FUSION OF ALGORITHMS FOR MULTIPLE MEASUREMENT VECTORS

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ABSTRACT

We consider the recovery of sparse signals that share a common support from multiple measurement vectors. The performance of several algorithms developed for this task depends on parameters like dimension of the sparse signal, dimension of measurement vector, sparsity level, measurement noise. We propose a fusion framework, where several multiple measurement vector reconstruction algorithms participate and the final signal estimate is obtained by combining the signal estimates of the participating algorithms. We present the conditions for achieving a better reconstruction performance than the participating algorithms. Numerical simulations demonstrate that our fusion algorithm often performs better than the participating algorithms.

Index Terms— Compressed sensing, Fusion, Sparse signal reconstruction, multiple measurement vectors

1. INTRODUCTION

Consider the standard Compressed Sensing (CS) measurement setup where a K-sparse signal $\mathbf{x} \in \mathbb{R}^{N\times 1}$ is acquired through M linear measurements via

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{M\times N}$ denotes the measurement matrix, $\mathbf{b} \in \mathbb{R}^{M\times 1}$ represents the measurement vector, and $\mathbf{w} \in \mathbb{R}^{M\times 1}$ denotes the additive measurement noise present in the system. The reconstruction problem, estimating $\mathbf{x}$ from (1) using $\mathbf{A}$ and $\mathbf{b}$, is known as Single Measurement Vector (SMV) problem. In this work, we consider the Multiple Measurement Vector (MMV) problem [1] where we have L measurements: $\mathbf{b}^{(1)} = \mathbf{A}\mathbf{x}^{(1)} + \mathbf{w}^{(1)}$, $\mathbf{b}^{(2)} = \mathbf{A}\mathbf{x}^{(2)} + \mathbf{w}^{(2)}$, $\cdots$, $\mathbf{b}^{(L)} = \mathbf{A}\mathbf{x}^{(L)} + \mathbf{w}^{(L)}$. The vectors $\{\mathbf{x}^{(l)}\}_{l=1}^{L}$ are assumed to have a common sparse support-set. The problem is to estimate $\mathbf{x}^{(l)}$ $(l = 1, 2, \ldots, L)$. Instead of recovering the L signals individually, the attempt in the MMV problem is to simultaneously recover all the L signals. MMV problem arises in many applications such as the neuromagnetic inverse problem in Magnetoencephalography (a modality for imaging the brain) [2, 3], array processing [4], non-parametric spectrum analysis of time series [5], and equalization of sparse communication channels [6].

Recently many algorithms have been proposed to recover signal vectors with a common sparse support. Some among them are algorithms based on diversity minimization methods like $\ell_{2,1}$ minimization [7], and M-FOCUSS [1], greedy methods like M-OMP and M-ORMP [1], and Bayesian methods like MSBL [8] and T-MSBL [9].

However it has been observed that the performance of many algorithms depends on many parameters like the dimension of the measurement vector, the sparsity level, the statistical distribution of the non-zero elements of the signal, the measurement noise power etc. [9]. Thus it becomes difficult to choose the best sparse reconstruction algorithm without *a priori* knowledge about these parameters.

Suppose we have the sparse signal estimates given by various algorithms. It may be possible to merge these estimates to form a more accurate estimate of the original. This idea of fusion of multiple estimators has been proposed in the context of signal denoising in [10] where fusion was performed by plain averaging. Recently, Ambat *et al.* [11–15] proposed fusion of the estimates of sparse reconstruction algorithms to improve the sparse signal reconstruction performance of SMV problem.

In this paper, we propose a framework which uses several MMV reconstruction algorithms and combines their sparse signal support estimates to determine the final signal estimate. We refer to this scheme as *MMV-Fusion of Algorithms for Compressed Sensing* (MMV-FACS). We present an upper bound on the reconstruction error by MMV-FACS. We also present a sufficient condition for achieving a better reconstruction performance than any participating algorithm. By Monte-Carlo simulations we show that fusion of viable algorithms leads to improved reconstruction performance for the MMV problem.

