ON TARGET LOCALIZATION WITH COMMUNICATION COSTS VIA TENSOR COMPLETION: A MULTI-MODAL APPROACH

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ABSTRACT

The problem of active target detection using low rank methods is explored. In prior work, a strategy was proposed based on matrix completion for randomly sampling a field combined with binary search to localize a target. Herein, two innovations are explored: the consideration of tensor-completion in order to exploit multi-modal data and the examination of the costs associated with communication. In particular, the random samples are collected in neighborhoods wherein the quality of the observation is a function of the distance of the sampling point to the centroid of the neighborhood. Due to the tradeoff between communication quality and sampling quality, there is an optimal neighborhood size.

***Index Terms*—** target detection, rank-one matrix completion, low-rank tensor completion, exploration-exploitation tradeoff, communication costs

1. INTRODUCTION

The identification of the location of a phenomenon of interest, a target, is needed in a variety of applications such as surveillance, anomaly detection, cyber-security, medical imaging and environmental monitoring. In a prior series of works [1, 2], we have examined the use of low-rank matrix completion [3] to derive an algorithm that *does not* need the knowledge of the target field decay profile, by exploiting the *a priori* assumption that the field in question is separable. By combining this approach with binary search, an active target localization algorithm was formulated. An analytical tradeoff between the sampling complexity and the target localization error, in the presence of noise, was provided under the assumption of a spatially uniform random sampling strategy.

Herein, we make two extensions. First, we tackle *multi-modal* data. That is, imagine that the vehicle is sampling multiple sensor systems at each location (acoustic, chemical, *etc.*). As a result, there is a concatenation of *maps* from which target localization should occur. A depiction of this environment is provided in Figure 1. Applying our previous methodology, we are now interested in *tensor completion* versus matrix completion. However, there are challenges to the consideration of tensor completion. In contrast to sparse approximation or low-rank approximation of a matrix, for many low-rank equivalent decompositions of tensors (such as the Kronecker decomposition), r-term approximations may not form a closed set [4, 5] and many related optimizations are NP-complete [6, 7]. The definition of rank is also up to interpretation and dependent on the decomposition. Second, we consider the impact of the communication channel on the tensor completion. For this evaluation, we provide a modified sampling strategy.

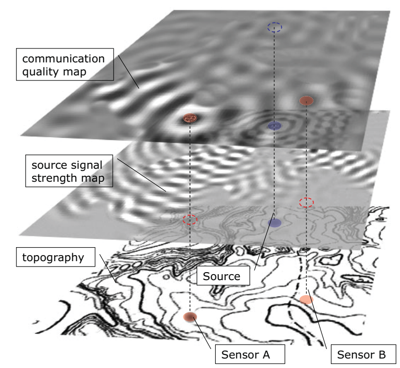

Fig. 1: Depiction of multi-modal maps.

Thus, a novel tensor based field model for incorporating multiple fields jointly for detecting the target location is developed. Using low-rank tensor completion methods in this framework, we design a hierarchical target localization algorithm We further show through numerical simulations the advantages of using the tensor model over using data from individual fields separately for target detection. It is seen that the detection probability improves by 10% when the number of samples is less than 20%. For a given detection probability of 80%, our model requires only half the number of samples as compared to target recovery using each field separately. Furthermore, if the quality of communication is considered, there is an optimal sampling radius to be considered for minimizing reconstruction error.

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1.1. Related Work

Target detection has been a persistently active area of work; [17] surveys methods that assume availability of the *full* target field/signature. A form of active localization is related to the exploration-exploitation tradeoff and path planning in robotics [19, 20]. In particular, [20] uses compressed sensing, via greedy optimization of the empirical restricted isometry measure) for field reconstruction (versus localization) with an emphasis on the impact of navigation costs and stopping times in contrast to the work herein. While distilled sensing (*e.g.* [21]) has a similar algorithmic philosophy to the work here; therein the field is assumed to be sparse rather than low-rank, and are thus sensitive to basis mismatch errors which are not a factor in the current approach

While there has been a flurry of recent work on tensor completion, much of it is not truly multi-modal in nature (*e.g.* hyperspectral data [12, 13] or video imagery (*e.g.* [10, 11]) or is completely agnostic to how the underlying tensor is created [**?**, 14].

