# SPARSITY BASED MULTI-TARGET TRACKING USING MOBILE SENSORS

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**ABSTRACT**

Sensor mobility and sparse sensor data correlations are exploited in this work for the problem of tracking multiple targets. Sparse matrix decomposition is integrated with the design of proper kinematic rules to identify informative sensors, associate them with the targets and enable them to follow closely the moving targets. The modified barrier method is employed to minimize proper error covariance matrices obtained by extended Kalman filtering recursions. Distributed updating recursive rules are obtained enabling only the informative sensors to update their location, and follow closely the corresponding targets while staying connected. This is to be contrasted with existing alternatives where all sensors need to move constantly. Numerical tests corroborate that the proposed scheme outperforms existing alternatives while accurately tracks multiple targets by imposing mobility only on target-informative sensors.

***Index Terms***— Mobile sensor networks, multi-target tracking, sparse decomposition, distributed processing

## 1. INTRODUCTION

Sensor networks find wide applicability due to the low cost of sensing units, the ability to cover large areas and robustness. Further, the incorporation of mobility in the sensors enables further flexibility, especially when tracking time-varying and non-stationary targets. The focus here is in deriving proper kinematic mechanisms along with association and tracking techniques to allow accurate tracking of multiple targets. Existing tracking schemes that utilize sensor mobility require all the sensing units in the network to acquire measurements, move and perform tracking [3, 9, 13]. Such approaches are expected to be resource-consuming despite the fact that the field targets are localized and affect a small percentage of the available sensors. The goal is to design an algorithmic framework that associates targets with a few sensors, and then properly determine kinematic strategies only for the informative sensors which will closely follow the field targets.

Single target tracking using mobile sensors has been studied for a wide variety of scenarios [7, 8, 15, 17]. Existing tracking schemes control the movement of *all* sensors by minimizing the estimation error covariance, [15], [3], while [17] manages the sensor mobility based on a Bayesian estimation model and restricting sensors to move only on a grid of locations.

In the presence of multiple-targets, the approach in [9] focuses on moving robotic sensors at fixed locations determined by exhaustive search on a grid of possible coordinates, while there is a need for a central fusion center to perform the processing. The scheme in [4], designs a Kalman filtering approach with gradient descent based kinematic rules under the assumption that it is known which targets every sensor observes bypassing in that way the essential sensor-to-target association step. Further, all sensors need to move at every time instant making the approach resource-demanding.

Exploiting statistical correlation between sensor measurements that sense the same target, sensor-to-target association is achieved via the sparse matrix decomposition scheme in [10, 11] originally designed for stationary (immobile) sensors. This sparsity-based approach is extended here to mobile sensors by integrating it with proper sensor kinematic strategies and tracking techniques. The modified barrier method [2, pg. 423] is employed to obtain distributed kinematic rules by minimizing proper error covariance matrices obtained by extended Kalman filtering recursions. In contrast to existing approaches, the novel framework identifies and controls the movement *only* of target-informative sensors allowing for accurate tracking.

## 2. PROBLEM STATEMENT

An ad hoc sensor network conformed with $p$ mobile sensors is considered here. The sensors monitor a field where an unknown and possibly time-varying number of moving targets is present. Each sensor communicates only with its neighboring sensors which are within its communication range and are able to exchange information via single-hop communications.

Each target, say the $\rho$th is characterized by a $4 \times 1$ state vector which contains both its position and velocity information, namely $\mathbf{p}_\rho(t) = [p_{\rho,x}(t), p_{\rho,y}(t)]^T$ and $\mathbf{v}_\rho(t) = [v_{\rho,x}(t), v_{\rho,y}(t)]^T$. At discrete-time $t = 0, 1, 2, \ldots$ the state vector can be written as: $\mathbf{s}_\rho(t) = [\mathbf{p}_\rho^T(t)\mathbf{v}_\rho^T(t)]^T$, while it evolves according to the following constant velocity model, see e.g., [1]:

$$\mathbf{s}_\rho(t+1) = \mathbf{F}\mathbf{s}_\rho(t) + \mathbf{u}_\rho(t), \tag{1}$$

where $\mathbf{F}$ is the $4 \times 4$ state transition matrix and $\mathbf{u}_\rho(t)$ the zero mean Gaussian state noise with variance $\mathbf{\Sigma}_u$. Matrices $\mathbf{F}$ and $\mathbf{\Sigma}_u$ for the constant velocity model are given as follows:

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{\Sigma}_u = \sigma_u^2 \begin{bmatrix} \frac{(\delta T)^3}{3} \cdot \mathbf{I}_2 & \frac{(\delta T)^2}{2} \cdot \mathbf{I}_2 \\ \frac{(\delta T)^2}{2} \cdot \mathbf{I}_2 & \delta T \cdot \mathbf{I}_2 \end{bmatrix}$$

where $\sigma_u^2$ is the noise variance and $\mathbf{I}_2$ is the identity matrix of size $2 \times 2$, while $\delta T$ denotes the sampling period.

Sensor $j$, senses the moving targets, by acquiring at time $t$ a scalar measurement depending on the target location according to the following nonlinear model:

$$x_j(t) = \sum\nolimits_{\rho=1}^{R} a_\rho(t) d_{j,\rho}^{-2}(t) + w_j(t), \; j = 1, \ldots, p, \tag{2}$$

This work is supported by the grants NSF CCF 1218079 and AFOSR FA9550-15-1- 0103.

where $a_\rho(t)$ denotes the intensity of signal emitted from the $\rho$th target, and $d_{j,\rho}(t) = \|\mathbf{p}_j(t) - \mathbf{p}_\rho(t)\|$ is the distance between sensor $j$, located at $\mathbf{p}_j(t)$, and the $\rho$th target at time $t$. The total number of targets which move in the field throughout the lifespan of the SN is indicated as $R$, and $w_j(t)$ denotes the white sensing noise with variance $\sigma_w^2$. In the measurement model in (2), it is assumed that targets act as transmitters and the signals emitted from the targets propagate via free space and are superimposed [5]. Among the summands in (2), one of them is assumed to be relatively stronger than the rest. This corresponds to a setting where almost one target is present within the sensing range of a sensor. The signal amplitudes $a_\rho(t)$ are assumed to be uncorrelated for different targets.

Stacking all the sensor measurements in (2) on an $p \times 1$ vector it follows:

$$\mathbf{x}_t = \mathbf{D}_t \mathbf{a}_t + \mathbf{w}_t, \text{ where } \mathbf{a}_t := [a_1(t)\ a_2(t) \ldots a_R(t)]^T, \quad (3)$$

while $\mathbf{D}_t$ is a $p \times R$ matrix with entries $\mathbf{D}_t(j,\rho) = d_{j,\rho}^{-2}(t)$ with $j = 1, \ldots, p$ and $\rho = 1, \ldots, R$. The noise $\mathbf{w}_t$ has covariance $\mathbf{\Sigma}_w = \sigma_w^2 \mathbf{I}_p$. Given that the entries of $\mathbf{a}_t$ are uncorrelated, it follows readily that the data covariance matrix is

$$\mathbf{\Sigma}_{x,t} = \mathbf{D}_t \mathbf{\Sigma}_a \mathbf{D}_t^T + \sigma_w^2 \mathbf{I}_p = \bar{\mathbf{D}}_t \bar{\mathbf{D}}_t^T + \sigma_w^2 \mathbf{I}_p, \quad (4)$$

where $\mathbf{\Sigma}_a$ is the diagonal covariance matrix of $\mathbf{a}_t$, while $\bar{\mathbf{D}}_t := \mathbf{D}_t \mathbf{\Sigma}_a^{1/2}$. Among the $R$ entries in $\mathbf{a}_t$, there will be $r(t)$ nonzero entries corresponding to the active targets moving in the sensed field at $t$.

The goal is to allow the mobile sensors to track an unknown number of targets present in the monitored field. Novel target association and sensor mobility strategies will be combined with tracking techniques to enable sensors to accurately track the different target trajectories.