2. PROBLEM FORMULATION

The MMV problem involves solving the following L under-determined systems of linear equations

$$\mathbf{b}^{(l)} = \mathbf{A}\mathbf{x}^{(l)} + \mathbf{w}^{(l)}, \quad l = 1, 2, 3, \ldots, L \tag{2}$$

where $\mathbf{A} \in \mathbb{R}^{M\times N}$ $(M \ll N)$ represents the measurement matrix, $\mathbf{b}^{(l)} \in \mathbb{R}^{M\times 1}$ represents the l^{th} measurement vector, and $\mathbf{x}^{(l)} \in \mathbb{R}^{N\times 1}$ denotes the corresponding K-sparse source vector. That is,

Notations: Matrices and vectors are denoted by bold upper case and bold lower case letters respectively. Sets are represented by upper case Greek alphabets and calligraphic letters. $\mathbf{A}_{\mathcal{T}}$ denotes the column sub-matrix of $\mathbf{A}$ where the indices of the columns are the elements of the set $\mathcal{T}$. $\mathbf{X}_{\mathcal{T},:}$ denotes the sub-matrix formed by those rows of $\mathbf{X}$ whose indices are listed in the set $\mathcal{T}$. $\mathbf{X}^K$ is the matrix obtained from $\mathbf{X}$ by keeping its K rows with the largest ℓ_2-norm and by setting all other rows to zero, breaking ties lexicographically. $\texttt{supp}(\mathbf{X})$ denotes the set of indices of non-zero rows of $\mathbf{X}$. For a matrix $\mathbf{X}$, $\mathbf{x}^{(l)}$ denotes the ℓ^{th} column vector of $\mathbf{X}$. $\hat{\mathbf{X}}_i$ denotes the reconstructed matrix by the i^{th} participating algorithm. The complement of the set $\mathcal{T}$ with respect to the set $\{1, 2, \ldots, N\}$ is denoted by $\mathcal{T}^c$. For two sets $\mathcal{T}_1$ and $\mathcal{T}_2$, $\mathcal{T}_1 \setminus \mathcal{T}_2 = \mathcal{T}_1 \cap \mathcal{T}_2^c$ denotes the set difference. $|\mathcal{T}|$ denotes the cardinality of set $\mathcal{T}$. $\mathbf{A}^\dagger$ and $\mathbf{A}^T$ denote the pseudo-inverse and transpose of matrix $\mathbf{A}$, respectively. The (p, q) mixed norm of the matrix $\mathbf{X}$ is defined as

$$\|\mathbf{X}\|_{(p,q)} = \left(\sum_i \|\mathbf{X}_{i,:}\|_p^q \right)^{1/q}$$

The Frobenius norm of matrix $\mathbf{A}$ is denoted as $\|\mathbf{A}\|_F$.

$|\mathtt{supp}(\mathbf{x}^{(l)})| \leq K$ and $\mathbf{x}^{(l)}$ share a common support-set for $l = 1, 2, \ldots, L$. $\mathbf{w}^{(l)} \in \mathbb{R}^{M \times 1}$ represents the additive measurement noise. We can rewrite (2) as

$$\mathbf{B} = \mathbf{A}\mathbf{X} + \mathbf{W} \tag{3}$$

where $\mathbf{X} = [\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(L)}]$, $\mathbf{W} = [\mathbf{w}^{(1)}, \mathbf{w}^{(2)}, \ldots, \mathbf{w}^{(L)}]$, and $\mathbf{B} = [\mathbf{b}^{(1)}, \mathbf{b}^{(2)}, \ldots, \mathbf{b}^{(L)}]$. For a matrix $\mathbf{X}$, we define $\mathtt{supp}(\mathbf{X}) = \bigcup_{i=1}^{L} \mathtt{supp}(\mathbf{x}^i)$. In (3), we assume that $\mathbf{X}$ is jointly K-sparse. That is, $|\mathtt{supp}(\mathbf{X})| \leq K$. There are at most K rows in $\mathbf{X}$ that contain non-zero elements. We assume that $K < M$ and K is known *a priori*.

