RECONSTRUCTION OF 3D SURFACE FROM 2D HOLOGRAPHIC SIGNAL BASED ON KALMAN FILTER

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Problem Domain— Image, video & multidimensional signal processing / Computational Imaging

1. INTRODUCTION

Holographic [1] signals are one of the visual signals which are important and popular, because after proper processing they enable viewers to watch 3D scenes without wearing any external equipment. Holographic technology, though not mature yet, is thus believed to be the ultimate way of 3D display. However, current optical viewing methods are not mature in accuracy. Under this circumstance, research on holographic signal processing is mostly carried out in computers. A complete system can be set up in computers to simulate the whole procedure. However, in such a system, there is a major problem. That is how to view reconstructed 3D objects on a 2D computer screen so that the output of the system can be evaluated.

One commonly-used method is to obtain 2D projection images at different distances from hologram plane. However, this method can only show the front views of reconstructed 3D objects, thus is not suitable for comprehensively evaluation. Another method is to solve the problem by surface reconstruction, which can view the reconstructed 3D objects from different angle of view.

Surface reconstruction can be done by estimating phase in light field [3]. There are several methods dealing with it. For example, there are norm based methods like least-squares, path-following methods like branch-cut, Fourier method, and energy minimization method. However, most of them don't perform very well because their anti-noise characteristics are not so good. In [4], authors use Unscented Kalman Filter (UKF) to solve noise problem and prove that UKF is effective in removing AWGN noise. But they don't give demonstration about its specific application in holographic signal reconstruction.

In this paper, we propose a new method to reconstruct surface of a 3D object by processing its 2D holographic signal. The method is based on Extended Kalman Filter (EKF). EKF has good performance on removing noise [2], and has been successfully used in recovering topography in InSAR [5]. The anti-noise characteristics of EKF will extend our method to various scenes. Our method involves pre-filtering, EKF, and self-adaptive weighting. With our method, the reconstructed 3D object can be viewed on a 2D screen. This will certainly facilitate future research on signal processing of 3D holograms.

2. BRIEF PRINCIPLES OF 3D HOLOGRAMS

The procedures of obtaining hologram and reconstructing object are showed in Figure 1 and Figure 2.

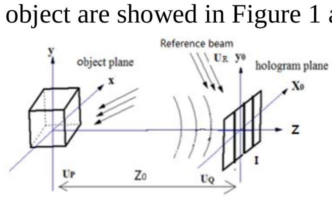


Fig.1. Obtain hologram I.

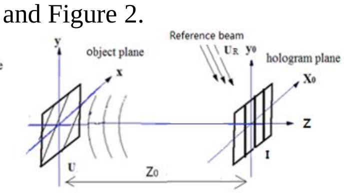


Fig.2. Obtain object light field U.

Here, U_P is the complex amplitude of light field of the original 3D object near object plane. U_Q is the light field at hologram plane after U_P go through Fresnel diffraction with a distance of z_0.Then, U_Q is added with reference beam U_R at hologram plane and hologram I is obtained [1] as follow:

$$\mathrm{I} = |\mathrm{U}_Q + \mathrm{U}_R|^2 \tag{1}$$

When the hologram I is illuminated by the same reference beam and another Fresnel diffraction is applied, light field at object plane U can be obtained which contains all information of the original 3D object. Figure 3 shows the section of a 3D object. When the surface height $\mathrm{h(x,y)}$ is relatively small compared with $\mathrm{z_0}$, $\mathrm{h(x,y)}$ is related to $\varphi(x,y)$ [2] as follow, where $\varphi(x,y)$ is the true phase of $\mathrm{U(x,y)}$:

$$\mathrm{h(x,y)} \propto \frac{\varphi(x,y)}{4\pi} \tag{2}$$

However, we can only directly get wrapped phase $\phi(\mathrm{x,y})$ which is among $[-\pi,\pi]$ from $\mathrm{U(x,y)}$ by arctan. And $\varphi(x,y)$ has the relation with $\phi(\mathrm{x,y})$ as follow:

$$\phi(\mathrm{x,y}) = \varphi(x,y) + \mathrm{k(x,y)} * 2\pi \tag{3}$$

Here, $\mathrm{k(x,y)} \epsilon Z$ which makes $\phi(\mathrm{x,y}) \epsilon [-\pi,\pi]$. It has different value for different $(\mathrm{x,y})$. So we need to obtain true phase $\varphi(x,y)$ from wrapped phase $\phi(x,y)$ to reconstruct surface.

