MEASUREMENT PARTITIONING AND OBSERVATIONAL EQUIVALENCE IN STATE ESTIMATION

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ABSTRACT

This paper studies observability of linear systems from both algebraic and graph-theoretic arguments, and further draws a parallel between the two. We show that a set of critical measurements (for state-space observability) can be partitioned into two types: α and β . This partitioning is driven by different graphical (or algebraic) methods used to define the corresponding measurements. Subsequently, we describe observational equivalence, i.e., given an α (or β) measurement, say y_i , what is the set of measurements equivalent to y_i , such that only one measurement in this set is required? Since α and β measurements are cast using different algebraic and graphical characteristics, their equivalence sets are also derived using different algebraic and graph-theoretic principles. The need to make such equivalence arises in areas, e.g., meter placement for power systems, where relevant lines of study include: (a) to guarantee state-space observability with as few sensors as possible; and, (b) to find candidate replacement measurements when a sensor incurs a fault. We illustrate the related concepts on a simple, yet insightful, system digraph.

Index Terms— State estimation, Observability of LTI systems, Sensor placement, Fault diagnosis.

1. INTRODUCTION

The concept of observability is a determining factor in state estimation. In linear but static cases, observability defines the solvablity of the set of p measurement equations to recover an n-dimensional parameter, subsequently requiring at least as many measurements as the number of unknowns, $p \ge n$, in general. Observability in LTI dynamics is more interesting since the number, p, of measurements may be less than the number, n, of states. Simply, an observable system possesses enough dependencies among the states, via the system matrix, that can be exploited towards state estimation with a smaller number of measurements. There are different approaches to check for observability: (i) algebraic method of finding the Gramian rank [1]; (ii) the Popov-Belevitch-Hautus (PBH) test [2]; and, (iii) graph-theoretic analysis [3–6]. In LTI state-space observability, a significant question is to find a set of critical measurements to ensure observability. Recent literature [6–8] discusses different aspects and approaches towards this problem; see our prior work [9] for rank-deficient systems. In these works, the LTI systems are modeled as digraphs and graph-theoretic algorithms are adapted to find the corresponding critical measurements. Since these results are structural (only depend on the nonzeros of the system matrices), they ensure *generic observability*, i.e.,the underlying LTI systems are observable for almost all choices of non-zeros in the corresponding matrices. The values for which the results do not hold lie on an algebraic variety with zero Lebesgue measure [4, 5].

In this paper, we first show that the set of critical measurements can be partitioned into two types distinct in both *algebraic* and *graph-theoretic* sense. Algebraically, these partitions belong to different methods of recovering the rank of the observability Gramian. Graph-theoretically, these partitions are related to the Strongly Connected Components (SCC) and contractions in the system digraphs [8–10]. We then introduce the notion of observational equivalence in state estimation. In particular, we derive a set of alternatives for each measurement such that if any critical measurement is not available then an equivalent measurement can be chosen to recover the loss of system observability. Clearly, since the measurements may have different algebraic and graph-theoretic characteristics, their equivalence sets are also derived using different algebraic and graph-theoretic principles.

Related work includes [9–12] with applications in fault diagnosis and observer isolation. Also related are dynamical systems interpreted as graphs, e.g., power systems where the nodes represent voltages and phases, and the edges are defined by the topology and impedance parameters; see e.g., [13–15] where our results on measurement partitioning are applicable and [16,17] on sensor placement scenarios. Our results on observational equivalence have relevance to fault detection in sensors where after the identification of faults, observational equivalence provides a list of new sensing locations [18–20] to recover the loss of observability. This equivalence further leads to characterizing measurements that may incur lower cost or improved estimation accuracy, e.g., extending the work in [11].

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We now describe the rest of the paper. Section 2 covers the preliminaries on LTI state-spaces and generic observability. Using this setup, we formulate the problem in Section 3. The main results on measurement partitioning are derived in Section 4, while observational equivalence is characterized in Section 5. Finally, an illustrative example and concluding remarks are given in Sections 6 and 7, respectively.