3. FUSION OF ALGORITHMS FOR MULTIPLE MEASUREMENT VECTOR PROBLEM

In this paper, we propose to employ multiple sparse reconstruction algorithms independently for estimating $\mathbf{X}$ from (3) and fuse the resultant estimates to yield a better sparse signal estimate. Let $P \geq 2$ denote the number of different participating algorithms employed to estimate the sparse signal. Let $\hat{\mathcal{T}}_i$ denote the support-set estimated by the i^{th} participating algorithm and let $\mathcal{T}$ denote the true-support-set. Denote the union of the estimated support-sets as Γ, i.e., $\Gamma \triangleq \cup_{i=1}^{P} \hat{\mathcal{T}}_i$, assume that $R \triangleq |\Gamma| \leq M$. We hope that different participating algorithms work on different principles and the support-set estimated by each participating algorithm includes a partially correct information about the true support-set $\mathcal{T}$. It may be also observed that the union of the estimated support-sets, Γ, is richer in terms of the true atoms as compared to the support-set estimated by any participating algorithm. Also note that, once the support-set is estimated, the non-zero magnitudes of $\mathbf{X}$ can be estimated by solving a Least-Squares (LS) problem on an over-determined system of linear equations. Hence if we can identify all the true atoms included in the joint support-set Γ, we can achieve a better sparse signal estimate.

Since we are estimating the support atoms only from Γ, we need to only solve the following problem which is lower dimensional as compared to the original problem (3):

$$\mathbf{B} = \mathbf{A}_{\Gamma}\mathbf{X}_{\Gamma,:} + \tilde{\mathbf{W}}, \tag{4}$$

where $\mathbf{A}_{\Gamma}$ denotes the sub-matrix formed by the columns of $\mathbf{A}$ whose indices are listed in Γ, $\mathbf{X}_{\Gamma,:}$ denotes the submatrix formed by the rows of $\mathbf{X}$ whose indices are listed in Γ, and $\tilde{\mathbf{W}} = \mathbf{W} + \mathbf{A}_{\Gamma^c}\mathbf{X}_{\Gamma^c,:}$. The matrix equation (4) represents a system of L linear equations which are over-determined in nature. We use the method of LS to find an approximate solution to the overdetermined system of equations in (4). Let $\mathbf{V}_{\Gamma,:}$ denote the LS solution of (4). We choose the support-set estimate of MMV-FACS as the support of $\mathbf{V}^K$, i.e., indices of those rows having the largest ℓ_2-norm. Once the non-zero rows are identified, solving the resultant overdetermined solution using LS we can estimate the non-zero entries of $\hat{\mathbf{X}}$. The proposed MMV-FACS is summarized in Algorithm 1.

3.1. Theoretical Results

In this section, we will state some theoretical results for the performance of MMV-FACS. We consider the general case for an arbitrary signal matrix. We also state the average case performance of MMV-FACS subsequently. For brevity we state only the results and the proofs are available in the extended version of this work [16].

Algorithm 1 : MMV-FACS

Inputs: $\mathbf{A} \in \mathbb{R}^{M \times N}$, $\mathbf{B} \in \mathbb{R}^{M \times L}$, K, and $\left\{\hat{\mathcal{T}}_i\right\}_{i=1:P}$.

Assumption: $|\bigcup_{i=1}^{P} \hat{\mathcal{T}}_i| \leq M$.

Initialization: $\mathbf{V} = \mathbf{0} \in \mathbb{R}^{N \times L}$.

Fusion:

1. $\Gamma = \cup_{i=1}^{P} \hat{\mathcal{T}}_i$;
2. $\mathbf{V}_{\Gamma,:} = \mathbf{A}_{\Gamma}^{\dagger}\mathbf{B}$, $\mathbf{V}_{\Gamma^c,:} = \mathbf{0}$;
3. $\hat{\mathcal{T}} = \mathtt{supp}(\mathbf{V}^K)$;

Outputs: $\hat{\mathcal{T}}$ and $\hat{\mathbf{X}}$ (where $\hat{\mathbf{X}}_{\hat{\mathcal{T}},:} = \mathbf{A}_{\hat{\mathcal{T}},:}^{\dagger}\mathbf{B}$ and $\hat{\mathbf{X}}_{\hat{\mathcal{T}}^c,:} = \mathbf{0}$)

For the theoretical analysis, we use the Restricted Isometry Property (RIP) [17] of the measurement matrix which is defined as follows.

Definition 1. *A matrix* $\mathbf{A}$ *satisfies Restricted Isometry Property (RIP) if for some* $\delta_K \in [0, 1)$

$$(1 - \delta_K) \|\mathbf{x}\|_2^2 \leq \|\mathbf{A}\mathbf{x}\|_2^2 \leq (1 + \delta_K) \|\mathbf{x}\|_2^2 \tag{5}$$

holds for all K-sparse vectors $\mathbf{x}$. *The Restricted Isometry Constant (RIC) is the smallest constant* $\delta_K \in [0, 1)$ *such that* (5) *holds for all K-sparse vectors* $\mathbf{x}$.