## 3. SENSOR MOBILITY, ASSOCIATION AND TRACKING

### 3.1. Target-Informative Sensor Selection

Due to the presence of multiple target in the monitored field, the first goal is to determine sets of sensors, namely $\mathcal{S}_{\rho,t}$, that acquire information bearing measurements about the $\rho$th target (association step). From the observation model in (3), note that the strong-amplitude entries of the $\rho$ column in $\mathbf{D}_t$, namely $\{\mathbf{D}_{t,:\rho}\}_{\rho=1}^R$, can reveal the sensors within subset $\mathcal{S}_{\rho,t}$. Recall that $\mathbf{D}_t(j,\rho) = d_{j,\rho}^{-2}(t)$, thus when sensor $j$ and target $\rho$ are close in distance then the corresponding entry is expected to have large amplitude, while the further away they get from each other the smaller the corresponding entry becomes. The matrix $\mathbf{D}_t$ can be assumed approximately sparse. To identify the informative sensors for every time step, we resort to the sparse matrix decomposition method in [11]. First the data covariance matrix $\mathbf{\Sigma}_{x,t}$ is estimated in real-time using exponentially-weighted averaging

$$\hat{\mathbf{\Sigma}}_{x,t} = \frac{1-\gamma}{1-\gamma^{t+1}} \textstyle\sum_{\tau=0}^{t} \gamma^{t-\tau} (\mathbf{x}_\tau - \bar{\mathbf{x}}_t)(\mathbf{x}_\tau - \bar{\mathbf{x}}_t)^T, \quad (5)$$

where $\gamma \in (0,1)$ denotes a forgetting factor and

$$\bar{\mathbf{x}}_t = (1-\gamma)(1-\gamma^{t+1})^{-1} \textstyle\sum_{\tau=0}^{t} \gamma^{t-\tau} x_\tau,$$

is an estimate for the expectation. Then, the sparse columns of $\bar{\mathbf{D}}_t$, namely $\{\hat{\mathbf{D}}_{t,:\rho}\}_{\rho=1}^{\hat{r}(t)}$, at time instant $t$ are estimated by minimizing the following norm-one/norm-two regularized formulation:

$$\left(\hat{\mathbf{M}}_t, \{\hat{\sigma}_j\}_{j=1}^m\right) := \arg\min_{\mathbf{M}_t, \{\sigma_j\}_{j=1}^m} \|\mathbf{E} \odot \left(\hat{\mathbf{\Sigma}}_{x,t} - \mathbf{M}_t \mathbf{M}_t^T \right. \quad (6)$$
$$\left. - \operatorname{diag}(\sigma_{1,t}^2, \ldots, \sigma_{m,t}^2)\right) \|_F^2 + \sum_{\ell=1}^{L} \left(\lambda_\ell \|\mathbf{M}_{t,:\ell}\|_1 + \phi \|\mathbf{M}_{t,:\ell}\|_2\right),$$

where $\odot$ denotes the Hadamard operator (entry-wise matrix product), $\mathbf{E}$ the sensor network adjacency matrix, while $\sigma_j^2$ is the sensing noise variance estimate at sensor $j$, and $L$ is an upper bound for the number of active sensed targets $r(t)$ ($L \geq r(t)$) and $\mathbf{M}_t \in \mathbb{R}^{p \times L}$ contains $L$ columns that estimate the sparse columns of $\bar{\mathbf{D}}_t$. $\mathbf{M}_{t,:\ell}$ denotes the $\ell$th column of $\mathbf{M}_t$. The coefficient $\lambda_\ell$ denotes the nonnegative sparsity-controlling coefficient used to adjust the number of zeros in $\hat{\mathbf{M}}_{t,:\ell}$, see e.g., [16]. The coefficient $\phi \geq 0$ in the last term of (6) promotes group sparsity among rows [14], this is done to zero-out unnecessary columns in $\hat{\mathbf{M}}_t$ when the number of active targets in the field is smaller than $R$. The cost in (6) is minimized by an iterative distributed minimization scheme based on coordinate descent [2, 12], where sensor $j$ is responsible for updating the $j$th row of $\mathbf{M}_t$, namely $\mathbf{M}_{t,j:}$ and variance $\sigma_j^2$. Details can be found in [10, 11] where stationary (immobile) sensor networks are considered.