2. PRELIMINARIES

Consider the LTI state-space dynamics as follows:

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + \mathbf{v}_k, \qquad \mathbf{y}_k = H\mathbf{x}_k + \mathbf{r}_k, \quad (1)$$

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{v}, \qquad \mathbf{y} = H\mathbf{x} + \mathbf{r}, \qquad (2)$$

where the former is the discrete-time description and the latter is the continuous-time description. Since observability in either case is identical, the treatment in this paper is applicable to both cases. Using the standard terminology, $\mathbf{x} = [x_1 \dots x_n]^\top \in \mathbb{R}^n$ is the state-space, $\mathbf{y} = [y_1, \dots, y_p] \in \mathbb{R}^p$ is the measurement vector; the noise variables, \mathbf{v} and \mathbf{r} , have appropriate dimensions with the standard assumptions on Gaussianity and independence. It is well-known that the LTI descriptions above, Eqs. (1) and (2), lead to a bounded error if and only if the observability Gramian is full-rank [1].

For estimation purposes, the system must be observable $(\operatorname{rank}(\mathcal{O}) = n)$ with the given measurements. Algebraic tests for observability check the rank of the Gramian, \mathcal{O} , or the invertibility of $\mathcal{O}^{\top}\mathcal{O}$, [1], also see the PBH test [2]. These approaches rely on the knowledge of exact values of system and measurement matrices, i.e.,the numerical values of all of the elements in A and H. However, it is often the case that the zeros and non-zeros of the system matrix are fixed while the non-zero elements change; e.g., consider electrical networks where the component values (e.g., line, transformer, or generator parameters) may be subject to variation (due to heating, magnetic saturation etc.), are inaccurate, or approximately equal to their nominal values to within some tolerance.

2.1. Graph-theoretic Observability

Instead of the algebraic tests, an alternate is a graphical approach described as follows. Let $\mathcal{X} = \{x_1, \ldots, x_n\}$ and $\mathcal{Y} = \{y_1, \ldots, y_p\}$ denote the set of states and measurements, respectively. The system digraph is a *directed graph* defined as $\mathcal{G}_{sys} = (\mathcal{V}_{sys}, \mathcal{E}_{sys})$, where $\mathcal{V}_{sys} = \mathcal{X} \cup \mathcal{Y}$ is the set of nodes and \mathcal{E}_{sys} is the set of edges; this digraph is induced by the structure of the system and measurement matrices, $A = \{a_{ij}\}$, and $H = \{h_{ij}\}$. An edge, $x_j \rightarrow x_i$, in \mathcal{E}_{sys} exists from x_j to x_i if $a_{ij} \neq 0$. Similarly, an edge, $x_j \rightarrow y_i$, in \mathcal{E}_{sys} exists from x_j to x_i if $h_{ij} \neq 0$. A *path* from x_j to x_i (or y_i) is denoted as $x_j \xrightarrow{\text{path}} x_i$. A path is called \mathcal{Y} -connected (denoted by $\xrightarrow{\text{path}} \mathcal{Y}$) if it ends in a measurement. A cycle is a path where the begin and end nodes are the same. A cycle family is a set of mutually disjoint cycles; see [5] for details.

Graph-theoretic (or generic) observability is based on structured systems theory; it relies on the system structure (zeros and non-zeros) and is valid for *almost all* values of the system parameters; the values for which generic observability does not hold lie on an algebraic variety of zero Lebesgue measure, see [5] and references therein. Following is a wellknown result on generic observability [4].

Theorem 1. A system is generically (A, H)-observable if and only if both of the following conditions are true in its digraph: (i) Every state x_i is the begin node of a \mathcal{Y} -connected path, i.e., $x_i \xrightarrow{\text{path}} \mathcal{Y}, \forall i \in \{1, ..., n\}$; (ii) There exist a family of disjoint \mathcal{Y} -connected paths and cycles that includes all states.