The performance analysis is characterized by a measure called *Signal-to-Reconstruction-Error Ratio* (SRER) defined as

$$\text{SRER} \triangleq \frac{\|\mathbf{X}\|_F^2}{\left\|\mathbf{X} - \hat{\mathbf{X}}\right\|_F^2} \tag{6}$$

where $\mathbf{X}$ and $\hat{\mathbf{X}}$ denote the actual and reconstructed signal matrix respectively.

3.1.1. Performance Analysis for Arbitrary Signals under Measurement Perturbations

It is seen that many signals found in practice are not exactly sparse, but are compressible in nature [18]. We give an upper bound on the reconstruction error by MMV-FACS in Theorem 1. We also state a sufficient condition to get an improved performance of MMV-FACS scheme over any given participating algorithm.

Theorem 1. *Let* $\mathbf{X}$ *be an arbitrary signal with supp(*$\mathbf{X}^K$*)=*$\mathcal{T}$*. Consider the MMV-FACS setup discussed in Section 3, and assume that the measurement matrix* $\mathbf{A}$ *satisfies RIP with RIC* δ_{R+K}*. We have the following results :*

1. *Upper bound on reconstruction error : Defining* $\nu = \dfrac{3 - \delta_{R+K}^2}{(1 - \delta_{R+K})^2}$, $C_1 = \left(1+\nu\sqrt{1 + \delta_{R+K}}\right)$, $C_2 = \dfrac{\nu\sqrt{1 + \delta_{R+K}}}{\sqrt{R + K}}$ *and* $C_3 = \dfrac{1 + \delta_{R+K}}{(1 - \delta_{R+K})^2}$, *we have the result that*

$$\left\|\mathbf{X} - \hat{\mathbf{X}}\right\|_F \leq C_1 \left\|\mathbf{X} - \mathbf{X}^K\right\|_F + C_2 \left\|\mathbf{X} - \mathbf{X}^K\right\|_{2,1} + C_3 \left\|\mathbf{X}_{\Gamma^c,:}\right\|_F + \nu \left\|\mathbf{W}\right\|_F$$

2. *SRER Gain : Assume that* $\left\|\mathbf{X}_{\hat{\mathcal{T}}_i^c,:}\right\|_F \neq 0$, $\|\mathbf{X}_{\Gamma^c,:}\|_F \neq 0$.

   Define $\eta_i = \dfrac{\|\mathbf{X}_{\Gamma^c,:}\|_F}{\left\|\mathbf{X}_{\hat{\mathcal{T}}_i^c,:}\right\|_F}$, $\zeta = \dfrac{\|\mathbf{W}\|_F}{\|\mathbf{X}_{\Gamma^c,:}\|_F}$, *and*

$$\xi = \left(3\sqrt{1+\delta_{R+K}}+1\right)\frac{\left\|\mathbf{X}-\mathbf{X}^K\right\|_F}{3\left\|\mathbf{X}_{\Gamma^c,:}\right\|_F} + \frac{\sqrt{1+\delta_{R+K}}}{\sqrt{R+K}}\frac{\left\|\mathbf{X}-\mathbf{X}^K\right\|_{2,1}}{\|\mathbf{X}_{\Gamma^c,:}\|_F}$$

   MMV-FACS provides at least SRER gain of $\left(\dfrac{(1-\delta_{R+K})^2}{(1+\delta_{R+K}+3\zeta+3\xi)\eta_i}\right)^2$ *over the* i^{th} *participating algorithm*

   if $\eta_i < \dfrac{(1-\delta_{R+K})^2}{(1+\delta_{R+K}+3\zeta+3\xi)}$.

3.1.2. Exactly K-sparse matrix

Theorem 1 considered the case when $\mathbf{X}$ is an arbitrary matrix. If $\mathbf{X}$ is a K-sparse matrix then we have $\mathbf{X} = \mathbf{X}^K$. Thus it follows from part 1 of Theorem 1 that if $\mathbf{X}_{\Gamma^c,:} = \mathbf{0}$ (union set contains all the correct atoms) and $\mathbf{W} = \mathbf{0}$ (clean measurement case) MMV-FACS provides exact reconstruction.

