NON-CONVEX SPARSE OPTIMIZATION FOR PHOTON-LIMITED IMAGING

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ABSTRACT

While convex optimization for low-light imaging has received some attention by the imaging community, non-convex optimization techniques for photon-limited imaging are still in their nascent stages. In this thesis, we developed a stage-based non-convex approach to recover high-resolution sparse signals from low-dimensional measurements corrupted by Poisson noise. We incorporate gradient-based information to construct a sequence of quadratic subproblems with an ℓ_p-norm ($0 \leq p < 1$) penalty term to promote sparsity. The proposed methods lead to more accurate and high strength reconstructions in medical imaging applications such as bioluminescence tomography and fluorescence lifetime imaging.

Index Terms— Photon-limited imaging, Poisson noise, ℓ_p-norm, time-dependent bioluminescence tomography, fluorescence lifetime imaging

1. INTRODUCTION

Acquisition of a sparse signal from an undersampled set of linear measurements is the main problem of compressed sensing (CS). Within the CS community, minimizing the ℓ_1-penalized least-squares problem also known as LASSO, is the most popular approach for sparse signal recovery. The least-squares data-fidelity term assumes a Gaussian noise model. However, there are many real world applications that do not follow Gaussian noise statistics. For an instance, when the number of observed photon counts is relatively low at the camera detector, they follow a Poisson distribution. This phenomena can be seen in a variety of different applications including atmospheric imaging, astronomy, night vision, and medical imaging such as bioluminescence tomography and fluorescence lifetime imaging. Accurate recovery of sparse signals from Poisson noise corrupted measurements is notoriously more difficult than the LASSO problem. It requires the development of new methods and algorithms that exploit the sparsity of the signal and model the system noise more accurately. In this work, we explicitly model noise using Poisson statistics and further enforce sparsity and structure in the solution using the ℓ_p-norm ($0 \leq p < 1$), which can be viewed as a bridge between the convex ℓ_1-norm and ℓ_0 counting seminorm.

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Significance: We build upon the recent Sparse Poisson Intensity Reconstruction ALgorithm (SPIRAL) [1] framework for solving photon-limited imaging problems. Our approach is different in the following manner: (1) We incorporate a non-convex ℓ_p-norm regularization to promote further sparsity in the solution, (2) the p-value can be tuned to highlight different structural properties of the signal, and (3) we solve time-dependent sparse recovery problems in several steps; in particular, we recover the support of the signal using the time-averaged data and reconstruct the signal intensity using the time-dependent data.

2. PROBLEM FORMULATION

The arrival of photons at the charge-couple device (CCD) camera is modeled by the Poisson process: $\mathbf{y} \sim \text{Poisson}(\mathbf{A}\mathbf{f}^*)$, where $\mathbf{f}^* \in \mathbb{R}^n_+$ is the true signal or image of interest, $\mathbf{A} \in \mathbb{R}^{m\times n}_+$ is the linear projection matrix ($m \ll n$), and $\mathbf{y} \in \mathbb{Z}^m_+$ is a vector of observed photon counts. Under the Poisson process model, $\mathbf{f}^*$ is estimated by minimizing the following constrained optimization problem:

$$\begin{aligned} \hat{\mathbf{f}} \quad = \quad & \underset{\mathbf{f}\in\mathbb{R}^n}{\text{minimize}} \quad \Phi(\mathbf{f}) \equiv F(\mathbf{f}) + \tau\, \text{pen}(\mathbf{f}) \\ & \text{subject to} \quad \mathbf{f} \succeq 0, \end{aligned} \tag{1}$$

where $F(\mathbf{f})$ is the negative Poisson log-likelihood function $F(\mathbf{f}) = \mathbf{1}^T A\mathbf{f} - \sum_{i=1}^m y_i \log(\mathbf{e}_i^T A\mathbf{f} + \beta)$, where $\mathbf{1}$ is the m-vector of ones, $\mathbf{e}_i$ is the i-th canonical basis unit vector, $\beta > 0$ (typically $\beta \ll 1$), pen : $\mathbb{R}^n \to \mathbb{R}$ is a sparsity-promoting penalty functional, and $\tau > 0$. In this work, we consider $\text{pen}(\mathbf{f})$ as $\|\mathbf{f}\|_p^p$ ($0 \leq p < 1$) to enhance the sparsity of the reconstruction better than the ℓ_1-norm regularization [1].