The first condition is known as *accessibility* and the second as the *S-rank* or *matching* condition. The above conditions, however, are known to have algebraic meanings, see [21]. We describe the algebraic connections in the following.

Proposition 1. An inaccessible node in the system digraph implies the existence of a permutation matrix P such that,

$$PAP^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}, PH = \begin{bmatrix} 0 & | H_1 \end{bmatrix}.$$
 (3)

Proposition 2. *S*-rank condition is related to the structural rank of the system, i.e. *S*-rank $\begin{bmatrix} A^{\top}H^{\top} \end{bmatrix}^{\top} = n$.

The definition of structural rank (*S*-rank) and its properties are as follows. The *S*-rank (or *generic rank*) is the maximal rank of a matrix, *A*, that can be achieved by changing its nonzero elements. In the system matrix, *A*, *S*-rank equals the maximum number of non-zero elements in the distinct rows and columns of *A*. In the system digraph, \mathcal{G}_{sys} , *S*-rank is the size of the *maximum matching*, see [21, 22] for details.

3. PROBLEM FORMULATION

In this paper, we show that a such of measurements required for observability can be partitioned into two types: α and β ; with different algebraic and graph-theoretic interpretations. Algebraically, Type- α measurements correspond to the *S*-rank condition in Prop. 2; while Type- β measurements are tied to the accessibility condition in Prop. 1. Graphtheoretically, Type- α and Type- β measurements belong to maximum matching and parent SCCs in the system digraph. We describe these results in Section 4.

Clearly, a set of measurements that ensures observability may not be unique motivating to search for all possible such sets. The second problem we consider is to define the states that are *equivalent* in terms of observability-the equivalence relation is denoted by ' \sim ': if two states, x_i and x_j are observationally equivalent, i.e., $x_i \sim x_j$, then measuring any one of them suffices. Hence, the corresponding measurements are also equivalent, i.e., $y_i \sim y_j$. In Section 5, we characterize this notion of observational equivalence towards state estimation in both algebraic and graph-theoretic sense.

4. MEASUREMENT PARTITIONING

We now describe measurement partitioning. Given a set, H, of observable measurements such that (A, H) is observable, we partition the measurements into three types: α , β , and γ . Type- α and Type- β are necessary for observability (assuming fixed H) while Type- γ measurements are redundant [10].

Definition 1. *Given system matrices, A and H, a measurement is necessary for observability if and only if removing that measurement renders the system unobservable.*

For a given H, let H_{α} , H_{β} , and H_{γ} denote the submatrices of H that represent each partition, α , β , and γ , respectively. Similarly, let $H_{\alpha,\beta}$ denote the submatrix corresponding to both α and β measurements. Using this notation the above definition can be summarized in following:

$$\operatorname{rank}\left(\mathcal{O}\left(A, \left[\begin{array}{c}H_{\alpha,\beta}\\H_{\gamma}\end{array}\right]\right)\right) = \operatorname{rank}\left(\mathcal{O}\left(A,H_{\alpha,\beta}\right)\right) = n. \quad (4)$$

In the rest of this section, we characterize the methods to arrive at these partitions. Note that the graph-theoretic interpretation is described in Theorem 1, while the algebraic interpretations are characterized in Prop.s 1 and 2.