1.2. Notation

We use lowercase boldface alphabets to denote column vectors (*e.g.* $\boldsymbol{z}$) and uppercase boldface alphabets to denote matrices (*e.g.* $\boldsymbol{A}$). The MATLAB® indexing rules will be used to denote parts of a vector/matrix (e.g. $\boldsymbol{A}(2:3, 4:6)$ denotes the sub-matrix of $\boldsymbol{A}$ formed by the rows $\{2, 3\}$ and columns $\{4, 5, 6\}$). The all zero, all one and identity matrices shall be respectively denoted by $\mathbf{0}$, $\mathbf{1}$ and $\mathbf{I}$ with dimensions dictated by context. $(\cdot)^{\mathrm{T}}$ denotes the transpose operation and $\langle\cdot,\cdot\rangle$ denotes the standard inner product on $\mathbb{R}^n$. The functions $\|\cdot\|_{\mathrm{F}}$ and $\|\cdot\|_*$ respectively return the Frobenius and nuclear norms of their matrix argument. The function $|\cdot|$ applied to a scalar (repectively a set) returns its absolute value (respectively cardinality). The symbol $\otimes$ represents the vector outer product.

2. PRELIMINARIES

We focus on three dimensional tensors (see [8]) , which are multi-dimensional arrays, or given our application, the concatenation of matrices. In order to describe a low-rank tensor, we focus on the CANDECOMP/PARAFAC, or CP, decomposition which factorizes into a sum of rank one tensors (see *e.g.* [9, 22]) For example, a third-order tensor $\boldsymbol{\chi} \in \mathbb{R}^{I\times J\times K}$ which admits a CP decomposition can be written as,

$$\boldsymbol{\chi} \approx \sum_{r=1}^{R} \boldsymbol{a_r} \otimes \boldsymbol{b_r} \otimes \boldsymbol{c_r} \tag{1}$$

where R is a positive integer and $\boldsymbol{a_r} \in \mathbb{R}^I, \boldsymbol{b_r} \in \mathbb{R}^J$, and $\boldsymbol{c_r} \in \mathbb{R}^K$ for $r = 1, \dots, R$. Each element of the tensor can be written as follows,

$$\boldsymbol{\chi}_{ijk} \approx \sum_{r=1}^{R} a_{ir} b_{jr} c_{kr} \forall i, j, k \tag{2}$$

$\boldsymbol{A} = [\boldsymbol{a_1}\ \boldsymbol{a_2}\ \dots\ \boldsymbol{a_R}], \boldsymbol{B} = [\boldsymbol{b_1}\ \boldsymbol{b_2}\ \dots\ \boldsymbol{b_R}]$, and $\boldsymbol{C} = [\boldsymbol{c_1}\ \boldsymbol{c_2}\ \dots\ \boldsymbol{c_R}]$ are called the *factor* matrices of the tensor $\boldsymbol{\chi}$.

For our tensor completion problem, out of IJK entries of an order-three tensor $T \in \mathbb{R}^{I\times J\times K}$, let a subset Ω be revealed. We use $\mathcal{P}_\Omega(T)$ to denote projection onto the revealed set such that:

$$\mathcal{P}_\Omega(T) = \begin{cases} T_{ijk}, & \text{if } (i,j,k) \in \Omega \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

The problem of low-rank tensor completion can be stated as:

$$\begin{aligned} \underset{\hat{T}}{\text{minimize}} \quad & \text{rank}(\hat{T}) \\ \text{subject to} \quad & \mathcal{P}_\Omega\left(\hat{T}\right) = \mathcal{P}_\Omega(T) \end{aligned} \tag{4}$$

