MAGNETIC BEAMFORMING FOR WIRELESS POWER TRANSFER

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ABSTRACT

Magnetic resonant coupling (MRC) is an efficient method for realizing the near-field wireless power transfer (WPT). The use of multiple transmitters (TXs) each with one coil can be applied to enhance the WPT performance by coherently combining the magnetic fields induced by all TX coils in a beam toward the receiver (RX) coil, a technique termed "magnetic beamforming". In this paper, we study the optimal magnetic beamforming design for an MRC-WPT system with multiple TXs and a single RX. We formulate a problem to jointly optimize the currents flowing through different TXs so as to minimize the total power drawn from their voltage sources, subject to the minimum power required by the RX load as well as the practical constraints on the peak voltage and current at all TXs. For the special case of identical TX resistances and without the peak voltage and current constraints, we show that the optimal current at each TX should be proportional to the mutual inductance between its TX coil and the RX coil. In general, the problem is a non-convex quadratically constrained quadratic programming (QCQP), which is reformulated as a semidefinite programming (SDP) with rank-one constraint. We show that the semidefinite relaxation (SDR) of the reformulated problem is tight and hence the problem is solved optimally. Numerical results show that the optimal magnetic beamforming design significantly enhances the deliverable power as well as the power efficiency over the uncoordinated WPT benchmark with equal current allocation over TXs.

Index Terms— Wireless power transfer, magnetic resonant coupling, magnetic beamforming, semidefinite relaxation

1. INTRODUCTION

Near-field wireless power transfer (WPT) has drawn significant interest, due to its high efficiency for delivering power to electric loads without the need of any wire. Near-field WPT can be realized by inductive coupling (IC) [1, 2] for short-range applications in centimeters, or magnetic resonant coupling (MRC) [3, 4] for mid-range applications up to a couple of meters. Although the short-range WPT has been in widely commercial use, the mid-range WPT is still largely under research and has received growing attention recently. The recent progress on mid-range WPT research and applications can be found in e.g., [4] and the references therein.

The MRC-WPT system with multiple transmitters (TXs) and/or multiple receivers (RXs) has been studied in the literature [5–8]. The MRC-WPT system with only two TXs and one RX is studied in [5, 6], while the analytical results therein cannot apply to the case of more than two TXs. Recently, an "Magnetic MIMO" wireless charging system is reported in [7] which can charge a phone 40cm away from the array of TX coils, independent of the phone's orientation. For an MRC-WPT system with multiple RXs, the load resistances of RXs are jointly optimized in [8] to minimize the total transmit power while achieving fair power delivered to the loads regardless of their near-far distances to the TX. Deploying multiple TXs can help focus their generated magnetic fields more efficiently toward the RX [7], hence achieving a magnetic beamforming gain, in a manner analogous to multi-antenna beamforming in the far-field wireless communications [9]. However, to our best knowledge, there is limited work that optimizes the magnetic beamforming design under practical circuit constraints, for an MRC-WPT system with arbitrary number of TXs, which thus motivates this paper.

In this paper, as shown in Fig. 1, we consider an MRC-WPT system with a single RX and multiple TXs where their source currents (or equivalently voltages) can be adjusted such that the induced magnetic fields are optimally combined at the RX, to maximize the amount of power transferred. We formulate a problem to minimize the total power drawn from all TXs by jointly optimizing the currents at all TXs, subject to the minimum power required by the RX load as well as the practical constraints on the peak voltage and current at all TXs. For the special case of identical TX resistances and without the TXs' voltage and current constraints, the optimal current at each TX is shown to be proportional to the mutual inductance between its TX coil and the RX coil. In general, our formulated problem is a non-convex quadratically constrained quadratic programming (QCQP). By recasting the problem as a semidefinite programming (SDP) with rank-one constraint, we show that its semidefinite relaxation (SDR) [10] is tight by exploiting the structure of the problem, i.e., the optimal solution to the SDR problem is always rank-one. The optimal solution to the original QCQP problem can thus be obtained efficiently via standard convex optimization software [11]. Numerical results show that the optimal magnetic beamforming solution significantly enhances the performance of WPT over the benchmark scheme with equal current allocation over all TXs.

