DETECTION OF CYCLOSTATIONARITY IN THE PRESENCE OF TEMPORAL OR SPATIAL STRUCTURE WITH APPLICATIONS TO COGNITIVE RADIO

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ABSTRACT

One approach to spectrum sensing for cognitive radio is the detection of cyclostationarity. We extend an existing multi-antenna detector for cyclostationarity proposed by Ramírez et al. [1], which makes no assumptions about the noise beyond being (temporally) wide-sense stationary. In special cases, the noise could be uncorrelated among antennas, or it could be temporally white. The performance of a general detector can be improved by making use of a priori structural information. We do not, however, require knowledge of the exact *values* of the temporal or spatial noise covariances. We develop an asymptotic generalized likelihood ratio test and evaluate the performance by simulations.

Index Terms— cognitive radio, cyclostationarity, detection, GLRT, spectrum sensing

1. INTRODUCTION

Cognitive radio is a promising technology for improving the usage of radio frequencies. It is already part of specifications such as IEEE 802.22 (WRAN) and may become more popular in the future. Here, we are concerned with a special type, called *interweave* cognitive radio, where a frequency band may be used when no primary user is occupying it. This reduces the amount of *white spaces* in the spectrum, where a licensed user is temporally or geographically not utilizing its right to use a band. Studies have shown that this indeed happens [2]. Therefore, a key component of interweave cognitive radio is the ability to perform *spectrum sensing*, which determines which frequency bands to use.

For that purpose, numerous detectors exist, as listed in reviews, e.g. [3]. Generally speaking, this is a *signal detection* task. For the use in cognitive radio, the challenges are a low SNR, fading channels, and limited knowledge of signal parameters, for example the noise level. One popular approach is to make use of the fact that digital communication signals are *cyclostationary* [4]. In this approach, the goal becomes the detection of a primary user by detecting cyclostationarity in the observed signal.

The detection of cyclostationary signals has been the subject of research since the early 90s. An asymptotic detector was proposed by Dandawaté [5] and another detector was developed by Enserink [6]. Based on these detectors, there have been some detectors that simultaneously evaluate different cycle frequencies [7] or extend the ideas to multiple antennas [8]. In contrast to most previous detectors, the detectors proposed in [1, 9, 10] are based on established statistical techniques such as the generalized likelihood ratio test (GLRT) and the locally most powerful invariant test. Even though they require the assumption of Gaussian input data, they perform well even for non-Gaussian communication signals. These tests are about the *structure* of the signal's covariance matrix, and knowledge of the matrix elements is actually not required.

This paper aims to extend the idea of [1, 9] for different assumptions about the noise. In [1, 9], the noise is assumed wide-sense stationary (WSS) with arbitrary spatio-temporal correlation. Spatial correlation in this context is the correlation between the signals at different antennas. Arbitrary spatial correlation and wide-sense stationarity are very general. In many cases, more specific information is available. Here we consider temporally white noise, spatially uncorrelated noise, or a combination thereof. Spatially uncorrelated noise is a good model if the ambient noise is low compared to the noise at the receivers. Since we do not require calibration, we allow for different noise levels at the antennas. We propose detectors based on the GLRT that have knowledge of the temporal or spatial structure of the noise.

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2. PROBLEM FORMULATION

The multiantenna cognitive monitor observes NP samples of the zero-mean discrete-time signal $\mathbf{x}[n] \in \mathbb{C}^L$, which is cyclostationary with known cycle period P, and L is the number of receiver antennas. In our model, we assume that in the absence of a primary user we observe only noise, which is WSS. The noise could be temporally *white* or *colored*, and spatially it could be either *correlated* or *uncorrelated* among the antennas. The temporal and spatial structure can occur in any combination. The general temporally colored and spatially correlated case was covered by [1]. The remaining three cases are

(I) temporally colored and spatially uncorrelated,

(II) temporally white and spatially correlated,

(III) temporally white and spatially uncorrelated.