Also, it follows from part 2 of Theorem 1 that MMV-FACS provides at least SRER gain of $\left(\dfrac{(1-\delta_{R+K})^2}{(1+\delta_{R+K}+3\zeta)\eta_i}\right)^2$ over i^{th} algorithm if $\eta_i < \dfrac{(1-\delta_{R+K})^2}{(1+\delta_{R+K}+3\zeta)}$. Thus, the improvement in the SRER gain provided by MMV-FACS over the i^{th} Algorithm for a K-sparse matrix is greater than that of an arbitrary matrix by a factor of $\left(1+\dfrac{3\xi}{(1+\delta_{R+K}+3\zeta)}\right)^2$.

3.1.3. Average Case Analysis

Intuitively, we expect multiple measurement vector problem to perform better than the single measurement vector case. However, if each measurement vector is the same, i.e., in the worst case, we have $\mathbf{x}^{(i)} = \mathbf{c}\ \forall\ i = 1,\ldots,L$ then we do not have extra information on $\mathbf{X}$ than provided by a single vector $\mathbf{x}^{(1)}$.

The theoretical results presented till now are worst case analysis, i.e., conditions under which the algorithm is able to recover any joint sparse matrix $\mathbf{X}$. This approach does not provide insight into the superiority of sparse signal reconstruction with multiple measurement vectors compared to the single measurement vector case.

To notice a performance gain with multiple measurement vectors, we proceed with an average case analysis. For average case analysis, on the support set $\mathcal{T}$, we impose that $\mathbf{X}_{\mathcal{T},:} = \Sigma\Phi$, where Σ is a $K \times K$ diagonal matrix with positive diagonal entries and Φ is a $K \times L$ random matrix with i.i.d. Gaussian entries. Our goal is to show that under this signal model the typical behaviour of MMV-FACS is better than in the worst case.

Theorem 2. *Consider the MMV-FACS setup discussed in Section 3. Assume a Gaussian signal model, i.e.,* $\mathbf{X}_{\mathcal{T},:} = \Sigma\Phi$, *where* Σ *is a* $K \times K$ *diagonal matrix with positive diagonal entries and* Φ *is a* $K \times L$ *random matrix with i.i.d. Gaussian entries. Let* $\mathbf{e}_i$ *denote the* $|\Gamma| \times 1$ *vector with a '1' in the* i^{th} *coordinate and '0' elsewhere. Let* $\eta = \min_{i\in(\mathcal{T}\cap\Gamma)}\left\|\mathbf{e}_i^T\mathbf{A}_\Gamma^\dagger\mathbf{W}\right\|_2 + \max_{j\in(\Gamma\setminus\mathcal{T})}\left\|\mathbf{e}_j^T\mathbf{A}_\Gamma^\dagger\mathbf{W}\right\|_2$ *and*

$$\gamma = \frac{\min_{i\in(\mathcal{T}\cap\Gamma)}\left\|\mathbf{e}_i^T\mathbf{A}_\Gamma^\dagger\mathbf{A}_\mathcal{T}\Sigma\right\|_2 - \max_{j\in(\Gamma\setminus\mathcal{T})}\left\|\mathbf{e}_j^T\mathbf{A}_\Gamma^\dagger\mathbf{A}_\mathcal{T}\Sigma\right\|_2 - \dfrac{\eta}{C_2(L)}}{\min_{i\in(\mathcal{T}\cap\Gamma)}\left\|\mathbf{e}_i^T\mathbf{A}_\Gamma^\dagger\mathbf{A}_\mathcal{T}\Sigma\right\|_2 + \max_{j\in(\Gamma\setminus\mathcal{T})}\left\|\mathbf{e}_j^T\mathbf{A}_\Gamma^\dagger\mathbf{A}_\mathcal{T}\Sigma\right\|_2}$$

where $C_2(L) = \mathbb{E}\|Z\|_2$ *with* $Z = (Z_1,\ldots,Z_L)$ *being a vector of independent standard Gaussian variables. Assume that* $\min_{i\in(\mathcal{T}\cap\Gamma)}\left\|\mathbf{e}_i^T\mathbf{A}_\Gamma^\dagger\mathbf{A}_\mathcal{T}\Sigma\right\|_2 - \max_{j\in(\Gamma\setminus\mathcal{T})}\left\|\mathbf{e}_j^T\mathbf{A}_\Gamma^\dagger\mathbf{A}_\mathcal{T}\Sigma\right\|_2 > \dfrac{\eta}{C_2(L)}$. *Let* Θ *denote the event that MMV-FACS picks all correct indices from the union set* Γ. *Then, we have*