2. SYSTEM MODEL

As shown in Fig. 1, we consider a multiple-input single-output (MISO) MRC-WPT system with $N \geq 1$ TXs each equipped with a single coil, and one single-coil RX. Each TX n, $n \in \{1, \cdots, N\}$, is connected to a stable energy source supplying sinusoidal voltage over time given by $\widetilde{v}_n(t) = \mathrm{Re}\{v_n e^{jwt}\}$, with v_n denoting the complex voltage and $w > 0$ denoting the operating angular frequency. Let $\widetilde{i}_n(t) = \mathrm{Re}\{i_n e^{jwt}\}$ denote the steady-state current flowing through TX n, with the complex current i_n. This current produces a time-varying magnetic flux in the n-th TX coil, which passes through the RX coil and induces time-varying currents in it. Let $\widetilde{i}_0(t) = \mathrm{Re}\{i_0 e^{jwt}\}$ denote the steady-state current in the RX coil, with the complex current i_0.

Let M_{n0} and M_{nk} denote the mutual inductance between the n-th TX coil and the RX coil, and the mutual inductance between the n-th TX coil and the k-th TX coil with $k \neq n$, respectively. The mutual inductance is a real number that depends on the physical characteristics of each pair of coils such as their relative distance, orientations, etc. [7]

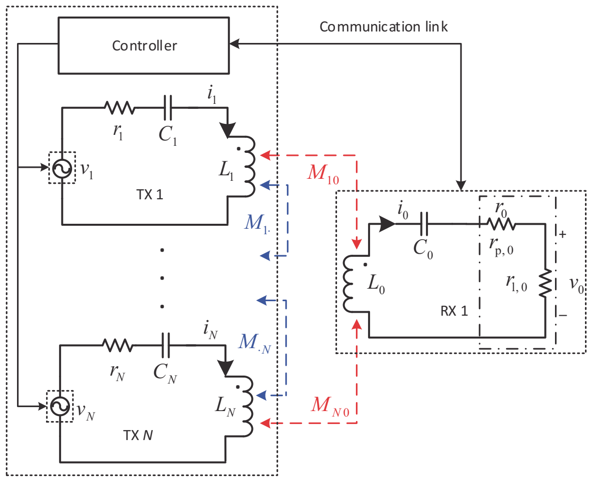

Fig. 1. System model of an MISO MRC-WPT system

We denote the self-inductance and the capacitance of the n-th TX coil (RX coil) by $L_n > 0$ ($L_0 > 0$) and $C_n > 0$ ($C_0 > 0$), respectively. The capacitance values are set as $C_n = \frac{1}{L_n w^2}, n = 1, \ldots, N$, and $C_0 = \frac{1}{L_0 w^2}$, such that all TXs and the RX operate at the same resonant angular frequency w. Let $r_n > 0$ denote the total source resistance (including the internal parasitic resistance) of the n-th TX. Define the diagonal resistance matrix as $\mathbf{R} \triangleq \mathrm{diag}\{r_1, \ldots, r_N\}$. The resistance of the RX, denoted by $r_0 > 0$, consists of the parasitic resistance $r_{\mathrm{p},0} > 0$ and the load resistance $r_{\mathrm{l},0} > 0$, i.e., $r_0 = r_{\mathrm{p},0} + r_{\mathrm{l},0}$. The load is assumed to be purely resistive.

In this paper, we assume that there is a controller that can communicate with all TXs as well as the RX such that it can obtain the information of all system parameters required to design the magnetic beamforming over TXs. For convenience, we treat the complex TX current values i_n's as design variables,[1] which can be optimally set by the controller to realize magnetic beamforming.

