A NOVEL FEEDFORWARD NOISE SHAPING FOR WORD-LENGTH REDUCTION

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ABSTRACT

In this paper, we present a novel approach to shape the quantization noise during word-length reduction. In comparison to the traditional feedback noise shaping, our approach is feedforward and thus inherently stable. It can achieve one or multiple frequency notches in the quantization noise spectrum with controlled notch width and notch depth while keeping the out-of-band noise level lower than the feedback noise shaping. Results from a digital communication example demonstrate the performance of this new noise shaping method.

Index Terms— quantization, noise shaping, feedforward noise shaping

1. INTRODUCTION

With the growing shrinkage of digital circuits [1], more and more analog components are being replaced by digital ones. Their size and energy consumption are directly related to the bit resolution (word length) of the digital signal [2]. Reducing the word length will reduce the size and energy consumption, but increase the quantization noise. In order to control the noise level, sigma-delta converters have been widely used [3, 4]. They are oversampled converters because the sampling rate is much higher than the Nyquist frequency. In high-bandwidth applications, it may be impossible to increase the sampling rate further and, thus, inefficient Nyquist converters are used [5]. For a long time, feedback noise shaping has been used in order to move quantization noise from critical frequency bands to others. However, they suffer from limit cycles [6], high out-of-band (out of desired frequency band) noise and difficulty to adapt to different spectrum shaping requirements.

In this paper, we propose a novel noise shaping approach by adding small corrections to the quantized signal in order to achieve one or multiple frequency notches in the noise shaped signal. It results in a feedforward (and thus inherently stable) and adaptable noise shaper with controlled notch width and notch depth while keeping the out-of-band noise level lower than the traditional feedback noise shaper.

One potential application is noise shaping during word-length reduction in the transmitter of a digital communication system. In order to minimize the size and energy consumption of the transmitter, a word-length reduction as much as possible is desired. The resulting high noise level would, however, distort the spectrum of different downlink signals like speech communication, WLAN, Bluetooth, GPS signal etc. In this case, it is highly desirable to have a flexible noise shaper to place multiple frequency notches of varying width and depth in order to mask the quantization noise spectrum and not to distort the downlink signals.

The paper is organized as follows. After a short review of feedback noise shaping in Section 2, we present our feedforward noise shaping in Section 3. Section 4 shows some noise shaping results for a communication example and compares the new approach against the old one.

2. FEEDBACK NOISE SHAPING

Fig. 1 shows a traditional feedback noise shaping during digital word-length reduction. The high-resolution signal $x[n]$ at discrete time n is quantized resulting in the low-resolution signal $y[n]$. The quantization error $e_q[n]$ is extracted from the quantizer Q, filtered by a noise shaping filter $H(\omega)$, and then added back to $x[n]$. Without noise shaping, the quantized signal has an added quantization noise $e_q[n]$ which has typically a flat spectrum [3]. By using the noise shaping in Fig. 1, we obtain

$$Y(\omega) = X(\omega) + (1 + H(\omega))E_q(\omega), \tag{1}$$

where $X(\omega), Y(\omega), E_q(\omega)$ are the Fourier transform of the corresponding time signals $x[n], y[n], e_q[n]$, respectively. $H(\omega)$ is the frequency response of the noise shaping filter. The shaped quantization noise $e[n]$ has thus the Fourier transform

$$E(\omega) = Y(\omega) - X(\omega) = (1 + H(\omega))E_q(\omega). \tag{2}$$

By choosing a suitable filter $H(\omega)$, the flat spectrum of $E_q(\omega)$ can be shaped.

Note that any noise shaping will increase the total noise energy of the shaped signal $y[n]$. A reduction of the noise level inside a desired frequency band will always lead to an increase of the noise level outside the band, the so called out-of-band noise. Thus noise shaping is a controlled nonuniform spread of total noise energy over frequency, see Fig. 3.