For each of these cases, we are interested in the binary hypothesis test

$$\begin{aligned} &\mathcal{H}_1 : \mathbf{x}[n] \text{ is cylostationary with cycle period } P,\\ &\mathcal{H}_0 : \mathbf{x}[n] \text{ is WSS with temporal and/or spatial structure.} \end{aligned} \tag{1}$$

Assuming that $\mathbf{x}[n]$ is a proper complex normal vector, both hypotheses result in a special structure of the covariance matrix. Following the idea of [1], we stack the NP observations as

$$\mathbf{y} = \left[\mathbf{x}^T[0] \dots \mathbf{x}^T[NP-1]\right]^T \tag{2}$$

to obtain the $NPL \times 1$ vector $\mathbf{y}$. To find the covariance matrix of $\mathbf{y}$ under $\mathcal{H}_1$, we define the process

$$\mathbf{x}_P[n] = \left[\mathbf{x}^T[nP] \dots \mathbf{x}^T[(n+1)P-1]\right]^T$$

to represent $\mathbf{y}$ as

$$\mathbf{y} = \left[\mathbf{x}_P^T[0] \dots \mathbf{x}_P^T[N-1]\right]^T.$$

Since $\mathbf{x}_P[n]$ is WSS [11], the autocorrelation function only depends on the lag:

$$\mathrm{E}_{\mathcal{H}_1}\left[\mathbf{x}_P[n]\mathbf{x}_P^H[n-k]\right] = \mathbf{Q}[k] \in \mathbb{C}^{LP \times LP}$$

This results in a block-Toeplitz structure of the covariance matrix $\mathbf{R}_1 = \mathrm{E}_{\mathcal{H}_1}\left[\mathbf{y}\mathbf{y}^H\right]$, with a block size of $LP \times LP$ [1]:

$$\mathbf{R}_1 = \begin{bmatrix} \mathbf{Q}[0] & \dots & \mathbf{Q}[-N+1] \\ \vdots & \ddots & \vdots \\ \mathbf{Q}[N-1] & \dots & \mathbf{Q}[0] \end{bmatrix}$$

The same procedure is followed to find the covariance matrix $\mathbf{R}_0$ under $\mathcal{H}_0$: Find the autocorrelation function and fill $\mathbf{R}_0$ with it. Since we do this for different spatial or temporal structures, this will be performed in the following subsections. Then we obtain the following hypothesis test, which is equivalent to the one in (1):

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_1)\\ \mathcal{H}_0 &: \mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_0) \end{aligned} \tag{3}$$

Since the exact values of the covariance matrices are unknown, this hypothesis test is about the *structure* of the covariance matrix.

The problem with this test is that we need a maximum likelihood (ML) estimate to obtain the generalized likelihood ratio, but there is no closed-form ML estimate of a block-Toeplitz matrix [12]. Therefore, we approximate the block-Toeplitz matrices by block-circulant matrices in the frequency domain [1, 9], and the observations in $\mathbf{y}$ are transformed to $\mathbf{z}$ [10]:

$$\mathbf{z} = (\mathbf{L}_{NP,N} \otimes \mathbf{I}_L)(\mathbf{F}_{NP} \otimes \mathbf{I}_L)^H \mathbf{y}, \tag{4}$$

where $\mathbf{F}_{NP}$ is a Fourier matrix and $\mathbf{L}_{NP,N}$ is the commutation matrix [13]. This approximation and the transformation are designed such that the covariance matrix of $\mathbf{z}$ becomes block-*diagonal* under both hypotheses [1]. This yields the following hypothesis test, which is asymptotically ($N \to \infty$) equivalent to (3):

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_1)\\ \mathcal{H}_0 &: \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_0) \end{aligned} \tag{5}$$

Here, $\mathbf{S}_1$ is block-diagonal with positive definite blocks of size $LP \times LP$ and $\mathbf{S}_0$ is a block-diagonal matrix with blocks of size $L \times L$. These blocks depend on the temporal and spatial structure and will be derived in the following.