By applying Kirchhoff's circuit law to the RX, we obtain its current i_0 as follows

$$i_0 = \frac{jw}{r_0} \sum_{n=1}^{N} M_{0n} i_n. \tag{1}$$

Define the vector of all TX currents as $\mathbf{i} = [i_1, \ldots, i_N]^T$. Define the vector of mutual inductances between the RX coil and all TX coils as $\mathbf{m} = [M_{01}\ M_{02}\ \cdots\ M_{0N}]^T$. From (1), the power delivered to the RX load is obtained as

$$p_0 = \frac{1}{2}|i_0|^2 r_{\mathrm{l},0} = \frac{w^2 r_{\mathrm{l},0}}{2r_0^2} \mathbf{i}^H \mathbf{m}\mathbf{m}^T \mathbf{i}. \tag{2}$$

By applying Kirchhoff's circuit law to each TX n, we obtain its source voltage as

$$v_n = r_n i_n + jw \sum_{k=1,\neq n}^{N} M_{nk} i_k - jw M_{n0} i_0$$

$$= \left(r_n + \frac{M_{n0}^2 w^2}{r_0}\right) i_n + \sum_{k \neq n} \left(jw M_{nk} + \frac{M_{n0} M_{0k} w^2}{r_0}\right) i_k. \tag{3}$$

We define an $N \times N$ square matrix $\mathbf{B}$ as follows,

$$\mathbf{B} = \overline{\mathbf{B}} + j\widehat{\mathbf{B}}, \tag{4}$$

where the elements in $\overline{\mathbf{B}}$ and $\widehat{\mathbf{B}}$ are respectively given by

$$\overline{B}_{nk} = \begin{cases} r_n + \frac{M_{n0}^2 w^2}{r_0}, & \text{if } k = n \\ \frac{M_{n0} M_{0k} w^2}{r_0}, & \text{otherwise} \end{cases} \tag{5}$$

$$\widehat{B}_{nk} = \begin{cases} 0, & \text{if } k = n \\ -w M_{nk}, & \text{otherwise} \end{cases} \tag{6}$$

Note that the matrices $\mathbf{B}$, $\overline{\mathbf{B}}$ and $\widehat{\mathbf{B}}$ are all symmetric, since $M_{nk} = M_{kn}, \ \forall n \neq k$. Denote the n-th column of the matrices $\mathbf{B}$, $\overline{\mathbf{B}}$, $\widehat{\mathbf{B}}$ by $\mathbf{b}_n$, $\overline{\mathbf{b}}_n$, $\widehat{\mathbf{b}}_n$, respectively. Moreover, the matrix $\overline{\mathbf{B}}$ is positive semi-definite (PSD), as it can be rewritten as

$$\overline{\mathbf{B}} = \mathbf{R} + \frac{w^2 \mathbf{m}\mathbf{m}^T}{r_0}. \tag{7}$$

The source voltage of each TX n given in (3) can be equivalently rewritten as

$$v_n = \mathbf{b}_n^H \mathbf{i}. \tag{8}$$

From (4) and (8), the total power drawn from all TXs, denoted by p, is given by

$$p = \frac{1}{2}\mathrm{Re}\left\{\sum_{n=1}^{N} \mathbf{i}^H \mathbf{b}_n i_n\right\} = \frac{1}{2}\mathbf{i}^H \overline{\mathbf{B}} \mathbf{i}. \tag{9}$$

Remark 1. From (5) and (9), it is observed that p depends on the mutual inductance M_{n0} between the coils of each TX n and the RX, but is not related to the mutual inductance M_{nk} between any pair of TX coils.