4.1. Graph-Theoretic: We note that $Type \cdot \alpha$ measurements are related to the maximum matching and contractions in the system digraph. Define the maximum matching, \mathcal{M} , as the largest subset of edges with no common end nodes; the maximum matching is not unique. This can be best defined over the bipartite representation, $\Gamma_A = (\mathcal{V}^+, \mathcal{V}^-, \mathcal{E}_{\Gamma_A})$, of system digraph, where $\mathcal{V}^+ = \mathcal{X}^+$ is the set of begin nodes and $\mathcal{V}^- = \mathcal{X}^- \cup \mathcal{Y}^-$ is the set of end nodes, with the edge set defined as $(v_i^+ \in \mathcal{V}^+, v_j^- \in \mathcal{V}^-) \in \mathcal{E}_{\Gamma_A}$ if $x_j \rightarrow x_i$ or $x_j \rightarrow y_i$. Given a maximum matching, \mathcal{M} , let $\delta \mathcal{M}^+$ represent the set of unmatched nodes defined as the nodes in \mathcal{V}^+ not incident to the edges in \mathcal{M} .

Definition 2. A Type- α measurement is the measurement of an unmatched node, $v_i \in \delta \mathcal{M}^+$, in the matching, \mathcal{M} .

On the other hand, $Type-\beta$ measurements are related to the Strongly-Connected Components (SCCs) in the system digraph. In a not strongly-connected digraph, define SCCs, S_i 's, as the largest SC sub-graphs. In addition, an SCC is matched, denoted by S_i^{\circlearrowright} , if it contains a family of disjoint cycles covering all its states. A cycle is a simple example of a matched SCC. An SCC is called *parent*, denoted by S_i^p , if it has no outgoing edges to any other SCC. Any SCC that is not parent is a *child*, S_i^c . In this regard, define *partial order*, \preceq , as the existence of edges from one SCC to another. Mathematically, $\{S_i \leq S_j\}$ if and only if $v_i \xrightarrow{path} v_j$ for some nodes $\{v_i \in S_i, v_j \in S_j\}$. Clearly we have, $S^c \leq S^p$.

Definition 3. A Type- β measurement is the measurement of a state in a matched parent SCC, $S_i^{\bigcirc p}$.

4.2. Algebraic: Note that Prop. 2 may not be satisfied by a measurement that satisfies Prop. 1, i.e., a measurement that recovers accessibility may not improve the *S*-rank of $[A^{\top} H^{\top}]^{\top}$. Hence, a Type- α measurement is the one that improves the *S*-rank of $[A^{\top} H^{\top}]^{\top}$ by 1. When the system matrix has a full *S*-rank, there are no Type- α measurements.

Definition 4. In the algebraic sense, the *i*th Type- α measurement, α_i , is formally defined as a measurement satisfying

$$S\text{-rank}\left(\left[\begin{array}{c}A^{\top} H_{\alpha_i}^{\top}\end{array}\right]^{\top}\right) = S\text{-rank}(A) + 1,$$
(5)

where H_{α_i} is a $1 \times n$ row with a non-zero at the α_i th location.

A Type- α agent thus improves the S-rank by exactly 1.

Definition 5. The *i*th Type- β measurement, β_i , does not improve the S-rank, *i.e.*

S-rank
$$\left(\left[A^{\top} H_{\beta_i}^{\top} \right] \right) = S$$
-rank $(A),$ (6)

But, from Def. 1, a Type- β_i measurement satisfies Eq. (6) and

$$nk\left(\mathcal{O}\left(A, \left[\begin{array}{c} H_{\alpha}^{\top} & H_{\beta_{i}}^{\top} \end{array} \right]^{\top}\right)\right) = rank\left(\mathcal{O}\left(A, H_{\alpha}\right)\right) + 1. \quad (7)$$

5. OBSERVATIONAL EQUIVALENCE

An equivalence relation, '~', has three properties: reflexivity, symmetry, and transitivity [23]. Towards observational equivalence, reflexivity implies that every state is equivalent to itself, symmetry implies that $x_i \sim x_j \Rightarrow x_j \sim x_i$, and transitivity implies that if $x_i \sim x_j$ and $x_j \sim x_m$, then $x_i \sim x_m$. We now define observational equivalence as follows.