$$P(\Theta) \geq 1 - K\exp(-2A_2(L)\gamma^2)$$

where

$$A_2(L) = \left(\frac{\ddot{\Gamma}((L+1)/2)}{\ddot{\Gamma}(L/2)}\right)^2 \approx L/2$$

where $\ddot{\Gamma}(\cdot)$ *denotes the Gamma function.*

Because $A_2(L) \approx L/2$ the probability that MMV-FACS selects all correct indices from the union set increases as L increases. Thus more than one measurement vector improves the performance.

4. NUMERICAL EXPERIMENTS AND RESULTS

We conducted numerical experiments using synthetic data to evaluate the performance of MMV-FACS. The performance is evaluated using Average SRER (ASRER) which is defined as

$$\text{ASRER} = \frac{\sum_{j=1}^{n_{trials}}\|\mathbf{X}_j\|_F^2}{\sum_{j=1}^{n_{trials}}\left\|\mathbf{X}_j - \hat{\mathbf{X}}_j\right\|_F^2}$$

where $\mathbf{X}_j$ and $\hat{\mathbf{X}}_j$ denote the actual and reconstructed jointly sparse signal matrix in the j^{th} trial respectively, and n_{trials} denotes the total number of trials.

We define the Signal-to-Measurement-Noise-Ratio (SMNR), as $\text{SMNR} \triangleq \mathbf{E}\left\{\left\|\mathbf{x}^{(i)}\right\|_2^2\right\}\Big/\mathbf{E}\left\{\left\|\mathbf{w}^{(i)}\right\|_2^2\right\}$, where $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation operator. For brevity, we show only a few simulation results in this paper. More results are available at [16].

4.1. Experimental Set-up

Following steps are involved in the simulation :

(i) Generate elements of $\mathbf{A}_{M\times N}$ independently from $\mathcal{N}(0, \frac{1}{M})$ and normalize each column norm to unity.

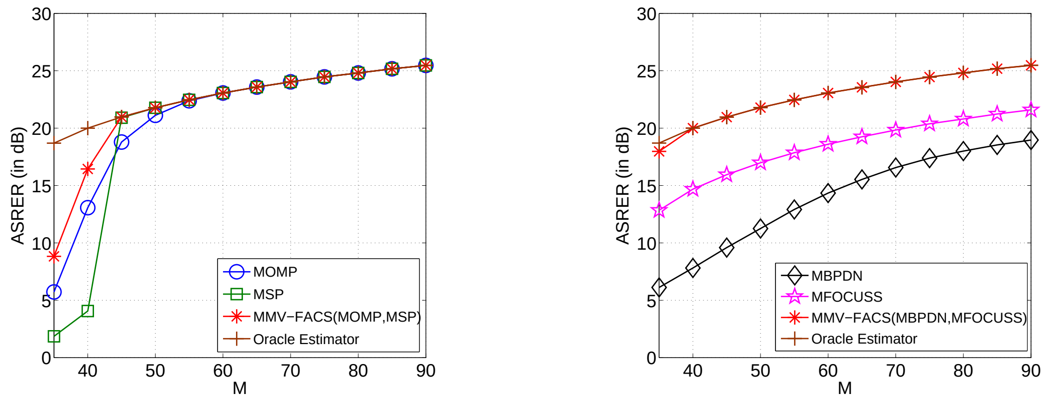
(ii) Choose K non-zero locations uniformly at random from the set $\{1, 2, \ldots, N\}$ and fill those K rows of $\mathbf{X}$ with values independently from $\mathcal{N}(0, 1)$. Remaining $N - K$ rows of $\mathbf{X}$ are made zero.

(iii) The MMV measurement matrix $\mathbf{B}$ is computed as $\mathbf{B} = \mathbf{AX} + \mathbf{W}$, where the columns of $\mathbf{W}$, $\mathbf{w}^{(i)}$'s are independent and their elements are i.i.d and Gaussian with variance determined from a specified SMNR.

(iv) Apply the MMV sparse recovery method.

(v) Repeat steps (1)-(4), 1000 times.

(vi) Calculate ASRER.