2.1. $\mathbf{S}_0$ for spatially uncorrelated noise

For the case (I), the autocorrelation is a function of the lag k:

$$\mathrm{E}_{\mathcal{H}_0}\left[\mathbf{x}[n]\mathbf{x}^H[n-k]\right] = \mathbf{M}[k] \in \mathbb{C}^{L \times L} \tag{6}$$

Since we consider the spatially uncorrelated case, the matrix-valued function $\mathbf{M}[k]$ is diagonal, but with unknown diagonal elements. Then the covariance matrix for the vector $\mathbf{y}$ as defined in (2) becomes block-Toeplitz (with block size $L \times L$) with *diagonal* blocks:

$$\mathbf{R}_0 = \begin{bmatrix} \mathbf{M}[0] & \dots & \mathbf{M}[-NP+1] \\ \vdots & \ddots & \vdots \\ \mathbf{M}[NP-1] & \dots & \mathbf{M}[0] \end{bmatrix}$$

In the frequency domain, this leads to a diagonal matrix $\mathbf{S}_0$, where we know only that the diagonal elements are positive.

2.2. $\mathbf{S}_0$ for white noise

Assuming white noise under $\mathcal{H}_0$, the autocorrelation function of $\mathbf{x}[n]$ is

$$\mathrm{E}_{\mathcal{H}_0}\big[\mathbf{x}[n]\mathbf{x}^H[n-k]\big] = \mathbf{M}_0\delta[k] \in \mathbb{C}^{L\times L}$$

with an unknown $\mathbf{M}_0$. This results in the covariance matrix $\mathbf{R}_0 = \mathrm{E}_{\mathcal{H}_0}\big[\mathbf{y}\mathbf{y}^H\big] = \mathbf{I}_{NP} \otimes \mathbf{M}_0$, which is block-diagonal with *identical* blocks $\mathbf{M}_0$. If we assume spatially correlated noise (case II), this matrix has no further structure beyond being positive definite. For case (III), $\mathbf{M}_0$ is a diagonal matrix with positive elements. Besides this structure, the exact correlations in $\mathbf{M}_0$ are unknown.

In the frequency domain this results in a block-diagonal matrix $\mathbf{S}_0 = \mathbf{I}_{NP} \otimes \tilde{\mathbf{S}}_0$, with identical blocks $\tilde{\mathbf{S}}_0$ of size $L \times L$. The blocks $\tilde{\mathbf{S}}_0$ are positive definite in any case, but only in case (III) are they diagonal.

3. DERIVATION OF THE GLRT

Since $\mathbf{S}_1$ and $\mathbf{S}_0$ are block-diagonal in all cases, we can obtain closed-form ML estimates for them and proceed with the derivation of the GLRT. We assume to have M independent and identically distributed (i.i.d.) realizations of $\mathbf{z}$, and we would like to decide whether or not the signal is cyclostationary. We cannot evaluate the likelihood ratio, as the covariance matrices are unknown. Instead, we perform a GLRT where the unknown covariance matrices are replaced by their ML estimates. To find the GLRT, we need the generalized likelihood ratio (GLR)

$$\mathscr{L} = \frac{\max\limits_{\mathbf{S}_0}\ p(\mathbf{z}_1,\dots,\mathbf{z}_M;\mathcal{H}_0)}{\max\limits_{\mathbf{S}_1}\ p(\mathbf{z}_1,\dots,\mathbf{z}_M;\mathcal{H}_1)},$$

and for this we need the ML estimates of the covariance matrices with the different structures. With the sample covariance matrix of $\mathbf{z}$,

$$\hat{\mathbf{S}} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{z}_m\mathbf{z}_m^H,$$

the estimator of $\mathbf{S}_1$ is the one from [1]:

$$\hat{\mathbf{S}}_1 = \operatorname{diag}_{LP}(\hat{\mathbf{S}})$$

The operation $\operatorname{diag}_{LP}(\hat{\mathbf{S}})$ returns the diagonal blocks of size $LP \times LP$ from $\hat{\mathbf{S}}$ and sets the off-diagonal blocks to zero.

