QUATERNION MULTIPLIERS BASED PARAUNITARY FILTER BANKS FOR TRANSFORM IMAGE CODING WITH FIXED POINT ARITHMETIC CONSTRAINTS

Nick A. Petrovsky

Department of Computer Engineering, Belarusian State University
of Informatics and Radioelectronics, Minsk, Belarus
nick@petrovsky.eu

ABSTRACT

In this work, we have introduced a systematic design of the integer-to-integer invertible quaternionic multiplier based on a generalized block-lifting structure using two alternative techniques: CORDIC architectures and distributed arithmetic (DA) as a block of M-band linear phase paraunitary filter banks LP Q-PUFB for the lossless-to-lossy (L2L) image coding. The low latency separable image processing in real-time is implemented on the Xilinx Zynq.

Index Terms— quaternion, filter bank, L2L image coding

1. BACKGROUND

A high-performance filter bank (FB) is typically at the heart of every state-of-the-art digital multimedia system as the one of the most efficient techniques to compress signals [1]. FBs are adopted in audio, image, and video coding standards such as JPEG, JPEG 2000, JPEG XR, MPEG, and H.264/AVC [2]. Image coding has lossy and lossless mode. Lossy image coding is adopted to Internet contents and mobile etc., and lossless image coding is used in high-end hardware for medical images, remote sensing, image archiving, satellite communications, and so on. Then lossless and lossy compressed data are usually independent from each other. With the rapid development of Internet and multimedia technology, the uniform of lossy and lossless compressed data (L2L mode) is an important problem and it's demanded to obtain higher quality and compression ratio. To apply FBs for L2L image coding, they are required to be integer-to-integer transforms. Several lifting-based FBs [3] are widely researched for the unification of lossy and lossless data [4, 5]. However, they are not practical because [4] can not achieve good coding performance due to many rounding operations, and [5] may have wide dynamic range of lifting coefficients due to inverse matrix. An efficient lifting structure for L2L image coding is implemented by the block parallel system [6] of DCT and inverse DCT devices [7]. One advantage of this structure is that any existing DCT device can be directly applied to L2L image coding even if they are designed for only lossy image coding. The main disadvantage of the given ladder structure is that the transformed signals by IDCT are not suitable for image compression because the frequency response of IDCT includes a lot of DC leakages. The realization of Integer DCT for L2L image coding has been proposed in [7] using an $M \times M$ side information block (SIB) initialized by null matrix and applied iteratively. SIB is also encoded with all forward transform coefficients. It's size is the same as that of DCT.

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2. THE MAIN OBJECTIVE OF THE WORK

The **main objective of this work** is to construct block-lifting-based FBs for L2L image coding. Perfect reconstruction and linear phase, regularity are essential desirable properties of FBs for image coding as it is associated with the smoothness of the related wavelet basis. The ability to design a FB is of an extreme importance. PUFBs have been of great interest over recent years, especially in image coding. New factorizations and structures appear regularly, offering new useful features, design flexibility and ease, or computational efficiency [1]. In this context, the authors have recently presented a novel concept of quaternionic building block suitable for many existing structures of FBs and transforms, especially for those with 4 and 8 channels (Q-PUFB), commonly used in image applications [8, 9]. The quaternion algebra $\mathbb{H}$ is an associative non-commutative four-dimensional algebra $\mathbb{H} = \{\mathbf{q} = q_1 + q_2 i + q_3 j + q_4 k | q_1, q_2, q_3, q_4 \in \mathbb{R}\}$ where the orthogonal imaginary numbers obey the following multiplicative rules: $i^2 = j^2 = k^2 = ijk = -1$, $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$. This becomes evident after identifying quaternions with vectors and writing the operation in matrix notation. The matrix-vector forms of non-commutative quaternion product use the multiplication matrices where the left- $\mathbf{M}^+(q)$ and the right-operand $\mathbf{M}^-(q)$ allowing modelling 4×4 orthogonal matrices $\underset{\mathbf{R} \in SO(4)}{\forall} \quad \underset{P,Q \in \text{ unit quat.}}{\exists} \quad \mathbf{R} = \mathbf{M}^+(P) \cdot \mathbf{M}^-(Q) = \mathbf{M}^-(Q) \cdot \mathbf{M}^+(P)$ directly (contrary to Givens rotations) to preserve their orthogonality in spite of quantization. Unlike the conventional algorithms, the proposed computational schemes of Q-PUFB maintain losslessness regardless of their coefficient quantization. Moreover, the one regularity conditions can be expressed directly in terms of the quaternion lattice coefficients and thus easily satisfied even in finite-precision arithmetic.

3. METHODOLOGICAL APPROACH

The **research goal** of the presented work is structurally orthogonal finite precision implementation of the quaternion multipliers-based PUFBs (Q-PUFB) for the L2L image coding.