### 3.2. Tracking

The target-informative sensor subsets $\mathcal{S}_{\rho_\ell,t}$ for $\ell = 1, \ldots, \hat{r}(t)$, where $\hat{r}(t)$ corresponds to an estimate of the number of targets at time instant $t$ obtained from the number of nonzero columns of $\hat{\mathbf{M}}_t := \hat{\mathbf{M}}_t^K$ at $t$ after applying $K$ coordinate cycles in (6). Extended Kalman filtering is employed to process the nonlinear observations and track each target's location using the observations of the corresponding set $\mathcal{S}_{\rho_\ell,t}$. The target state estimator and corresponding error covariance matrix, obtained by the extended Kalman filter using the observations in $\mathcal{S}_{\rho_\ell,t}$ for target $\rho_\ell$ are denoted by $\hat{\mathbf{s}}_{\rho_\ell}(t|t)$ and $\mathbf{M}_{\rho_\ell}(t|t)$, respectively. The prediction state, see e.g. [1, 6], involves the following updating recursions for the state estimator and covariance at time instant $t$

$$\hat{\mathbf{s}}_{\rho_\ell}(t+1|t) = \mathbf{F}\hat{\mathbf{s}}_{\rho_\ell}(t|t),\ \hat{\mathbf{M}}_{\rho_\ell}(t+1|t) = \mathbf{F}\hat{\mathbf{M}}_{\rho_\ell}(t|t)\mathbf{F}^T + \mathbf{\Sigma}_u. \quad (7)$$

The measurements of the sensors within set $\mathcal{S}_{\rho_\ell,t}$ will then be used to carry out the correction step of the extended Kalman filter which involves the following update recursions

$$\hat{\mathbf{s}}_{\rho_\ell}(t+1|t+1) = \hat{\mathbf{s}}_{\rho_\ell}(t+1|t) \quad (8)$$
$$+ \mathbf{K}_{\rho_\ell}(t+1) \cdot [\mathbf{x}(t+1) - a_\rho(t)\hat{\mathbf{D}}_{\mathcal{S}_{\rho_\ell,t}}]$$
$$\mathbf{M}_{\rho_\ell}(t+1|t+1) = \mathbf{M}_{\rho_\ell}(t+1|t)$$
$$+ \mathbf{D}_{\nabla,\rho_\ell}^T \cdot \sigma_w^2 \mathbf{I}_{|\mathcal{S}_{\rho_\ell,t}|} \cdot \mathbf{D}_{\nabla,\rho_\ell}, \quad (9)$$

for $\ell = 1, \ldots, \hat{r}(t)$ while the matrix $\mathbf{K}_{\rho_\ell}(t+1)$ corresponds to the Kalman gain given as

$$\mathbf{K}_{\rho_\ell}(t+1) = \mathbf{M}_{\rho_\ell}(t+1|t+1) \cdot \mathbf{D}_{\nabla,\rho_\ell} \cdot \sigma_w^2 \mathbf{I}_{|\mathcal{S}_{\rho_\ell,t}|} \quad (10)$$

where $\hat{\mathbf{D}}_{\mathcal{S}_{\rho_\ell,t}}$ is a $|\mathcal{S}_{\rho_\ell,t}| \times 1$ vector whose entries are given by $\{||\mathbf{p}_j(t) - \hat{\mathbf{p}}_{\rho_\ell}(t+1|t)||^{-2}\}_{j \in \mathcal{S}_{\rho_\ell,t}}$, in which $\hat{\mathbf{p}}_{\rho_\ell}(t+1|t)$ is the $\rho_\ell$-th target position extracted from the state prediction $\hat{\mathbf{s}}_{\rho_\ell}(t+1|t)$, and $\mathbf{D}_{\nabla,\rho_\ell}$ is the $|\mathcal{S}_{\rho_\ell,t}| \times 4$ matrix whose rows constitute of the gradient $\nabla\mathbf{D}_t(j,\rho_\ell)$ with respect to the state vector $\mathbf{s}_{\rho_\ell}$ and evaluated at $\hat{\mathbf{s}}_{\rho_\ell}(t+1|t)$ for $j \in \mathcal{S}_{\rho_\ell,t}$. Within each informative subset of sensors $\mathcal{S}_{\rho_\ell,t}$, the sensor closest in distance to the predicted position of the $\rho_\ell$-th target, namely $\hat{\mathbf{s}}_{\rho_\ell}(t+1|t)$, is set as the subset head sensor that will gather the measurements of all other sensors in $\mathcal{S}_{\rho_\ell,t}$ and perform the EKF tracking recursions.