For $\mathbf{S}_0$, we have different estimators depending on the structure: In general, the likelihood function is

$$\begin{aligned} p(\mathbf{z}_1,\dots,\mathbf{z}_M) &= \pi^{-LMNP}(\det\mathbf{S}_0)^{-M} \\ &\quad\times\exp\left\{-M\operatorname{tr}\left(\mathbf{S}_0^{-1}\hat{\mathbf{S}}\right)\right\} \\ &= \pi^{-LMNP}\prod_{k=1}^{NP}(\det\mathbf{S}_k)^{-M} \\ &\quad\times\exp\left\{-M\operatorname{tr}\left(\sum_{k=1}^{NP}\mathbf{S}_k^{-1}\hat{\mathbf{S}}_k\right)\right\}, \end{aligned}$$

where we used the kth $L \times L$ diagonal blocks $\hat{\mathbf{S}}_k$ and $\mathbf{S}_k$ of the matrices $\hat{\mathbf{S}}$ and $\mathbf{S}_0$, respectively. For spatially uncorrelated noise (case I), the blocks $\hat{\mathbf{S}}_k$ and $\mathbf{S}_k$ become diagonal. For temporally white noise (cases II and III), the blocks $\mathbf{S}_k = \tilde{\mathbf{S}}_0$ are identical, and we can further simplify the likelihood:

$$\begin{aligned} p(\mathbf{z}_1,\dots,\mathbf{z}_M) &= \pi^{-LMNP}(\det\tilde{\mathbf{S}}_0)^{-MNP} \\ &\quad\times\exp\left\{-M\operatorname{tr}\left(\tilde{\mathbf{S}}_0^{-1}\sum_{k=1}^{NP}\hat{\mathbf{S}}_k\right)\right\} \end{aligned}$$

The ML estimators for all three cases of $\mathbf{S}_0$ can be derived using complex matrix derivatives [14] (see Table 1). Now, we can find the GLR: Using the respective estimate from Table 1, we define the *sample coherence matrix* as

$$\hat{\mathbf{C}} = \hat{\mathbf{S}}_0^{-1/2}\hat{\mathbf{S}}_1\hat{\mathbf{S}}_0^{-1/2},$$

and then the GLR for the hypothesis test (5) can be compactly written as $\mathscr{L} \propto \det(\hat{\mathbf{C}})$. The corresponding test is obtained by comparing the statistic with a threshold η:[1]

$$\mathscr{L} \propto \det(\hat{\mathbf{C}}) \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \eta$$

The sample coherence matrix can be interpreted as the estimate under $\mathcal{H}_1$ normalized by the estimate under $\mathcal{H}_0$. This requires the covariance matrix $\hat{\mathbf{S}}_0$ to be full-rank.

Table 1: Estimate $\hat{\mathbf{S}}_0$ for the covariance matrix of noise with special temporal or spatial structure.

temporally	*spatially*	$\hat{\mathbf{S}}_0$
colored	uncorrelated	$\operatorname{diag}(\hat{\mathbf{S}})$
white	correlated	$\mathbf{I}_{NP} \otimes \left[\frac{1}{NP}\sum_{k=1}^{NP}\hat{\mathbf{S}}_k\right]$
white	uncorrelated	$\mathbf{I}_{NP} \otimes \left[\frac{1}{NP}\sum_{k=1}^{NP}\operatorname{diag}(\hat{\mathbf{S}}_k)\right]$

4. SIMULATION

To demonstrate the performance of the proposed detectors, we ran Monte Carlo simulations. As a benchmark detector

[1] The selection of the threshold η was discussed in detail by [1].