We solve the ℓ_p-norm penalized optimization problem (1) by minimizing a sequence of quadratic models. In particular, the resulting subproblems are uncoupled into a sequence of scalar minimization problems in the general form:

$$f_s^* \quad = \quad \underset{f\geq 0}{\arg\min} \quad \frac{1}{2}(f-s)^2 + \lambda|f|^p, \tag{2}$$

where s and λ scalars are known [2]. The resulting scalar optimization problem (2) is solved using the *generalized soft-thresholding function* – a zero finding method such as Newton's method or fixed-point iteration method is used along with a threshold value to find the global minimum of (2) [3].

3. APPLICATIONS

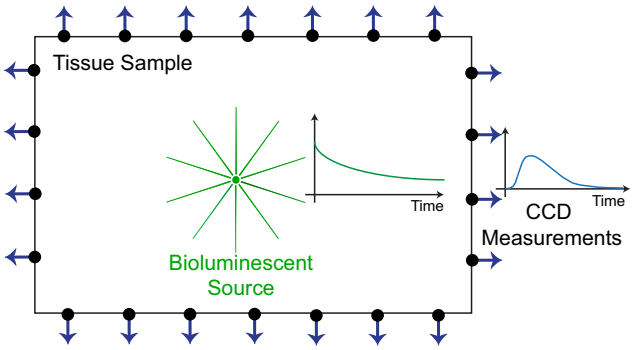

Fig. 1. Schematic diagram of the time-dependent bioluminescence tomography. CCD camera captures the decaying photon-count measurements at all four boundaries.

Time-dependent bioluminescence tomography: In time-dependent bioluminescence tomography (BLT), we seek to recover the decaying light sources within a tissue sample from boundary measurements captured by a CCD camera (see Fig.1). Here, we propose a novel two-stage method to solve the ill-posed time-dependent BLT inverse problem [4]. Unlike previous methods, the first stage of our approach uses our nonconvex Poisson noise-based sparsity promoting method to recover the support using the time-averaged data. In the second stage, we use the determined support from stage one to recover the characteristic time decay using the time-dependent data. In the experiment with 5% Poisson noise corrupted boundary measurements, we recovered the support accurately and the characteristic decay rate is approximated as 1.53, while the true decay rate is 1.50 (see Fig. 2).

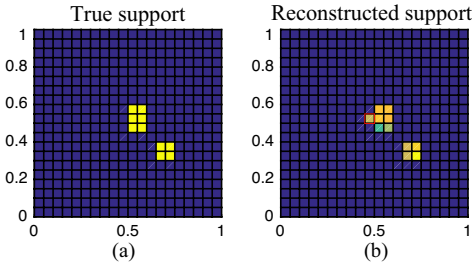

Fig. 2. Spatial support of two group of bioluminescent sources from the time-averaged data. (a) True locations of the sources. (b) Reconstructed support using our nonconvex sparsity promoting approach. Note that there is a spurious support in the reconstruction which is marked by red color box.

Fluorescence lifetime imaging: To solve the fluorescence lifetime imaging problem using the CCD camera measurements with lower exposure time, we propose a three-stage based method [5]. Similar to the BLT approach, we recover the support of the fluorophores in the first-stage using the time-averaged measurements. In the second stage, we recover the excited fluorescence source amplitude with the given support and time-dependent data (see Fig. 3). In the third-stage, we apply a nonlinear least-squares solver to recover the fluorophore concentration and the lifetime τ. We evaluate the propose method using two experiments: (1) with two point sources, $\tau \approx 5.64$, (2) with the two island of sources, $\tau \approx 5.76$ (true $\tau^* = 5.7$ for both experiments).

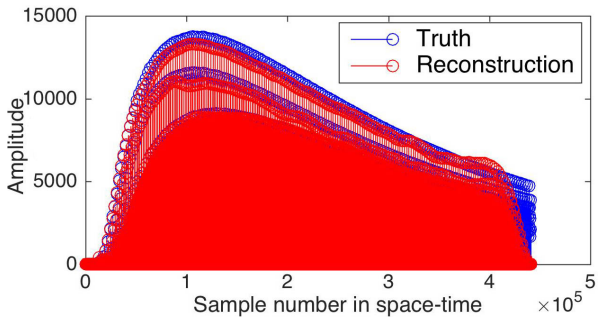

Fig. 3. Fluorescence point source amplitude reconstruction with the given support and 7.5% Poisson noise corrupted time-dependent measurements.

4. CONCLUSION

We have developed and implemented a fast stage-based nonconvex sparsity promoting method that leads to more accurate and high strength reconstructions with the applications to medical imaging. Results for applications to real phantom data measurements in fluorescence molecular tomography are underway.

5. REFERENCES

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