### 3.3. Informative Sensors' Kinematic Strategy

Kinematic rules are derived here for the informative sensors selected in Sec. 3.1 in order to follow closely the moving targets and give accurate position estimates. Having a few sensors moving allows tracking of the targets even when they move away from the original field being monitored by the sensors. Toward this end, the informative sensors in each subset $\mathcal{S}_{\rho_\ell}$ will be placed/move in locations that minimize the trace of the predicted error covariance associated with the estimator $\hat{\mathbf{s}}_{\rho_\ell}(t+1|t)$. This will ensure that the informative sensors associated with each target move to a location that will provide measurements that result good tracking accuracy. Existing kinematic strategies require all sensors to move [8, 15, 17]. In contrast, here kinematic strategies are derived in the presence of multiple sensors, while a judiciously selected small percentage of informative sensors will be moving potentially reducing resource consumption.

In detail, the position of sensor $j \in \mathcal{S}_{\rho_\ell,t}$ at time instant $t+1$ is determined by minimizing the covariance in (8) which results i.e., the new location $\mathbf{p}_j(t+1)$ is found as

$$\arg\min \frac{4}{([\mathbf{p}_{j,x} - \hat{\mathbf{p}}_{\rho_\ell,x}(t+1|t)]^2 + [\mathbf{p}_{j,y} - \hat{\mathbf{p}}_{\rho_\ell,y}(t+1|t)]^2)^3}$$
$$\text{s. to } \|\mathbf{p}_j - \hat{\mathbf{p}}_{\rho_\ell}(t+1|t)\|_2^2 < R_j^2 \tag{11}$$

where minimization is performed with respect to (wrt) $\mathbf{p}_j := [\mathbf{p}_{j,x}, \mathbf{p}_{j,y}]^T$. Note that the inequality constraint in (11) ensures that the new location of the moving sensors $j \in \mathcal{S}_{\rho_\ell}$ will be within distance $R_j$ from the latest target location estimate $\hat{\mathbf{p}}_{\rho_\ell}(t+1|t)$. This inequality further ensures that all sensors in $\mathcal{S}_{\rho_\ell}$ i) will move to new locations which are 'close' to the target; and ii) will be within distance $\sqrt{2}R_j$ from each other which can ensure connectivity as long as the communication range is sufficiently high.

The modified barrier method (MBM) is utilized [2, pg. 423] to allow every sensor $j \in \mathcal{S}_{\rho_\ell}$ to solve (11) and determine its next location. To this end, let $f(\mathbf{p}_j)$ denote the cost in (11) and $g(\mathbf{p}_j)$ denote the left hand side function of the inequality constraint in (11). MBM involves an iterative application of the following unconstrained minimization problem (where $\kappa$ denotes the iteration index within time instant $t+1$):

$$\mathbf{p}_j^\kappa(t+1) = \arg\min_{\mathbf{p}_j}\{f(\mathbf{p}_j) + \mu^\kappa(c^\kappa)^{-1}\phi[c^\kappa g(\mathbf{p}_j)]\}, \tag{12}$$

where the barrier function $\phi[\tau]$ is chosen as $\phi(\tau) = -ln(1-\tau)$ and the Lagrange multiplier-like scalar $\mu^\kappa$ is updated as

$$\mu^{\kappa+1} = \frac{\mu^\kappa}{1 - c^\kappa g(\mathbf{p}_j^\kappa(t+1))}, \tag{13}$$

while $c^\kappa$ is a penalty parameter associated with the inequality constraint in (11) that is updated according to the recursion $c^\kappa = \frac{\omega^\kappa}{\mu^\kappa}$, where $\{\omega^\kappa\}$ is a positive monotonically increasing scalar sequence [2].