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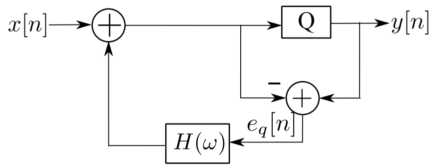

Fig. 1. Traditional feedback noise shaping

3. FEEDFORWARD NOISE SHAPING

3.1. Basic idea

In Fig. 1, we see that the noise shaping is done by adding a correction signal $h[n] * e_q[n]$ to the high-resolution signal $x[n]$. In our approach, we propose to add a correction signal to the low-resolution signal $y[n]$, thus avoiding the feedback.

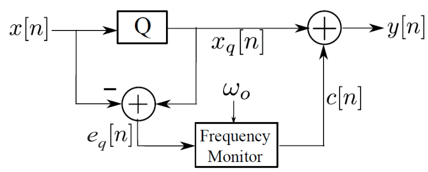

Fig. 2. Principle of feedforward noise shaping

Fig. 2 shows the basic idea of our feedforward noise shaping. $x[n]$ is the original high-resolution signal. $x_q[n]$ is the quantized low-resolution signal. $e_q[n] = x_q[n]-x[n]$ is the unshaped quantization error with the least significant bit (LSB) q_0. Based on $e_q[n]$ and the notch frequency ω_0 where we need to place a notch in the spectrum of $e_q[n]$, we calculate a suitable correction signal $c[n]$ and add it to $x_q[n]$ resulting in

$$y[n] = x_q[n] + c[n] = x[n] + e_q[n] + c[n] = x[n] + e[n]. \quad (3)$$

The effective quantization noise after noise shaping is

$$e[n] = e_q[n] + c[n]. \quad (4)$$

Clearly, in order to guarantee that $y[n]$ has the low-resolution of LSB q_0 as well, the correction signal $c[n]$ must be an integer multiple of q_0:

$$c[n] = i \cdot q_0, \quad i \in \mathbb{Z}. \quad (5)$$

$c[n]$ is then chosen such to minimize the magnitude of the Fourier transform $E(\omega)$ of $e[n]$ at $\omega = \omega_0$ subject to Eq. (5):

$$\min_{c(n)=iq_0, i\in\mathbb{Z}} |E(\omega_0)|. \quad (6)$$

3.2. Calculation of the correction signal

In order to calculate the correction signal $c[n]$, the frequency monitoring block in Fig. 2 should be able to update $E(\omega_0)$ on a sample-by-sample basis. For this purpose, $E(\omega_0)$ is calculated time recursively

$$E_n(\omega_0) = \sum_{l=-\infty}^{n} e[l]e^{-j\omega_0 l} = E_{n-1}(\omega_0)+(e_q[n]+c[n])e^{-j\omega_0 n} \quad (7)$$

where $E_n(\omega)$ is the Fourier transform of $e[n]$ calculated from samples until time n only. Given $\omega_0, e_q[n], E_{n-1}(\omega_0)$,

$$c[n] = -E_{n-1}(\omega_0)e^{j\omega_0 n} - e_q[n] \quad (8)$$

would eliminate $E_n(\omega_0)$ completely. Considering the constraint (5), the best choice of $c[n]$ is

$$c[n] = Q_{q_0}(-E_{n-1}(\omega_0)e^{j\omega_0 n} - e_q[n]) \quad (9)$$

where the quantizer $Q_{q_0}(z)$ maps a complex value z to its nearest neighbor subject to (5). Putting this non-perfect correction signal $c[n]$ back into Eq. (7) results in a non-zero residual value $E_n(\omega_0)$ which is then used in Eq. (9) to calculate $c[n+1]$.

3.3. Minimization of out-of-band noise

In order to place a frequency notch while minimizing the out-of-band noise at the same time, the total energy of the additive correction signal $c[n]$ should be as small as possible. For this purpose, we impose the more restrictive constraint

$$c[n] \in \{-q_0, 0, q_0\} \quad (10)$$

instead of Eq. (5).

Up to now, we have only presented the basic feedforward noise shaping algorithm which is able to place only one notch. Below we extend this basic idea to a more general and adaptable feedforward noise shaper with a controlled notch width, notch depth and even multiple controlled notches.

3.4. Control of notch width

The received downlink signal can have different bandwidths. For example, WLAN has a larger bandwidth than Bluetooth. So it would be useful to have an easy method to control the width of the notch.