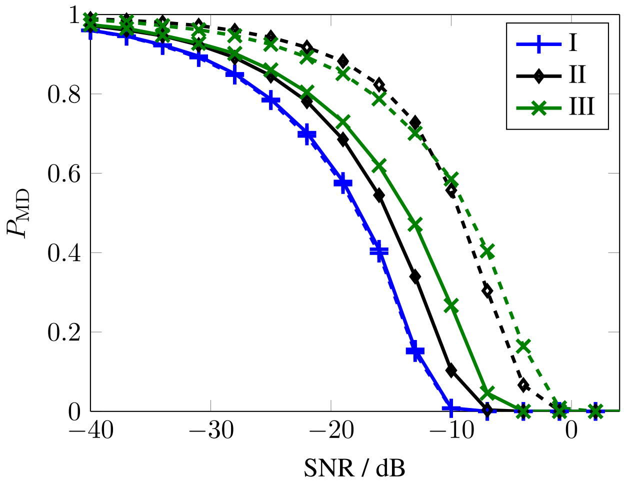

Figure 1: P_{MD} vs. SNR: benchmark detector [1] (dashed lines) and proposed detector (full lines)

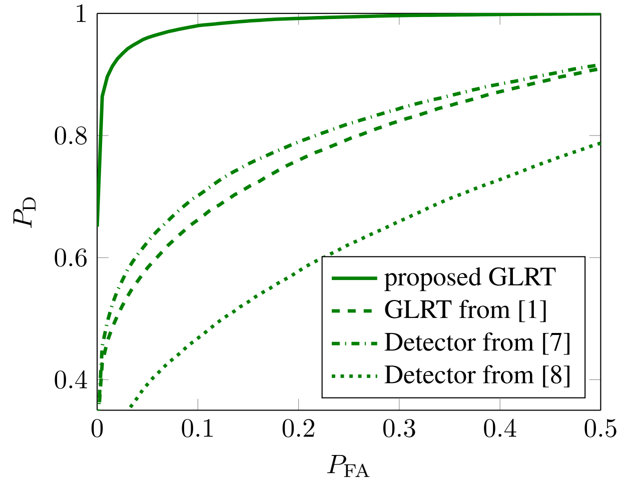

Figure 2: ROC curves of four detectors for the case (III)

we used the GLRT from [1] and, with a few modifications, we used the same simulation setup: Under the different hypotheses, the generated signals $\mathbf{x}[n] \in \mathbb{C}^L$ are

$$\mathcal{H}_1 : \mathbf{x}[n] = (\mathbf{H} * \mathbf{s})[n] + \mathbf{w}[n]$$
$$\mathcal{H}_0 : \mathbf{x}[n] = \mathbf{w}[n],$$

where $*$ means convolution. The noise process $\mathbf{w}[n]$ is either (un)correlated among antennas, temporally white or colored, or a combination thereof. To realize temporally colored noise, we pass white noise through a moving average filter of order 19. The signal $\mathbf{s}[n]$ is baseband QPSK with rectangular shaping and P samples per symbol. The channel $\mathbf{H}[n]$ is a Rayleigh fading channel, with exponential power delay profile, uncorrelated among antennas, and constant for each Monte Carlo experiment.

In Figure 1, we plot the performance of the detectors (measured by the probability of missed detection P_{MD}) as a function of the SNR. We use the parameters $P = 4$, $N = 16$, $L = 4$, $M = 20$, and a constant false alarm rate of $P_{\mathrm{FA}} = 10^{-3}$. For case (I), the difference between the benchmark and the proposed detector is negligible, but for the cases (II) and (III) with white noise, the proposed detectors perform much better. The main reason is that in (I) the number of unknown parameters under $\mathcal{H}_0$ is not reduced as much as in (II) or (III).

One more comment concerning Figure 1 is in order. One may wonder why the detector in case (III), which uses the *most* a priori information about the noise, actually performs *worst* among the three cases. The answer is that these cases are not completely comparable in terms of SNR only, because the *kind* of colored noise (i.e., the exact covariance of the noise) also influences the performance.