3. PROBLEM FORMULATION

In this section, we formulate the magnetic beamforming design problem to minimize the total power drawn from all TXs by jointly optimizing the TX currents $\mathbf{i}$, subject to the following constraints: the power delivered to the RX load should meet a minimum level $\beta_0 > 0$, i.e., $p_0 \geq \beta_0$; the peak amplitude of the voltage v_n at each TX n is V_n, i.e., $|v_n| \leq V_n$; and the peak amplitude of the current i_n at each TX n is A_n, i.e., $|i_n| \leq A_n$. Let $\mathbf{Q}_n$ be the rank-one matrix with the n-th diagonal element equal to one and all other elements being zero. From (2), (8) and (9), the problem is formulated as follows.

$$(\mathrm{P1}): \ \min_{\mathbf{i} \in \mathcal{C}^N} \ \frac{1}{2}\mathbf{i}^H \overline{\mathbf{B}} \mathbf{i} \tag{10a}$$

$$\text{s. t.} \ \frac{w^2 r_{\mathrm{l},0}}{2r_0^2} \mathbf{i}^H \mathbf{m}\mathbf{m}^T \mathbf{i} \geq \beta_0 \tag{10b}$$

$$\mathbf{i}^H \mathbf{b}_n \mathbf{b}_n^H \mathbf{i} \leq V_n^2, \quad n = 1,\ 2,\ \cdots\ N \tag{10c}$$

$$\mathbf{i}^H \mathbf{Q}_n \mathbf{i} \leq A_n^2, \quad n = 1,\ 2,\ \cdots\ N \tag{10d}$$

(P1) is a non-convex QCQP problem [11]. Although solving such a problem is non-trivial in general [10], we obtain the optimal solution to (P1) in the next section.

[1] In practice, it is more convenient to use voltage source than current source. Therefore, after obtaining the TX current values i_n's, the resulting voltage values v_n's can be computed and set by the controller accordingly.

4. OPTIMAL SOLUTION

4.1. Optimal Solution to (P1) without Constraints (10c) and (10d)

In this subsection, we consider the simplified problem of (P1) without the peak voltage and current constraints given in (10c) and (10d), respectively, to obtain the performance limit of magnetic beamforming. Let $\mathbf{i} = \bar{\mathbf{i}} + j\hat{\mathbf{i}}$. It is then observed that the real-part $\bar{\mathbf{i}}$ and the imaginary-part $\hat{\mathbf{i}}$ contribute in the same way to the total TX power in (10a) as well as the delivered load power in (10b), since both $\overline{\mathbf{B}}$ and $\mathbf{m}\mathbf{m}^T$ are symmetric matrices. As a result, we can set $\hat{\mathbf{i}} = \mathbf{0}$ and adjust $\bar{\mathbf{i}}$ only, i.e., we need to solve

$$(\mathrm{P2}): \quad \min_{\bar{\mathbf{i}} \in \mathbb{R}^N} \ \frac{1}{2}\bar{\mathbf{i}}^T \overline{\mathbf{B}}\bar{\mathbf{i}} \tag{11a}$$

$$\text{s. t.} \ \frac{w^2 r_{\mathrm{l},0}}{2r_0^2}\bar{\mathbf{i}}^T \mathbf{m}\mathbf{m}^T \bar{\mathbf{i}} \geq \beta_0 \tag{11b}$$

The optimal solution to (P2) is given by the following theorem.

Theorem 1. *The optimal solution to* (P2) *is* $\bar{\mathbf{i}}^\star = \alpha \mathbf{u}_1$*, where* α *is a constant such that the constraint* (11b) *holds with equality, and* $\mathbf{u}_1$ *is the eigenvector associated with the minimum eigenvalue, denoted by* γ_1*, of the matrix*

$$\mathbf{T} = \mathbf{R} + \frac{w^2(r_0 - v^\star r_{\mathrm{l},0})}{r_0^2}\mathbf{m}\mathbf{m}^T, \tag{12}$$

where $v^\star$ *is chosen such that* $\gamma_1 = 0$.