In the traditional feedback noise shaping, the notch width is fixed for a given noise shaping filter $H(\omega)$. The only way to change the notch width is to use a different noise shaping filter, say a notch filter of 4th order instead of 2nd order. In our case, the notch width can be controlled easily by changing the frequency of corrections, i.e. how often $c[n] \neq 0$. We will see later that the less often the corrections, the smaller the notch width. We can control the frequency of corrections by scaling the exact correction value in Eq. (9) by a notch width factor $0 < \beta \leq 1$. The correction signal in Eq. (9) then becomes

$$c[n] = Q_{q_0}(\beta(-E_{n-1}(\omega_0)e^{j\omega_0 n} - e_q[n])). \quad (11)$$

The smaller β is, the smaller the value inside $Q_{q_0}()$ and the larger the probability of $c[n] = 0$. This implies less often corrections and a narrower notch. Tuning the parameter β to control the notch width without changing the circuit of feedforward noise shaper is thus much easier than calculating a new filter for the feedback noise shaper.

3.5. Control of notch depth

The signal-to-noise ratio (SNR) of the downlink signal is known to vary in time and space for different communication protocols. Sometimes the downlink signal at ω_0 has a low SNR. Then we need to cancel the frequency component $E(\omega_0)$ as much as possible, which in turn increases the out-of-band noise. In other situations, the SNR of the downlink signal is high and the notch does not need to be so deep. This has the advantage of a smaller out-of-band noise. Controlling the depth of the notch is thus a useful instrument to control the out-of-band noise and to achieve a desired compromise "deep notch vs. low out-of-band noise".

In the traditional feedback noise shaping, a control of the notch depth is difficult because it requires different noise shaping filters $H(\omega)$. In our proposed approach, we can control the notch depth by simply tuning a second parameter, the notch depth factor $0 < \lambda \leq 1$. This factor is used in the recursive calculation of $E_n(\omega_0)$

$$E_n(\omega_0) = \lambda E_{n-1}(\omega_0) + (e_q[n] + c[n])e^{-j\omega_0 n} \tag{12}$$

instead of Eq. (7). By choosing $0 < \lambda < 1$, the correction value $c[n]$ cancels only a portion of the frequency component at ω_0. Therefore, a larger residual frequency component remains which corresponds to a less depth notch.

3.6. Multiple notches

In some applications, it is required to have more than one notch in the noise shaped signal. For example, in mobile communication systems, it is usually required to have a notch at the data receiving frequency of the transceiver and additional notches for WLAN, GPS signals etc.

In feedback noise shaping, a new higher-order multi-notch noise shaping filter $H(\omega)$ has to be designed and used. In our case, we can easily extend our basic noise shaping algorithm for one notch to multiple notches by tracking the Fourier transform $E(\omega)$ of the shaped noise $e[n]$ at multiple frequencies ω_i:

$$E_n(\omega_i) = \lambda_i E_{n-1}(\omega_i) + (e_q[n] + c[n])e^{-j\omega_i n}. \tag{13}$$

A choice of $c[n]$ to minimize $|E_n(\omega_i)|$ for all ω_i subject to (10) is difficult. We propose to choose $c[n]$ such to reduce the largest frequency component $|E_n(\omega_i)|$ among all ω_i:

$$k = \arg\max_i \left|\beta_i(-E_{n-1}(\omega_i)e^{j\omega_i n} - e_q[n])\right|, \tag{14}$$

$$c[n] = Q_{q_0}(\beta_k(-E_{n-1}(\omega_k)e^{j\omega_k n} - e_q[n])). \tag{15}$$

k is the index of that frequency component with the largest amplitude to be canceled. Note that in Eq. (13) to (15), different notch width factors β_i and different notch depth factors λ_i are used for different notch frequencies ω_i in order to achieve an individual notch width and depth for different downlink signals.

The complete feedforward noise shaper consists of a time-recursive calculation of Eq. (13) to (15). Its behavior is controlled by the set of notch frequencies $\{\omega_i\}$, the set of notch width and depth parameters $\{\beta_i, \lambda_i\}$, and the amount of word-length reduction Q.