(a) Fusion of MOMP and MSP

(b) Fusion of MBPDN and MFOCUSS

Fig. 1. Performance of MMV-FACS, averaged over $1,000$ trials, for Gaussian sparse signal matrix with SMNR $=$ 20 dB. Sparse signal dimension $N = 500$, Sparsity level $K = 20$ and number of measurement vectors $L = 20$.

4.2. Results

We used M-OMP, M-SP, M-BPDN [19] and M-FOCUSS [3] as the participating algorithms in MMV-FACS. The software code of M-BPDN was taken from [20]. Since M-FOCUSS and M-BPDN algorithms may not yield an exact K-sparse solution, we estimate the support-set as the indices of the K rows with largest l_2 norm. We fixed the sparse signal dimension $N = 500$ and sparsity level $K = 20$ in the simulations. As an upper bound on the performance, we use an *Oracle estimator*, which is aware of the true support set and finds the non-zero entries of the sparse matrix by solving least-squares.

The empirical performance of MMV reconstruction algorithms for different values of M is shown in Fig.1. The simulation parameters are $L = 20$, SMNR=20 dB and $\mathbf{X}$ is chosen as Gaussian sparse signal matrix. The results of fusion of MOMP and MSP are shown in Fig. 1(a). It may be observed that for $M = 35$ and $M = 40$ MOMP shows a better ASRER than MSP. However for $M = 45$ MSP resulted in a better ASRER than MOMP. Interestingly, MMV-FACS(MOMP, MSP) always resulted in a better ASRER than both MOMP and MSP in these cases.

Fig. 1(b) depicts the results of fusion of MBPDN and MFOCUSS. Here also MMV-FACS yielded a better ASRER than the participating algorithms MBPDN and MFOCUSS. For example, for $M = 35$, MMV-FACS (MBPDN,MFOCUSS) gives 10.67 dB and 4.27 dB improvement over MBPDN and MFOCUSS respectively.

4.2.1. Reproducible Research

We provide necessary Matlab codes to reproduce the figures, publicly downloadable from `http://www.ece.iisc.ernet.in/~ssplab/Public/MMVFACS.tar.gz`.

4.3. Real Compressible Signals

To evaluate the performance of MMV-FACS on compressible signals and real world data, we used the data set '*05091.dat*' from MIT-BIH Atrial Fibrillation Database [21]. We used a randomly generated Gaussian sensing matrix of size $M \times 250$, with different values of M in the experiment. We assumed sparsity level $K = 50$ and used M-OMP and M-SP as the participating algorithms. The reconstruction results are shown in Figure 2.

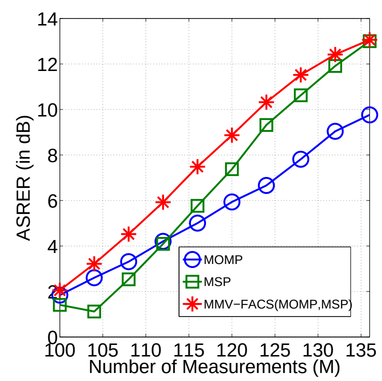


Fig. 2. Performance of MMV-FACS for 2-channel ECG signals from MIT-BIH Atrial Fibrillation Database [21].

Similar to synthetic signals, MMV-FACS shows a better ASRER compared to the participating algorithms M-OMP and M-SP. This demonstrates the advantage of MMV-FACS in real-life applications, requiring fewer measurement samples to yield an approximate reconstruction.

5. CONCLUSIONS

We proposed a fusion framework for MMV problem where we employ multiple MMV sparse reconstruction algorithms and fuse the resultant estimates to yield a better sparse signal estimate which is often better than the best sparse reconstruction algorithm among the participating algorithms. We provided theoretical results for improvement in sparse signal reconstruction. The performance of the proposed method was also verified using Monte Carlo simulations.

6. REFERENCES

[1] S. F. Cotter, B. D. Rao, K. Engan, and K. Kreutz-Delgado, "Sparse solutions to linear inverse problems with multiple measurement vectors," *IEEE Trans. Signal Process.*, vol. 53, no. 7, pp. 2477–2488, July 2005.

[2] I. F. Gorodnitsky, J. S. George, and B. D. Rao, "Neuromagnetic source imaging with focuss : A recursive weighted minimum norm algorithm," *J. Electroencephalog. Clinical Neurophysiol.*, vol. 95, no. 4, pp. 231–251, Oct. 1995.