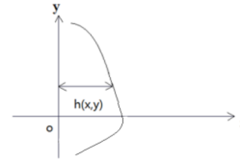


Fig.3. surface of 3D object.

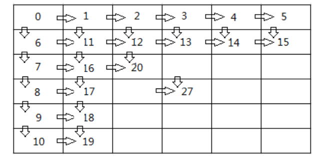


Fig.4. Calculation order

3. PROPOSED METHOD

3.1. Pre-filtering

For any 3D object, when sampling adequately, $\varphi(x-1,y)$ and $\varphi(x,y)$ can be assumed to have minimal value difference which is among $[-\pi,\pi]$. The same is true for $\varphi(x,y-1)$ and $\varphi(x,y)$. So we define $\Delta_R\phi(\mathrm{x},\mathrm{y})$ and $\Delta_C\phi(\mathrm{x},\mathrm{y})$ as follow. Here, $k_R,\ k_C \epsilon Z$ which make $\Delta_R\phi(\mathrm{x},\mathrm{y}),\ \Delta_C\phi(\mathrm{x},\mathrm{y})\epsilon[-\pi,\pi]$.

$$\begin{aligned}\Delta_R\phi(\mathrm{x},\mathrm{y}) &= \phi(\mathrm{x},\mathrm{y})-\phi(\mathrm{x},\mathrm{y}-1)+k_R(x,y)*2\pi \\ \Delta_C\phi(\mathrm{x},\mathrm{y}) &= \phi(\mathrm{x},\mathrm{y})-\phi(\mathrm{x}-1,\mathrm{y})+k_C(x,y)*2\pi\end{aligned} \tag{4}$$

The pre-filtering result $\phi'(\mathrm{x},\mathrm{y})$ is:

$$\phi'(\mathrm{x},\mathrm{y})=\begin{cases}\phi(1,1) & \mathrm{x}=1,\mathrm{y}=1\\ \phi(1,\mathrm{y}-1)+\Delta_R\phi(\mathrm{x},\mathrm{y}) & \mathrm{x}=1,\mathrm{y}>1\\ \phi(\mathrm{x}-1,1)+\Delta_C\phi(\mathrm{x},\mathrm{y}) & \mathrm{x}>1,\mathrm{y}=1\\ \frac{1}{2}\{[\phi(\mathrm{x},\mathrm{y}-1)+\Delta_R\phi(\mathrm{x},\mathrm{y})] & \mathrm{x}>1,\mathrm{y}>1\\ \quad +[\phi(\mathrm{x}-1,\mathrm{y})+\Delta_C\phi(\mathrm{x},\mathrm{y})]\} & \end{cases} \tag{5}$$

The order of calculation is one row and one column a time as showed in Figure 4.

3.2. Extended Kalman Filtering (EKF)

We calculate φ following the same order as ϕ' to transfer the 2 dimensional problem into 1 dimensional. So we will use $\phi'_n,\ \varphi_n,\ U_n$ instead of $\phi'(x,y), \varphi(\mathrm{x},\mathrm{y}), \mathrm{U}(\mathrm{x},\mathrm{y})$ from now on.

The state model s[n] and the observation model x[n] we use in Extended Kalman Filter are as:

$$\mathrm{s}[\mathrm{n}]=\begin{bmatrix}\varphi_n\\ \dot{\varphi}_n\end{bmatrix}, \dot{\varphi}_\mathrm{n}=\varphi_n-\varphi_{n-1} \tag{6}$$

$$\mathrm{x}[\mathrm{n}]=\begin{bmatrix}cos\varphi_n\\ sin\varphi_n\end{bmatrix}+w[n]=\begin{bmatrix}Re(U_n)\\ Im(U_n)\end{bmatrix} \tag{7}$$

Here, $w[n]$ is observation noise and can be assumed as additive white Gaussian noise with zero mean.