4. SIMULATIONS AND RESULTS

In order to compare different noise shaping algorithms, we need a metric to measure the overall increase of noise energy. Below we use the signal-to-quantization-noise-ratio (SQNR) γ for comparison:

$$\gamma\ [\text{dB}] = 10\log_{10}\left(\frac{\sum_n (y[n])^2}{\sum_n (y[n] - x[n])^2}\right). \tag{16}$$

We distinguish between two different values:

- γ_b: SQNR with feedback noise shaping
- γ_f: SQNR with feedfoward noise shaping

The larger γ, the lower the overall increase of noise energy.

For the sake of comparison, a second order FIR(2) notch filter is used for feedback noise shaping:

$$1 + H(\omega) = 1 - 2\cos(\omega_0)e^{-j\omega} + e^{-2j\omega}. \tag{17}$$

It can be shown to have the lowest possible noise energy among all FIR noise shaping filters. Theoretically, it is also possible to use an IIR notch filter $H(\omega)$ in the feedback loop. However, such a feedback filter in a feedback loop is rarely used in practice due to stability reasons.

In the simulation, 1000 complex-valued 64-QAM baseband symbols are generated. They are filtered using a raised cosine pulse shaping filter to achieve a normalized baseband bandwidth of 0.08. We used an oversampling factor of 32 resulting in 32000 samples $x[n]$. Then we truncate the word length of $x[n]$ from 10 bits (high resolution) to 5 bits (low resolution).

Fig. 3 shows the power spectrum density (PSD) of the high-resolution signal $x[n]$ (blue), the low-resolution signal $Q(x[n])$ without noise shaping (green), the feedback noise-shaped signal (cyan), and the feedforward noise-shaped signal (red) for a normalized notch frequency $f_0 = \omega_0/(2\pi) = 0.15$. We see that for high frequency components, the PSD of the feedforward approach has a 4dB lower noise level than the feedback approach. This is also verified by $\gamma_f = 18.36, \gamma_b = 16.59$.

Fig. 4 compares γ_b and γ_f for a varying notch frequency $0 \leq f_0 \leq 0.5$. Obviously, the feedforward noise shaping is able to maintain a more or less constant γ_f for all notch frequencies, while the feedback noise shaping has a varying γ_b which is always smaller than γ_f.

Fig. 5 shows the PSD for two different notches at $f_0 = 0.1, f_1 = 0.15$. In this case, a 4th order FIR noise shaping filter $H(\omega)$ consisting of two cascaded sections of the type (17) is used to generate two notches at f_0, f_1. We see that the feedforward approach achieves two narrow notches as desired while the feedback approach also attenuates the frequency band between the two notches significantly. The values $\gamma_f = 15.559, \gamma_b = 8.21$ indicate a considerably larger noise energy in the feedback approach.

Fig. 6 shows the PSD for two different notches at $f_0 = 0.1, f_1 = 0.15$ with two different notch widths $\beta_0 = 0.8, \beta_1 = 0.2$. As predicted, a smaller value of β_1 leads to less corrections and a narrower notch. The SQNR values are $\gamma_f = 15.95, \gamma_b = 8.21$.

Fig. 7 shows the PSD for one notch at $f_0 = 0.3$ with a varying notch width by choosing different values $1, 0.6, 0.4$ for β. As β decreases, the notch becomes narrower while remaining its depth. As a result of the reduced notch width, the out-of-band noise is reduced. The corresponding SQNR values are $\gamma_f = 19.93, 21.53, 22.16$.

Finally, Fig. 8 shows the PSD for one notch at $f_0 = 0.3$ with a varying notch depth by choosing different values $1, 0.95, 0.87, 0.77$ for λ. As λ decreases, the notch becomes more shallow while remaining its width. This also leads to a reduction of the out-of-band noise. The corresponding SQNR values are $\gamma_f = 19.29, 19.88, 20.71, 22.75$.

