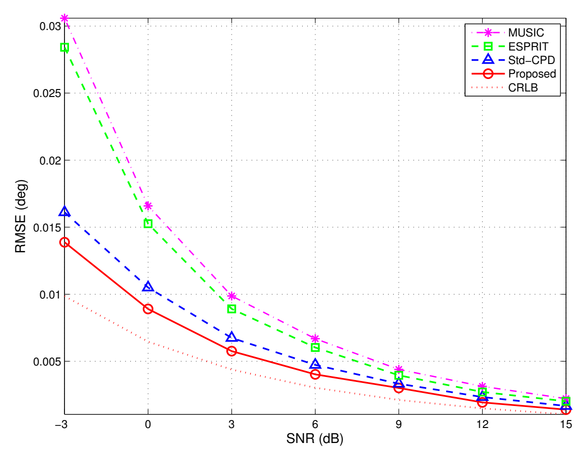

Fig. 1. RMSE versus SNRs (2D-AOA estimation).

III. SAR Imaging by TKD Modeling

Sparse reconstruction approach has been applied in synthetic aperture radar (SAR) imaging over the last decade, which can obtain necessary information by the sampling rate much lower than Nyquist limit [7]. However, the previous work only considered the sparsity of the signal without utilizing the structural characteristics of targets. In real-world applications of SAR imaging, the target scatterers always possess the block sparse feature. To ameliorate the above problems, we introduce a tensor decomposition framework which can exploit such geometry feature and maintain a lower resource requirement.

For a point scattering SAR imaging model, the phase history need to achieve decoupling by interpolation procedure. In consideration of the Kronecker structure in steering matrix, the received signal can be represented as a TKD model, $\boldsymbol{y} = (\boldsymbol{D}_N \otimes \boldsymbol{D}_{N-1} \otimes ... \otimes \boldsymbol{D}_1)\, \boldsymbol{s}$, where $\boldsymbol{s}$ is the scattering coefficients, $\boldsymbol{D}_n$ is the factor matrix including azimuth, pitch angles and frequency information. Since the target scatterers usually clustered together as blocks, we can reformulate the scattering coefficients estimation as multi-way block sparse reconstruction problem. Define $\boldsymbol{B}_n = \boldsymbol{D}_n\,(:, \mathscr{I}_n)$, where $\mathscr{I}_n = \left[i_n^1, i_n^2, ..., i_n^{K_n}\right]$ is a subset of index for mode-n. Then the received signal can be rewritten as $\hat{\boldsymbol{y}} = (\boldsymbol{B}_N \otimes \boldsymbol{B}_{N-1} \otimes ... \otimes \boldsymbol{B}_1)\, \boldsymbol{s}_{nz}$, where $\boldsymbol{s}_{nz} \in \mathbb{R}^K$ is the vectorization of all nonzero coefficients. As a consequence, the solution of the problem can be given as

$$\boldsymbol{s}_{nz} = \arg\min_{\boldsymbol{u}} \left\| (\boldsymbol{B}_N \otimes \boldsymbol{B}_{N-1} \otimes ... \otimes \boldsymbol{B}_1)\, \boldsymbol{u} - \boldsymbol{y} \right\|_2^2. \quad (3)$$

Besides, a more efficient calculation step can be utilized by introducing Cholesky decomposition [8]. All nonzero elements of core tensor will be included in a subset of index during iteration and the expected scattering coefficients can be recovered as well. The algorithm complexity analysis indicates that the proposed algorithm can achieve convergence with much fewer iterations comparing with the classic l_0 technique. Moreover, it has been proven that the reconstruction method using Kronecker structure has much less severe requirements for coherence than the classic matching pursuit methods with the same sparsity, and it is deduced that the former has the higher successfully recovery bound.

The realizations of SAR imaging based on practical measured data are shown in Fig. 2. The observation target is a crawler-type engineering vehicle. The data is collected from three pitch central angles: 30°, 40°, and 50°, and the sparsity is set to 200. The reference methods are PFA, OMP and CoSaMP.

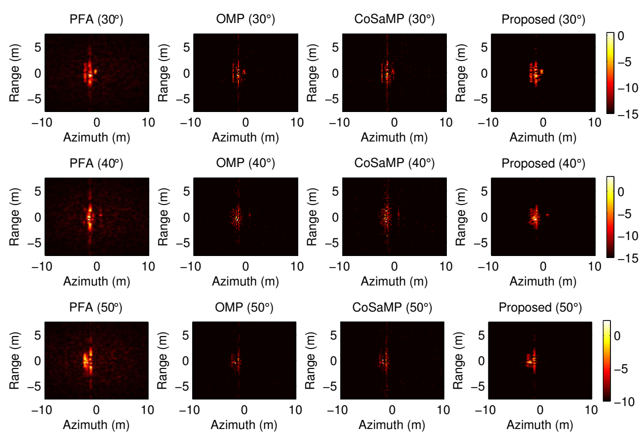

Fig. 2. SAR imaging of different observation angles.

The RMSE curves of the proposed algorithm and reference methods are shown in Fig. 3. The solid lines and dashed lines represent the simulations for 10 scatterers and 50 scatterers, respectively. It is demonstrated from the results that the performance of the proposed method is superior to the others for both the cases of small number and large number of scattering points, especially at low SNR. In addition, the computational cost experiments indicate that the proposed method could run not only faster one order of magnitude than CoSaMP, meanwhile maintain a better performance, which has significance in practical applications.

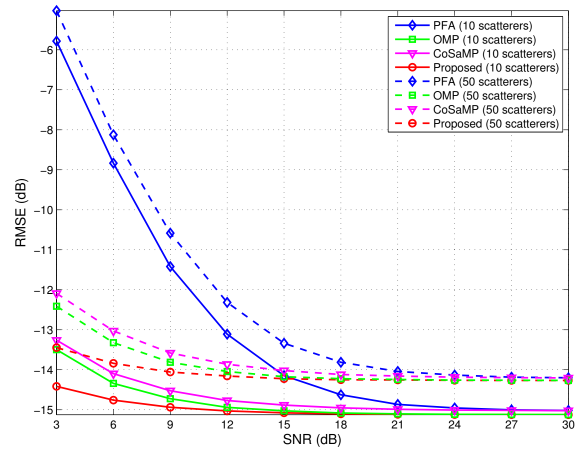

Fig. 3. RMSE versus SNRs for different number of scatterers (SAR imaging).

IV. Conclusion and Future Work

This Ph.D. work mainly investigates the parameter estimation algorithms based on tensor decomposition. Two application fields, array signal processing and SAR imaging, have been included in this research and achieved the expected results. The future work will focus on the multi-parameter estimation of polarization sensitive array and acoustic vector hydrophone based on block term decomposition.

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