We start filtering from the third point of ϕ'_n. The first two points are used as initial values. We obtain state $s[n]$ from state $s[n-1]$ and observation x[n], thus obtain φ_n. The procedure of this EKF is standard. We omit it for lack of space.

3.3. Self-adaptive Weighting

Considering that Kalman Filter may bring error of former points into latter points, and pre-filtering result ϕ'_n gives a rough description of the surface we try to reconstruct, we propose a self-adaptive weight scheme. The final result φ_{nf} combines the EKF result φ_n and ϕ'_n with weight t_n as:

$$\varphi_{nf}=t_n\cdot\varphi_n+(1-t_n)\cdot\ \phi'_n \tag{8}$$

And we define D_n as our criterion:

$$\mathrm{D}_n=\sqrt{([\mathrm{Im}(\mathrm{U_n})-\sin(\varphi_{nf})]^2+[\mathrm{Re}(\mathrm{U_n})-\cos(\varphi_{nf})]^2)} \tag{9}$$

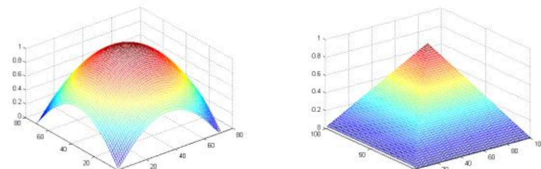
Then, t_n is decided as follow with $t_0=1$ and Δt invariant:

$$t_n=\begin{cases}t_{n-1}+\Delta t & \mathrm{D}_{n-1}<\mathrm{D}_{n-2}\\ t_{n-1} & \mathrm{D}_{n-1}=\mathrm{D}_{n-2}\\ t_{n-1}-\Delta t & \mathrm{D}_{n-1}>\mathrm{D}_{n-2}\end{cases} \tag{10}$$

4. ACHIEVED EXPERIMENTAL RESULTS

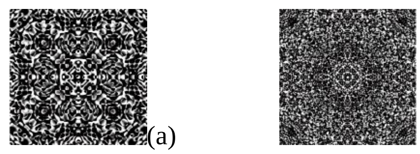
A 3D hemisphere and a 3D pyramid are created to evaluate the method. The surface of these two objects are showed in Figure 5. The holograms are illuminated by monochromatic light with wave-length of 600nm as showed in Figure 6. Figure 7 shows the wrapped phase. Figure 8 shows the reconstructed surface after calculation with our method.

It can be seen that the surface of hemisphere and pyramid has been reconstructed successfully. Compared these two surface, hemisphere is reconstructed better. This is because hemisphere surface is smooth and pyramid surface is not continuous. When applying EKF techniques, the latter one is smoothed to some extent. That also explains why the highest point in the original pyramid is not so obvious in the reconstructed result. But for smooth objects like hemisphere, the method works well.



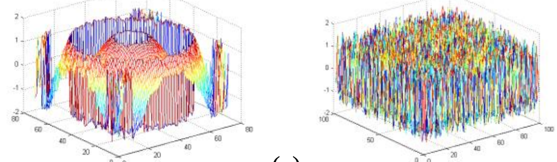
(a) (b)

Fig.5. Surface. (a) Hemisphere. (b) Pyramid.



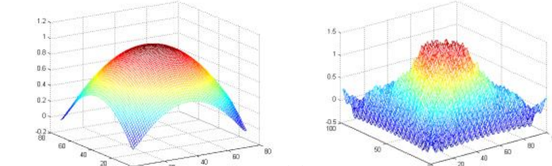
(a) (b)

Fig.6. Hologram. (a) Hemisphere. (b) Pyramid.



(a) (b)

Fig.7. Wrapped phase. (a) Hemisphere. (b) Pyramid.



(a) (b)

Fig.8. Reconstructed surface. (a) Hemisphere. (b) Pyramid.

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