Definition 6. Let H_i denote a row with a non-zero at the *i*th location denoting a measurement of the *i*th state. Observational equivalence among two states, $x_i \sim x_j$, is defined as

$$\operatorname{rank} \mathcal{O}(A, H_i) = \operatorname{rank} \mathcal{O}(A, H_j) = \operatorname{rank} \mathcal{O}\left(A, \begin{bmatrix} H_i \\ H_j \end{bmatrix}\right).$$

It can be easily verified that the above definition obeys the three requirements on transitivity, reflexivity, and symmetry.

5.1. Graph-Theoretic: In order to define observational equivalence using graph-theoretic arguments, we need to introduce the notion of a *contraction*. To this aim, we first define an auxiliary graph, $\Gamma_A^{\mathcal{M}}$, as the graph constructed by reversing all the edges of the maximum matching, \mathcal{M} , in the bipartite graph Γ_A ; recall Section 4.1. This auxiliary graph, $\Gamma_A^{\mathcal{M}}$, is used to find the contractions in the system digraph \mathcal{G}_{sys} as follows. In the auxiliary graph, $\Gamma_A^{\mathcal{M}}$, assign a contraction, \mathcal{C}_i , to each unmatched node, $v_j \in \delta \mathcal{M}^+$, as the set of all nodes in $\delta \mathcal{M}^+$ reachable by alternating paths from v_j . We denote $v_j(\mathcal{C}_i)$ as the unmatched node within the contraction, \mathcal{C}_i . An alternating path is a path with every second edge in \mathcal{M} . Equivalence among Type- α agents is thus defined using contractions and unmatched nodes within each contraction. We have the following result.

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Lemma 1. Given a contraction, C_i , the unmatched node, $v_j(C_i)$, within this contraction is not unique, i.e., for $v_j(C_i)$ and $v_g(C_i)$, $v_j(C_i) \in \delta \mathcal{M}_1^+$ and $v_g(C_i) \in \delta \mathcal{M}_2^+$, where \mathcal{M}_1 and \mathcal{M}_2 are two choices of maximum matching.

Proof. The proof is a direct result of non-uniqueness of maximum matching from Dulmage-Mendelson decomposition [24]. To find a contraction, we start with a particular unmatched node, e.g., in $\delta \mathcal{M}_1^+$. However, once we establish the contraction, C_i , within this contraction there may be multiple options for an unmatched node (in another unmatched set $\delta \mathcal{M}_2^+$). In other words, every contraction includes exactly one unmatched node for any choice of \mathcal{M} .

Lemma 2. All nodes that belong to the same contraction, C_i , are equivalent Type- α measurements.

Proof. The equivalence relation for the Type- α measurements is tied to unmatched nodes. Since a measurement of each unmatched node improves the *S*-rank by 1 and each contraction contributes to a single rank-deficiency, it can be shown that the equivalence properties are satisfied.

The following establishes Type- β equivalence.

Lemma 3. Two Type- β measurements, β_i and β_j , of states x_i and x_j , are equivalent, $\beta_i \sim \beta_j$, if they belong to the same parent SCC, $S_i^{\bigcirc p}$. An immediate corollary is that all of the states that belong to the same parent SCC are equivalent.

Proof. The proof follows form the strong connectivity of $S_i^{\bigcirc p}$, which implies accessibility of all nodes from a single node in the corresponding parent SCC, [10].

Since the parent SCCs are disjoint components in the system digraph, equivalent sets among Type- β measurements are disjoint. Notice that if a parent SCC is *unmatched*, the measurement is of Type- α . In such cases, a Type- α measurement recovers both conditions for observability in Theorem 1.