[3] I. F. Gorodnitsky and B. D. Rao, "Sparse signal reconstruction from limited data using focuss: a re-weighted minimum norm algorithm," *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 600–616, Mar 1997.

[4] D. Malioutov, M. Cetin, and A. S. Willsky, "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010–3022, Aug 2005.

[5] Petre Stoica and Randolph L. Moses, *Introduction to spectral analysis*, Upper Saddle River, N.J. Prentice Hall, 1997.

[6] S. F. Cotter and B. D. Rao, "Sparse channel estimation via matching pursuit with application to equalization," *IEEE Trans. Commun.*, vol. 50, no. 3, pp. 374–377, Mar 2002.

[7] Joel A. Tropp, Anna C. Gilbert, and Martin J. Strauss, "Algorithms for simultaneous sparse approximation: Part i: Greedy pursuit," *Signal Process.*, vol. 86, no. 3, pp. 572–588, Mar. 2006.

[8] D. P. Wipf and B. D. Rao, "An empirical bayesian strategy for solving the simultaneous sparse approximation problem," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3704–3716, July 2007.

[9] Zhilin Zhang and B. D. Rao, "Sparse signal recovery with temporally correlated source vectors using sparse bayesian learning," *IEEE J. Sel. Topics Signal Process.*, vol. 5, no. 5, pp. 912–926, Sept 2011.

[10] M. Elad and I. Yavneh, "A plurality of sparse representations is better than the sparsest one alone," *IEEE Trans. Inf. Theory*, vol. 55, no. 10, pp. 4701–4714, Oct 2009.

[11] Sooraj K. Ambat, Saikat Chatterjee, and K.V.S. Hari, "Fusion of algorithms for compressed sensing," *IEEE Trans. Signal Process.*, vol. 61, no. 14, pp. 3699–3704, July 2013.

[12] Sooraj K. Ambat, Saikat Chatterjee, and K.V.S. Hari, "Fusion of Algorithms for Compressed Sensing," in *Acoustics, Speech and Signal Processing (ICASSP), 2013 IEEE International Conference on*, May 2013, pp. 5860–5864.

[13] Sooraj K. Ambat, S. Chatterjee, and K.V.S. Hari, "A Committee Machine Approach for Compressed Sensing Signal Reconstruction," *IEEE Trans. Signal Process.*, Accepted 2014.

[14] Sooraj K. Ambat, Saikat Chatterjee, and K.V.S. Hari, "Progressive Fusion of Reconstruction Algorithms for Low Latency Applications in Compressed Sensing," *Signal Processing*, vol. 97, no. 0, pp. 146 – 151, 2014.

[15] Sooraj K. Ambat, Saikat Chatterjee, and K.V.S. Hari, "Fusion of Greedy Pursuits for Compressed Sensing Signal Reconstruction," in *20th European Signal Processing Conference 2012 (EUSIPCO 2012)*, Bucharest, Romania, Aug. 2012.

[16] Deepa K. G., Sooraj K. Ambat, and K. V. S. Hari, "Fusion of Sparse Reconstruction Algorithms for Multiple Measurement Vectors," *ArXiv e-prints*, Apr. 2015, http://arxiv.org/abs/1504.01705v1.

[17] Emmanuel J. Candès, Justin K. Romberg, and Terence Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Communications on Pure and Applied Mathematics*, vol. 59, no. 8, pp. 1207–1223, 2006.

[18] E. J. Candès and M. B. Wakin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 21–30, March 2008.

[19] E. van den Berg and M. P. Friedlander, "Probing the pareto frontier for basis pursuit solutions," *SIAM Journal on Scientific Computing*, vol. 31, no. 2, pp. 890–912, 2008.

[20] E. van den Berg and M. P. Friedlander, "SPGL1: A solver for large-scale sparse reconstruction," June 2007, http://www.cs.ubc.ca/labs/scl/spgl1.

[21] A. L. Goldberger, L. A. N. Amaral, L. Glass, J. M. Hausdorff, P. Ch. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C.-K. Peng, and H. E. Stanley, "PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals," *Circulation*, vol. 101, no. 23, pp. e215–e220, 2000 (June 13), Circulation Electronic Pages: http://circ.ahajournals.org/cgi/content/full/101/23/e215 PMID:1085218; doi: 10.1161/01.CIR.101.23.e215.