Further, letting $F(\mathbf{p}_j) := f(\mathbf{p}_j) + \mu^\kappa(c^\kappa)^{-1}\phi[c^\kappa g(\mathbf{p}_j)]$ the coordinates of the new sensor location $\mathbf{p}_j(t+1)$ are updated during MBM iteration $\kappa+1$ according to the following gradient descent way

$$\mathbf{p}_j^{\kappa+1}(t+1) = \mathbf{p}_j^\kappa(t+1) - \Gamma \cdot \nabla_{\mathbf{p}_j} F(\mathbf{p}_j)\Big|_{\mathbf{p}_j^\kappa(t+1)} \tag{14}$$

where $\Gamma$ is a step-size and $\nabla_{\mathbf{p}_j} F(\mathbf{p}_j)\Big|_{\mathbf{p}_j^\kappa(t+1)}$ denotes the gradient of $F(\mathbf{p}_j)$ wrt $\mathbf{p}_j$ and evaluated at point $\mathbf{p}_j^\kappa(t+1)$. During time instant $t+1$ sensors $j \in \mathcal{S}_{\rho_\ell,t}$ will keep updating their location until the difference between the cost function in (11) evaluated at two consecutive updating steps $\kappa, \kappa+1$ drops below a predefined threshold $\epsilon$. Each sensor in $j \in \mathcal{S}_{\rho_\ell,t}$, determines its new location using the MBM scheme. Sensors in $\mathcal{S}_{\rho_\ell,t}$ check their distances and if they are located too close they subtly adjust their coordinates to avoid collision when moving. Sensor $j \in \mathcal{S}_{\rho_\ell,t}$ updates its location $\mathbf{p}_j(t+1)$ while the remaining sensors in $\mathcal{S}_{\rho_\ell,t}$ are stationary and are waiting for their turn in a coordinate fashion.

### 3.4. Algorithmic Summary

During the start-up stage, fast sampling is used to acquire $Q$ measurements which allows to assume that the initial number of targets $r(0)$ are essentially immobile. By utilizing the $Q$ acquired data, the subsets of target-informative sensors $\{\mathcal{S}_{\rho_\ell,0}\}$ are initialized, where $\ell = 1, \ldots, \hat{r}(0)$ and $\hat{r}(0)$ is the estimated number of $r(0)$ sensed targets at time $t = 0$. One sensor within each $\mathcal{S}_{\rho_\ell,0}$ will be randomly selected as the head sensor, which will collect the measurements $x_j(0)$ and their positions $\mathbf{p}_j(0)$ from all the other sensors $j \in \mathcal{S}_{\rho_\ell,0}$. Each head sensor $C_{\rho_\ell,0}$ averages the positions of the informative sensors in subset $\mathcal{S}_{\rho_\ell,0}$ to be the initial estimate of the corresponding target $\rho_\ell$ which along with the measurements $x_j(0)$, for $j \in \mathcal{S}_{\rho_\ell,0}$ are utilized to initialize the recursions of the extended Kalman filtering carrying out the target tracking in Sec. 3.2.

At time $t$, every head sensor $C_{\rho_\ell,t}$ has available the state estimate for target $\rho_\ell$, namely $\hat{\mathbf{s}}_{\rho_\ell}(t|t)$, obtained via the EKF recursions in Sec. 3.2. Then, a group of 'candidate informative' sensors for target $\rho_\ell$, denoted as $\mathcal{J}_{\rho_\ell,t}$ is formed at $t$. This set is formed by having the head sensor transmit the estimated state $\hat{\mathbf{s}}_{\rho_\ell}(t|t)$ to its single-hop neighboring sensors which will also transmit the same information to their own neighbors. Every sensor $j$ who receives $\hat{\mathbf{s}}_{\rho_\ell}(t|t)$, from a neighboring sensor, subsequently forwards this estimate only to those sensors $j' \in \mathcal{N}_j$ whose current location is within radius $R_s$ from the estimated target location, i.e., $\|\mathbf{p}_{j'}(t) - \hat{\mathbf{p}}_{\rho_\ell}(t|t)\|_2 \leq R_s$. Radius $R_s$ can be selected sufficiently large such that all $\rho_\ell$-target informative sensors to be incorporated in subset $\mathcal{J}_{\rho_\ell,t}$. The subset $\mathcal{J}_{\rho_\ell,t}$ by construction is connected.