The performance for the scenario (III) is further studied in Figure 2, which shows a receiver operating characteristic (ROC). Here the simulation parameters are chosen as $L = 2$, the SNR is $-8\,\mathrm{dB}$, and everything else is as before. As additional detectors the tests from [7] and [8] are used. The test [7] can be used as an extension of the test from [5], in the case of multiple antennas. This detector requires the choice of parameters and for this simulation we use the first cycle frequency and the lags 0, 1, 2, and 3. For the test from [8] we also use the first cycle frequency, but we can only use one lag and we choose the lag equal to 2. In Figure 2, it is clearly visible that the proposed detector, which makes use of the structural information, outperforms the competing detectors.

5. CONCLUSION

We have derived an asymptotic GLRT that makes use of additional information. We assume to know whether the noise is uncorrelated or correlated among antennas and whether the noise is white or colored. For these cases, we developed tests and showed with simulations that this knowledge helps improve the more general detector from [1].

6. REFERENCES

[1] D. Ramírez, P. J. Schreier, J. Vía, I. Santamaría, and L. L. Scharf, “Detection of Multivariate Cyclostationarity,” *IEEE Trans. Signal Process.*, vol. 63, no. 20, pp. 5395–5408, 2015.

[2] M. A. McHenry, P. A. Tenhula, D. McCloskey, D. A. Roberson, and C. S. Hood, “Chicago spectrum occupancy measurements & analysis and a long-term studies proposal,” in *Proc. First Int. Work. Technol. Policy Access. Spectr.*, 2006.

[3] E. Axell, G. Leus, E. G. Larsson, and H. V. Poor, “Spectrum sensing for cognitive radio : State-of-the-art and recent advances,” *IEEE Signal Process. Mag.*, vol. 29, no. 3, pp. 101–116, 2012.

[4] W. A. Gardner, A. Napolitano, and L. Paura, “Cyclostationarity: Half a century of research,” *Signal Process.*, vol. 86, no. 4, pp. 639–697, 2006.

[5] A. V. Dandawaté and G. B. Giannakis, “Statistical tests for presence of cyclostationarity,” *IEEE Trans. Signal Process.*, vol. 42, no. 9, pp. 2355–2369, 1994.

[6] S. Enserink and D. Cochran, “A cyclostationary feature detector,” in *Proc. 1994 28th Asilomar Conf. Signals, Syst. Comput.*, 1994, vol. 2, pp. 806–810.

[7] J. Lundén, V. Koivunen, A. Huttunen, and H. V. Poor, “Collaborative cyclostationary spectrum sensing for cognitive radio systems,” *IEEE Trans. Signal Process.*, vol. 57, no. 11, pp. 4182–4195, 2009.

[8] P. Urriza, E. Rebeiz, and D. Cabric, “Multiple antenna cyclostationary spectrum sensing based on the cyclic correlation significance test,” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 11, pp. 2185–2195, 2013.

[9] D. Ramírez, L. L. Scharf, J. Vía, I. Santamaría, and P. J. Schreier, “An asymptotic GLRT for the detection of cyclostationary signals,” in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, Florence, Italy, 2014, pp. 3415–3419.

[10] D. Ramírez, P. J. Schreier, J. Vía, I. Santamaría, and L. L. Scharf, “An asymptotic LMPI test for cyclostationarity detection with application to cognitive radio,” in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, Brisbane, Australia, 2015, pp. 5669–5673.

[11] E. D. Gladyshev, “Periodically correlated random sequences,” *Sov. Math. Dokl.*, vol. 2, pp. 385–388, 1961.

[12] J. Burg, D. Luenberger, and D. Wenger, “Estimation of structured covariance matrices,” *Proc. IEEE*, vol. 70, no. 9, pp. 963–974, 1982.

[13] J. R. Magnus and H. Neudecker, *Matrix Differential Calculus with Applications in Statistics and Econometrics*, Wiley, 3rd edition, 2007.

[14] P. J. Schreier and L. L. Scharf, *Statistical Signal Processing of Complex-Valued Data*, Cambridge University Press, 2010.