However, even computing the rank of a tensor is NP-hard in general, where the rank is defined as the minimum R for which the CP-decomposition exists. So we instead fix the rank of $\hat{T}$ by explicitly modelling it as $\hat{T} = \sum_{r=1}^{R} \boldsymbol{a_r} \otimes \boldsymbol{b_r} \otimes \boldsymbol{c_r}$ and solve the following problem:

$$\underset{\hat{T}, rank(\hat{T})=R}{\text{minimize}} \quad \left\|\mathcal{P}_\Omega\left(\hat{T}\right) - \mathcal{P}_\Omega(T)\right\|_{\mathrm{F}}^2 \tag{5}$$

We employ alternating minimization to solve the non-convex problem above; this is a block coordinate descent method in which we minimize the objective function of Problem (5) with respect to each of the factor matrices alternately while keeping the others fixed [23]. Alternating minimization can get mired in local minima, underscoring the necessity for good initialization. To this end, we use the Robust Tensor Power Method (RTPM) [24], which yields a provably good approximate orthogonal tensor decomposition, to initialize the factor matrices.

3. PROBLEM DESCRIPTION

We shall make assumptions on the target field present in each plane (matrix) of the tensor.

3.1. Target Field Assumptions

Without loss of generality and for clarity, we assume that the target is located at the origin and noise is absent. However, in our simulations, we will not make these assumptions. We will consider two planar fields with distinct target signatures. Let the search region be the two dimensional unit square, and $\boldsymbol{y} = (y_c, y_r) \in [0, 1]^2$ denote an arbitrary location in the search space. Let $H_1 : \mathbb{R}^2 \to \mathbb{R}$ and $H_2 : \mathbb{R}^2 \to \mathbb{R}$ denote the *scalar valued* fields of the same target. We shall make the following key assumptions on the fields $H_1(\boldsymbol{y})$ and $H_2(\boldsymbol{y})$:

(A1) Both the fields are separable in some known basis of $\mathbb{R}^2$, independent of the true location of the target.

(A2) The magnitude of the fields, $|H_1(\boldsymbol{y})|$ and $|H_2(\boldsymbol{y})|$ is a monotonically non-increasing function of the distance from the target in every direction [1, 2].

(A3) $H_1(\boldsymbol{y})$ and $H_2(\boldsymbol{y})$ are spatially invariant relative to the target's position.

We assume separability of $H_1(\boldsymbol{y})$ and $H_2(\boldsymbol{y})$ in the y_c and y_r directions (*i.e.* in the canonical basis $\{[1,0],[0,1]\}$) as per (A1). This means that there exist functions $F_i\colon \mathbb{R} \to \mathbb{R}$ and $G_i\colon \mathbb{R} \to \mathbb{R}$ such that $H_i(\boldsymbol{y}) = F_i(y_\mathrm{c})G_i(y_\mathrm{r}), \forall (y_\mathrm{c}, y_\mathrm{r}) \in \mathbb{R}^2$ for $i \in \{1,2\}$.

Assumption (A2) is intuitively clear and can be mathematically described by the inequality:

$$|H_i(t_1(\boldsymbol{y}-\boldsymbol{y}_0))| \geq |H_i(t_2(\boldsymbol{y}-\boldsymbol{y}_0))|, \;\; i \in \{1,2\}, \quad (6)$$

holding $\forall \boldsymbol{y} \in \mathbb{R}^2, t_2 > t_1 > 0$, where $\boldsymbol{y}_0$ represents the unknown location of the target. This assumption hence implies that the peak in each of the fields is at the target's location.

Assumption (A3) implies that if the target were moved from $\boldsymbol{y}_0$ to a new position $\boldsymbol{y}_0'$, then the new fields at location $\boldsymbol{y}$ would be given by $H_i(\boldsymbol{y}-\boldsymbol{y}_0'+\boldsymbol{y}_0), \; i \in \{1,2\}$, thus ensuring that (A1) holds in the canonical basis, regardless of the target's position $\boldsymbol{y}_0$.