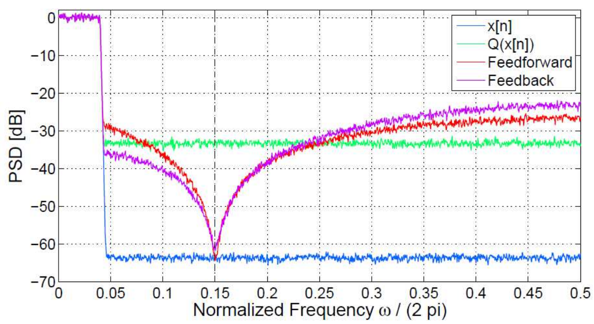

Fig. 3. Feedforward vs. feedback noise shaping for one notch

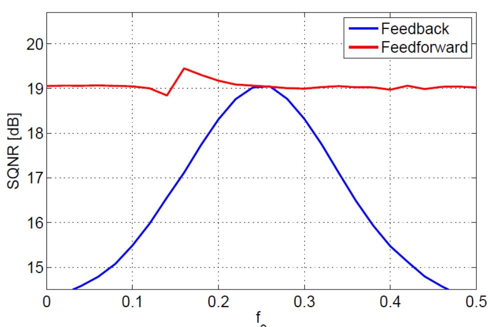

Fig. 4. Feedforward vs. feedback noise shaping for a varying notch frequency f_0

5. CONCLUSION

In this paper, we have presented a novel noise shaping approach. It has a feedforward structure and is inherently stable. It is simple to implement. It is more flexible than the feedback noise shaping because the same feedforward noise shaper can achieve multiple notches with individually controlled notch width and depth. In addition, it has a lower out-of-band noise than the traditional feedback noise shaping.

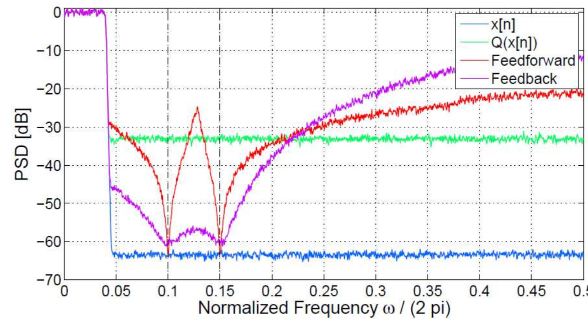

Fig. 5. Feedforward vs. feedback noise shaping for two notches

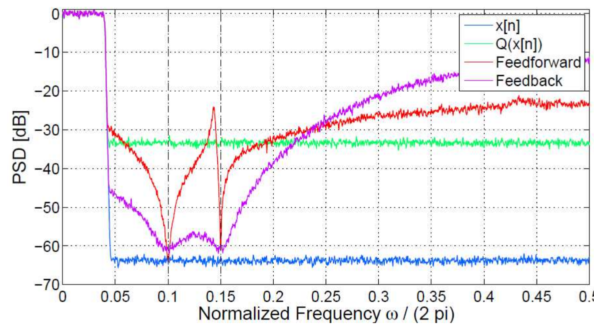

Fig. 6. Feedforward vs. feedback noise shaping for two notches of different widths

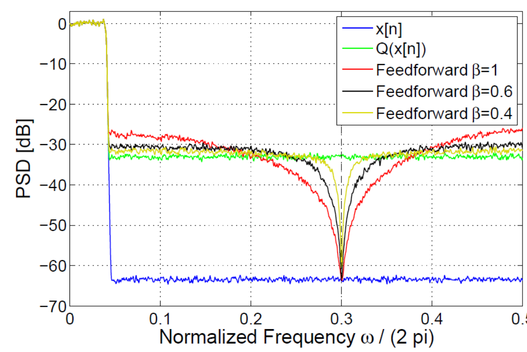

Fig. 7. Feedforward noise shaping with different notch widths

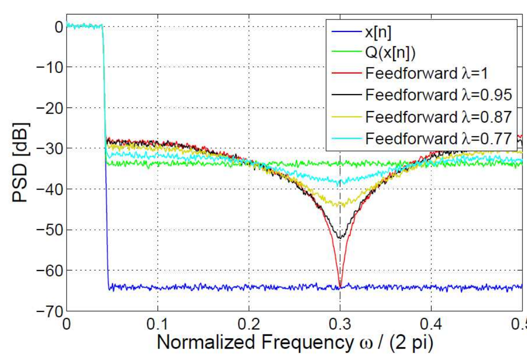

Fig. 8. Feedforward noise shaping with different notch depths

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