5.2. Algebraic: We now provide the algebraic interpretation of equivalence among the Type- α and Type- β measurements.

Lemma 4. Two Type- α measurements, α_i and α_j , are equivalent, $\alpha_i \sim \alpha_j$, if and only if

$$S\text{-rank}\left(\left[\begin{array}{c}A\\H_{\alpha_{i}}\end{array}\right]\right) = S\text{-rank}\left(\left[\begin{array}{c}A\\H_{\alpha_{j}}\end{array}\right]\right) = S\text{-rank}\left(\left[\begin{array}{c}A\\H_{\alpha_{i}}\\H_{\alpha_{j}}\end{array}\right]\right).$$
(8)

Proof. Reflexivity and symmetry are directly induced by Eq. (8). For transitivity, consider three Type- α measurements, $\alpha_i, \alpha_j, \alpha_m$, with $\alpha_i \sim \alpha_j$ and $\alpha_j \sim \alpha_m$. From Eqs. (5) and (8), we have $\text{span}([A^\top, H_{\alpha_i}^\top]^\top) = \text{span}([A^\top, H_{\alpha_j}^\top]^\top)$; similarly, $\text{span}([A^\top, H_{\alpha_j}^\top]^\top) = \text{span}([A^\top, H_{\alpha_k}^\top]^\top)$ and transitivity follows. Sufficiency also follows similarly.

It is noteworthy that the notion of (row) span in Lemma 4 is driven by S-rank and is to be considered as the maximal span over all possible non-zeros in the corresponding matrix. The following lemma establishes Type- β equivalence.

Lemma 5. Let H_{α} denote the Type- α measurement matrix. Two Type- β measurements, β_i and β_j , are equivalent, when

$$\operatorname{rank}\left(\mathcal{O}\left(A, \left[\begin{array}{c}H_{\alpha}\\H_{\beta_{i}}\end{array}\right]\right)\right) = \operatorname{rank}\left(\mathcal{O}\left(A, \left[\begin{array}{c}H_{\alpha}\\H_{\beta_{j}}\end{array}\right]\right)\right)$$
$$= \operatorname{rank}\left(\mathcal{O}\left(A, \left[\begin{array}{c}H_{\alpha}\\H_{\beta_{i}}\\H_{\beta_{j}}\end{array}\right]\right)\right) = \operatorname{rank}\left(\mathcal{O}\left(A, H_{\alpha}\right)\right) + 1. \quad (9)$$

Proof. Reflexivity and symmetry are trivial. Transitivity follows from the fact that equivalent Type- β measurements belong to the *same irreducible block* of A (see [24]).

6. ILLUSTRATIONS

Consider an LTI dynamical system with 20 states in Fig. 1 with three contractions, $\{\{2,7,9\},\{4,15\},\{10,12\}\}$, constituting the equivalent Type- α sets; and two matched parent SCCs, $\{\{11, 12, 13, 14\}, \{9\}\}$, constituting the equivalent Type- β sets (the SCC, $\{16, 17, 18\}$, e.g., has an outgoing edge and hence is not parent). Three unmatched nodes each from a contraction make the Type- α sets: $\alpha_1 \in \{2,7,9\}, \alpha_2 \in \{10, 12\}, \alpha_3 \in \{4, 15\}$. Notice that both Type- β sets share nodes with the Type- α sets. Therefore, at least three measurements, e.g., $\{4,9,12\}$, are necessary. In the case of not observing a shared α/β state, e.g., 12, more than three observations are required; for example, $\{4,9,10,13\}$ is another set of necessary measurements.



Fig. 1. (Left) Type β equivalence sets: red and green; (Right) Type- α equivalent sets: orange, purple, and green.

7. CONCLUSIONS

In this paper, we characterize measurement partitioning and observational equivalence in state estimation. We first derive both graph-theoretic and algebraic representations of two different classes of critical measurements, Type- α and Type- β , required for observability. This twofold construction of partitions leads to establishing the notion of equivalence among both Type- α and Type- β measurements with different graph-theoretic and algebraic interpretations. Notice that, there are combinatorial algorithms in *polynomial order* to find partial order of SCCs, maximum matching, and contractions in the system digraphs [24].

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