3.2. Formulation

By virtue of assumption (A2), detecting the target is synonymous with locating the peak of the induced field. In light of our assumptions, we can state the target detection problem as the following task: *To determine the location of the peak in the fields $H_1(\boldsymbol{y})$ and $H_2(\boldsymbol{y})$ from their values in only a few locations $\boldsymbol{y} \in [0,1]^2$.* We employ the *lifting* technique from optimization to demonstrate that the separability assumption (A1) implies a rank one structure on each of the fields.

Let $H_i(\boldsymbol{y}) = F_i(y_\mathrm{c})G_i(y_\mathrm{r}), \; i \in \{1,2\}$ be the canonical separable representation of the target fields and let $\boldsymbol{H_i}$ denote a high resolution discretized version of $H_i(\boldsymbol{y})$ on the $n \times n$ regular grid $\mathcal{V} \in [0,1]^2$. Let $\mathcal{V} = \{y_\mathrm{r}^1, y_\mathrm{r}^2, \ldots, y_\mathrm{r}^n\} \times \{y_\mathrm{c}^1, y_\mathrm{c}^2, \ldots, y_\mathrm{c}^n\}$ be the representation of the grid. The set of all possible sampled values of the field on the set $\mathcal{V}$ is given by $\left\{H_i\left(y_\mathrm{c}^i, y_\mathrm{r}^j\right) \,\middle|\, \left(y_\mathrm{c}^i, y_\mathrm{r}^j\right) \in \mathcal{V}\right\}$ and can be arranged in the form of the rank one matrix $\boldsymbol{H_i} \in \mathbb{R}^{n\times n}$, whose $(i,j)^\mathrm{th}$ entry $\boldsymbol{H_i}(i,j)$ is

$$\boldsymbol{H_i}(i,j) = H_i\big(y_\mathrm{c}^i, y_\mathrm{r}^j\big) = F_i\big(y_\mathrm{c}^i\big)G_i\big(y_\mathrm{r}^j\big). \quad (7)$$

where $\left(y_\mathrm{c}^i, y_\mathrm{r}^j\right)$ is the physical location of the $(i,j)^\mathrm{th}$ point in $\mathcal{V}$. The matrix $\boldsymbol{H_i}$ is clearly of rank one since we can express it as the outer product $\boldsymbol{H_i} = \boldsymbol{f_i}\boldsymbol{g_i}^\mathrm{T}$ where the vector $\boldsymbol{f_i} = \left[F_i\big(y_\mathrm{c}^1\big), F_i\big(y_\mathrm{c}^2\big), \ldots, F_i\big(y_\mathrm{c}^n\big)\right]^\mathrm{T}$ and $\boldsymbol{g_i} = \left[G_i\big(y_\mathrm{r}^1\big), G_i\big(y_\mathrm{r}^2\big), \ldots, G_i\big(y_\mathrm{r}^n\big)\right]^\mathrm{T}$. Without loss of generality, we assume that both $y_\mathrm{r}^1, y_\mathrm{r}^2, \ldots, y_\mathrm{r}^n$ and $y_\mathrm{c}^1, y_\mathrm{c}^2, \ldots, y_\mathrm{c}^n$ are sorted in ascending order, corresponding respectively to traversing the grid from top to bottom and from left to right. Because of the preceding derivation, we can refer to $\boldsymbol{H_i}$ as the target field with a slight abuse of terminology. Consequently, we can consider $\mathcal{V}$ in a rescaled sense to refer to the set of index pairs $\{1,2,\ldots,n\}^2$ for the matrix $\boldsymbol{H_i}$.

Let the order-three tensor T be formed by stacking the two scalar valued fields H_1 and H_2 such that: $T(i,j,1) = H_1(i,j)$ and $T(i,j,2) = H_2(i,j)$. Since both the fields have the same peak location and similar decay profiles, the tensor will have approximately rank-one structure in the third dimension. However, the deviation from a rank-one tensor will impact reconstruction performance. This effect is studied in Figures 3a and 3b.