Since not all sensors within the candidate subsets $\mathcal{J}_{\rho_\ell,t}$ maybe informative, the scheme in Sec. 3.1 is employed among the sensors in $\mathcal{J}_{\rho_\ell,t}$ to find out the target-informative sensor subset $\mathcal{S}_{\rho_\ell,t+1} \subseteq \mathcal{J}_{\rho_\ell,t}$ for all targets. Rather than running the target-sensor association scheme in Sec. 3.1 in the whole sensor network, it is performed independently in the different sensor subsets $\mathcal{J}_{\rho_\ell,t}$ associated with each target leading to much less computational and communication complexity.

Once the subsets $\mathcal{S}_{\rho_\ell,t+1}$ are determined, the head sensor in each of these subsets is designated as the sensor whose distance is the closest to the estimated position of the corresponding target $\rho_\ell$. The head sensor $C_{\rho_\ell,t+1}$ collects the sensor measurement $x_j(t+1)$ from $j \in \mathcal{S}_{\rho_\ell,t+1}$ to carry out the extended Kalman filtering recursions at time instant $t+1$ as outlined in Sec. 3.2. Then, the procedure in Sec. 3.3 is employed to enable all sensors in $\mathcal{S}_{\rho_\ell,t+1}$ to determine and move to their new locations $\mathbf{p}_j(t+1)$. The head sensor $C_{\rho_\ell,t+1}$ forwards the latest state estimate $\hat{\mathbf{s}}_{\rho_\ell}(t+1|t+1)$ to its single-hop neighbors and repeats the process described earlier to update the subsets of candidate informative sensors $\mathcal{J}_{\rho_\ell,t+1}$.

The kinematic strategy implemented at each sensor in Sec. 3.3 is fully distributed since each sensor requires knowledge only of its location and the estimated target position obtained from the head sensor of $\mathcal{S}_{\rho_\ell,t+1}$. In the same way connectivity of the candidate informative subsets $\mathcal{J}_{\rho_\ell,t+1}$ is ensured by construction irrespective of the sensor movement, allowing the sensor-to-target association scheme in Sec. 3.1 to be applied and determine the informative sensors.

## 4. NUMERICAL TESTS

The tracking performance of the novel scheme is tested in a network of $p = 80$ sensors, which are deployed randomly in the region of $[0, 15] \times [0, 15]m^2$ field. The radius $R_s$ to determine the candidate informative subset $\mathcal{J}_{\rho_\ell,t+1}$ is set to be $R_s = 5$. The threshold $R_j$ which controls the distance between the estimated targets' location and the mobile sensors is set as $R_j = 3$. The state noise variance is set as $\sigma_u^2 = 0.08$, while the observation noise variance is selected as $\sigma_w^2 = 0.08$ and corresponds to a sensing SNR of 11dB.

The tracking root-mean square error (RMSE) achieved by our novel mobile-sensor based tracking scheme is compared with the one attained by existing related tracking schemes that also employ mobile sensors [17] and [13]. The comparison will be done using one target since the aforementioned existing approaches can handle one target. The target is initialized at location $[5, 7]$ and set to be moving with velocities of $0.4$ and $0.13]m/s$ along the $x$ and $y$ axis, respectively. The tracking process is carried out for a total duration of $30s$. Fig. 1 depicts the logarithm of the tracking RMSE (for better display) of i) the novel approach proposed here; ii) the tracking scheme in [17]; and iii) the tracking approach in [13]. As depicted in Fig. 1, our tracking scheme exhibits the lowest tracking RMSE. The approach in [17] attains the worst performance since the sensors can only move on a grid which reduces accuracy. The scheme in [13] performs worse than our approach since it does not have an informative sensor selection scheme, and as a result all sensors have to move and participate in tracking which may reduce accuracy especially when noisy sensors are used in the tracking process.