These algorithms use quaternion multiplications in which one of the operands is a constant with unit magnitude. The left-operand multiplication matrix $\mathbf{M}^+(q)$ can be of the following structure ($\mathbf{M}^+(\overline{q})$ and $\mathbf{M}^-(q)$ factorized the same way): $\mathbf{M}^+(q) = \begin{bmatrix} \mathbf{C}(q) & -\mathbf{S}(q) \\ \mathbf{S}(q) & \mathbf{C}(q) \end{bmatrix}, \mathbf{C}(q) = \begin{bmatrix} q_1 & -q_2 \\ q_2 & q_1 \end{bmatrix}, \mathbf{S}(q) = \begin{bmatrix} q_3 & q_4 \\ q_4 & -q_3 \end{bmatrix}$. Taking into account the previously mentioned matrix and being inspired by the three-step lifting factorization for 2-D rotations, we have introduced its analogue for 4-D rotations [10]: $\mathbf{M}^+(q) =$

$$= \mathbf{U}(q)\mathbf{L}(q)\mathbf{V}(q) = \begin{bmatrix}\mathbf{I}_2 & \mathbf{F}(q)\\ 0 & \mathbf{I}_2\end{bmatrix}\begin{bmatrix}\mathbf{I}_2 & 0\\ \mathbf{G}(q) & \mathbf{I}_2\end{bmatrix}\begin{bmatrix}\mathbf{I}_2 & \mathbf{H}(q)\\ 0 & \mathbf{I}_2\end{bmatrix}.$$

A set of matrix equations can be defined for a given unit-norm hypercomplex coefficient q and multiplication matrix. At the same time the set can be solved uniquely for $\mathbf{F}(q)$, $\mathbf{G}(q)$, and $\mathbf{H}(q)$, provided that $\mathbf{S}(q)$ is nonsingular, or more specifically, nonzero: $\mathbf{F}(q) = (\mathbf{C}(q)-\mathbf{I}_2)\mathbf{S}(q)^{-1}, \mathbf{G}(q) = \mathbf{S}(q), \mathbf{H}(q) = \mathbf{S}(q)^{-1}(\mathbf{C}(q)-\mathbf{I}_2)$. Their elements represent real-valued lifting coefficients. Namely, we prove that the magnitudes of all lifting coefficients can always be limited to the range from zero to one, which simplifies scaling and word length determination.

There are pairs of lifting coefficients with the same absolute value, as well as the matrices $\mathbf{F}(q)$, $\mathbf{G}(q)$, and $\mathbf{H}(q)$ structure which turns out to be closely related to that of the 2D rotation matrix [11, 12]. Thus, the hybrid CORDIC-lifting parameterization or block-lifting structure of LP Q-PUBF allow integrating CORDIC-algorithm "inside" the multipliers lattice network, replacing the actual multiplication by microrotation of the CORDIC-algorithm: addition and shift, as well as they can allow getting hardware implementation of the perfectly reconstructed Q-PUBF. However, to achieve a high throughput, the multipliers based on the CORDIC approach have to be fully pipelined, thus leading to the high latency computation and at the same time to high hardware requirements.

The distributed arithmetic (DA) implementation of the block-lifting step has many advantages over the aforementioned methods. The DA is the efficient procedure for computing inner products of the block-lifting step (the matrices $\mathbf{F}(q)$, $\mathbf{G}(q)$, and $\mathbf{H}(q)$) and variable vectors data. In such case faster circuits can be constructed using digit (L-bit)-serial DA [13], which allows a digit representation using L bits to be processed in a single clock cycle (L-BAAT scheme). The main advantage of this structure to the one of the multiplier block-lifting using 2-D CORDIC algorithm [14] is that the pipeline stages are synchronous: DA operation time is constant for all pipeline stages. If we take the structure of L-BAAT DA module $L = B$ (B is word length), then the DA operation will be executed in a single processor clock cycle. It possesses the advantages inherent for such systems, which are particularly useful from the point of view of multiplierless VLSI implementations.

4. CONCLUSION

Thus, the Q-PUFBs based on these techniques have less rounding operations number and have a regular layout. Additionally the parallel-pipelined efficient architecture of Q-PUFB (P^2E_Q-PUFB) has been proposed [14]. The low latency separable image processing is implemented in the given architecture based on the Xilinx Zynq [15]. The P^2E_Q-PUFB processor based on the pipelined embedded DA-based quaternion multiplier processors network for the proposed 8×24 Q-PUFB ($CG = 9.60$ dB) with word length $B = 16$ bits has the absolutely perfect reconstruction property. The comparisons of PSNRs for compression ratio $1:32$, $1:16$ and $1:8$ (test images: 512×512 "Lena", "Barbara") between conventional structures of 8×24 PUFB ($CG = 9.49$ dB) and biorthogonal filter banks 8×24 BOFB ($CG = 9.68$ dB), and also 8-channel 24-tap PUFB ($CG = 9.39$ dB) based on lapped orthogonal transform (LOT) and proposed 8×24 Q-PUFB ($CG = 9.60$ dB) have shown that the 8×24 Q-PUFB has a better PSNR perfomance than the corresponding filter banks, especially for image with relatively strong highpass components. In addition, the superiority of our coder over JPEG2000 results in differences in the number of bands and the encoding algorithm.

5. REFERENCES

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