4. SAMPLING AND RECONSTRUCTION

We will assume H_1 and H_2 are positive scalar fields. The field is assumed to be on an $n \times n$ grid.

4.1. The Reconstruction Algorithm

Let the first round of sampling be on the Cartesian product of the index sets $\mathcal{T}_c, \mathcal{T}_r \subset \{1,2,\ldots,n\}, \mathcal{T}_k = \{1,2\}$ such that $|\mathcal{T}_c| = |\mathcal{T}_r| = m$ with $m \ll n$. We will denote this Cartesian product by the sub-tensor $T(\mathcal{T}_c, \mathcal{T}_r, \mathcal{T}_k)$. Without loss of generality, assume that the indices in $\mathcal{T}_c$, denoted by $\mathcal{T}_c(1), \mathcal{T}_c(2), \ldots, \mathcal{T}_c(|\mathcal{T}_c|)$ are in increasing order and for notational convenience we shall let $\mathcal{T}_c(0) = 1$ and $\mathcal{T}_c(|\mathcal{T}_c|+1) = n$ to denote the boundary indices of each of the field grids.

Let Ω be the sampled set on the sub-tensor $T(\mathcal{T}_c, \mathcal{T}_r, \mathcal{T}_k)$. Using the alternating minimization algorithm, we fit a rank-one CP model by solving the following problem:

$$\underset{\hat{T}, rank(\hat{T})=1}{\text{minimize}} \quad \left\| \mathcal{P}_\Omega\left(\hat{T}\right) - \mathcal{P}_\Omega(T(\mathcal{T}_c, \mathcal{T}_r, \mathcal{T}_k)) \right\|_\mathrm{F}^2 \quad (8)$$

The rank-one reconstruction can be written as $\hat{T}(\mathcal{T}_c, \mathcal{T}_r, \mathcal{T}_k) = \boldsymbol{a} \otimes \boldsymbol{b} \otimes \boldsymbol{c}$. Let the largest magnitude element of $\boldsymbol{a}$ be at index $j_0 \in \{1,2,\ldots,|\mathcal{T}_c|\}$ and that of $\boldsymbol{b}$ be at index $i_0 \in \{1,2,\ldots,|\mathcal{T}_r|\}$. The next round of sampling is on the Cartesian product of the index sets $\mathcal{T}_c' \subseteq \{\mathcal{T}_c(j_0-1), \ldots, \mathcal{T}_c(j_0+1)\}, \mathcal{T}_r' \subseteq \{\mathcal{T}_r(i_0-1), \ldots, \mathcal{T}_r(i_0+1)\}, \mathcal{T}_k = \{1,2\}$ such that $|\mathcal{T}_c'| = |\mathcal{T}_r'| = m'$ with $m' \ll n$.

4.2. Effect of Communication Channel

We next examine the impact of communication quality on the probability of target detection. To do so, we modify the sampling model. A sampling radius is selected and within this region, $|\Omega|$ uniformly random samples are considered; there are N_c sampling centers that are themselves, uniformly randomly selected. The sampling budget $|\Omega|$. Thus, the number of samples collected is $N_c|\Omega|$ In simulations, we will examine the effect of varying the radius size.

An additional consideration that is relevant is the limited communication range of the sampling agent from the center. The noise induced by the channel on the observations transmitted by the sampling agent depends on the distance to the closest sampling center. If x_i is the field intensity at a location l_i, the sampled value $\hat{x}_i$ would be: $\hat{x}_i = x_i + n_i$ where $n_i \sim \mathcal{N}(0, f(\min_j(\|l_i - c_j\|)))$ and $f(\cdot)$ is a monotonically increasing function which captures the attenuation in signal as a function of range often experienced in wireless systems [25, 26]. We will see that there is a tradeoff between the effects of the noise, which increase with the radius size and how uniformly we sample the region which improves with the sampling radius.