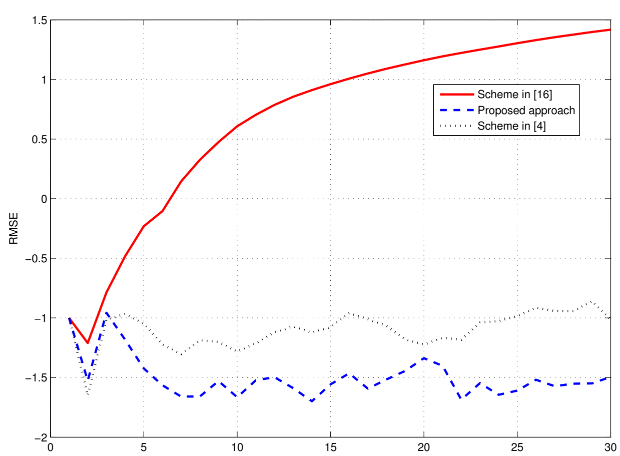


**Fig. 1**. RMSE versus time when tracking a single target.

The proposed tracking scheme is tested next in a setting with two targets where one of targets splits into two targets at a certain time. The estimated targets' trajectories as well as the moving sensors trajectories are studied in Fig. 2. The two targets $\rho = 1, 2$ start moving at positions $[1.5, 11.5]$, $[5, 7]$ (denoted by the blue stars), and follow the dynamics in (1), with velocities of $[0.15, 0.1]m/s$ and $[0.4, 0.13]m/s$ along the $x$-axis and $y$-axis respectively. After $30s$, the first target stops moving while the second targets splits into two separate objects denoted as targets $\rho = 3, 4$. Target $\rho = 3$ continues to move according to the dynamics of target $\rho = 2$, while target $\rho = 4$ moves with velocities $v_x = -0.4m/s$ and $v_y = 0.8m/s$ along the x-axis and y-axis. The two new targets move during the time interval $[31, 40]s$, while the splitting point is indicated by the green star in Fig. 2. The target trajectories in Fig. 2 are depicted by blue dashed lines for the time interval $[1, 30]s$, and by blue crossed lines after $t = 30s$). The estimated trajectories obtained using our novel tracking scheme are depicted using red lines. The estimated trajectories using immobile sensors are depicted via violet colored lines. The black dashed lines correspond to some of the mobile sensors' trajectories using the kinematic rules in Sec. 3.3, while the red crosses indicate the mobile sensors' starting location. Note that when sensors cannot move (immobile), the violet trajectories indicate that i) at $t = 30$ the split of targets can not be followed; and ii) target $\rho = 1$ cannot be tracked after a while since the target is moving away from the stationary sensors. This is not the case when mobile sensors are employed using the kinematic rules in Sec. 3.3 that enable the moving sensors to follow closely the targets as depicted by the black dashed sensor trajectories. Thus, the corresponding blue estimated trajectories provide accurate trajectories estimates for the multiple targets present in the field.

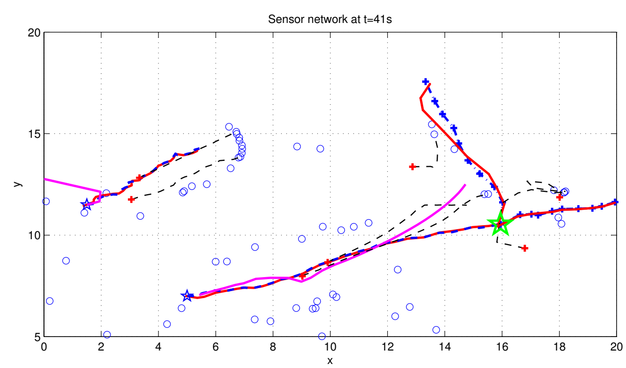


**Fig. 2**. Tracking multiple objects using mobile sensors.

## 5. CONCLUSIONS

A novel framework combining sparse decompositions with proper kinematic rules allow mobile sensors to track multiple targets. After associating sensors to targets via a sparsity-based minimization framework, optimal kinematic rules are obtained by minimizing the covariances of parallel extended Kalman filters that track multiple targets using only informative sensors. Numerical tests in multi-sensor networks, corroborate that our novel scheme outperforms related approaches and accurately tracks multiple targets having a few sensors closely following them.

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