5. SIMULATIONS

We consider two 100×100 Gaussian matrices as the target fields of the form:

$$G_k(i,j) = C_k e^{\left(-\frac{(i-i_0)^2 + (j-j_0)^2)}{2\sigma_k^2}\right)} \quad k = 1, 2$$

Both the fields peak at the same location but have different spread factors σ_1 and σ_2. These two matrices are stacked together to form a tensor; however, this tensor is not rank-one unless $\sigma_1 = \sigma_2$. Our proposed tensor completion method is compared to the case of performing matrix completion on each map/plane individually and then averaging the two selected peaks; this performance is shown in Figure 2 where it is clear that the joint method based on tensor completion achieves a better probability of detection. The fact that a larger ratio of spreads σ_1/σ_2 implies that the tensor is further away from rank-one is shown in Figure 3a, where the distance from a rank-one tensor is shown and correlates with the detection performance shown in Figure 3b, *i.e.* the further away from rank-one the tensor is, the worse the detection performance. The distance is the normalized error between the true tensor and the rank-one approximation. Finally, we consider the effect of the communication channel via the multi-agent sampling strategy described in Section 4.1. Figure **??** reveals the predicted tension. When the sampling radius is small; samples are received with low noise. However, the samples are spatially concentrated and thus the overall field of interest is not uniformly sampled. If the sampling radius is too large, the field is well-sampled, but the samples near the edge of the sampling regions have more noise. Thus, there is an optimal radius that balances between these two effects.

6. CONCLUSIONS

In this work, we have extended our prior strategy for active target detection via matrix completion to a multi-modal sensing environment. In this case, the object to be searched is a tensor of three dimensions. Each plane of the tensor corresponds to under sampled matrices whose components correspond to the sensor signal. As in our prior work, we only require that each sensor modality result in a matrix that is low rank in the absence of noise. Thus, we do not need to know the signature of the target for each sensor modality. It shown that the tensor approach improves upon localization using each sensor modality separately and then combining, i.e. a joint approach is better. We further show that there is an optimal radius size when considering the quality of samples collected by multiple agents. The size of the sampling radius impacts the quality of the measurements but also how uniformly the field can be sampled. Small radii imply good quality measurements, but a less uniformly sampled field; hence the existence of an optimal radius. Future work will develop theoretical analyses of these methods.

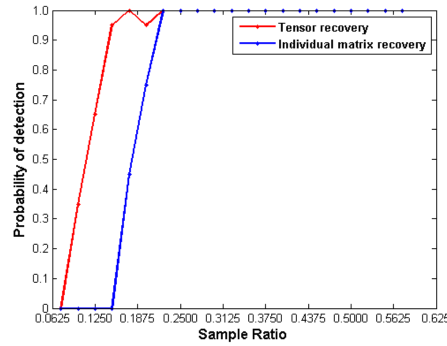
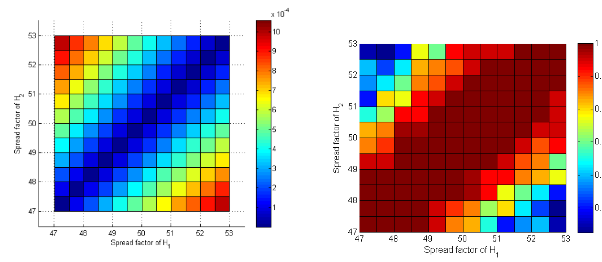


Fig. 2: Comparison of tensor and matrix methods for peak detection



(a) Reconstruction Error (b) Probability of detection

Fig. 3: Analysis of the variation of probability of detection and reconstruction error with spread factors of the two fields

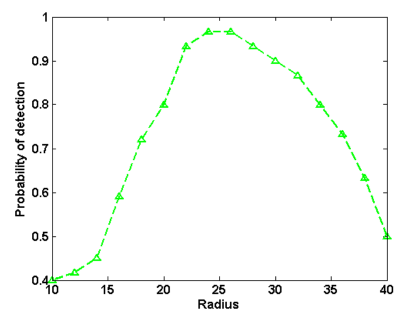


Fig. 4: Probability of target localization versus radius size.

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