Specifically, for the case of identical TX resistances, i.e., $\mathbf{R} \triangleq r\mathbf{I}$*, the optimal solution to (P2) is*

$$\bar{\mathbf{i}}^\star = \frac{\alpha \mathbf{m}}{\|\mathbf{m}\|_2}. \tag{13}$$

Proof. Due to the space limitation, the proof is omitted here, and given in an online technical report of this paper [12]. □

Remark 2. Theorem 1 indicates that for the special case of identical TX resistances, the optimal current of each TX n is proportional to the mutual inductance M_{0n} between the RX and TX n. This is analogous to the maximal-ratio-transmission (MRT) based beamforming in wireless communications [9]. However, magnetic beamforming operates over the near-field and thus the phase of the TX current only needs to be 0 or π (depending on positive or negative mutual inductance value); while in wireless communications, beamfoming operates over the far-field, and as a result, the beamforming weight at each transmit antenna needs to be set to be of the opposite phase of the wireless channel which can be arbitrarily distributed between 0 and 2π [13].

4.2. Optimal Solution to (P1) with All Constraints

Define $\mathbf{X} \triangleq \mathbf{i}\mathbf{i}^H$, $\mathbf{M} \triangleq \mathbf{m}\mathbf{m}^T$, and $\mathbf{B}_n \triangleq \mathbf{b}_n\mathbf{b}_n^H$. Thus, (P1) can be equivalently rewritten as the following SDP with rank-one constraint.

$$(\mathrm{P1{-}SDP}): \quad \min_{\mathbf{X} \in \mathbb{C}^{N\times N}} \ \frac{1}{2}\operatorname{Tr}\left(\overline{\mathbf{B}}\mathbf{X}\right) \tag{14a}$$

$$\text{s. t.} \ \operatorname{Tr}(\mathbf{M}\mathbf{X}) \geq \frac{2r_0^2\beta_0}{w^2 r_{\mathrm{l},0}} \tag{14b}$$

$$\operatorname{Tr}(\mathbf{B}_n\mathbf{X}) \leq V_n^2, \ \ n = 1, \cdots N \tag{14c}$$

$$\operatorname{Tr}(\mathbf{Q}_n\mathbf{X}) \leq A_n^2, \ \ n = 1, \cdots N \tag{14d}$$

$$\mathbf{X} \succcurlyeq 0, \ \operatorname{rank}(\mathbf{X}) = 1 \tag{14e}$$

where $\mathbf{X} \succcurlyeq 0$ indicates that $\mathbf{X}$ is PSD.

In general, (P1−SDP) is non-convex due to the rank constraint in (14e) [10]. By ignoring the rank-one constraint in (14e), we obtain the SDR of (P1−SDP), denoted by (P1−SDR), which is convex. Next, we show the following theorem on the optimal solution to (P1), by exploiting the special structure of the SDR of (P1−SDP).

Theorem 2. *The SDR of* (P1−SDP) *is tight, i.e., the optimal solution* $\mathbf{X}^\star$ *to* (P1−SDR) *is always rank-one with* $\mathbf{X}^\star = \mathbf{i}^\star(\mathbf{i}^\star)^H$*, where* $i^\star$ *is the optimal solution to* (P1).

Proof. Let $\lambda \geq 0$, $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_N]^T \geq 0$, and $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_N]^T \geq 0$ be the dual variables corresponding to the constraint(s) given in (10b), (10c), and (10d), respectively. Let the matrix $\mathbf{S} \succcurlyeq 0$ be the dual variable corresponding to the constraint $\mathbf{X} \succcurlyeq 0$ in (14e). The Lagrangian of (P1−SDR) is then written as

$$L(\mathbf{X}, \lambda, \boldsymbol{\rho}, \mathbf{S}) = \frac{1}{2}\operatorname{Tr}\left(\overline{\mathbf{B}}\mathbf{X}\right) - \lambda\left(\operatorname{Tr}(\mathbf{M}\mathbf{X}) - \frac{2r_0^2\beta_0}{w^2 r_{\mathrm{l},0}}\right) + \sum_{n=1}^{N}\rho_n\left(\operatorname{Tr}(\mathbf{B}_n\mathbf{X}) - A_n^2\right) + \sum_{n=1}^{N}\mu_n\left(\operatorname{Tr}(\mathbf{Q}_n\mathbf{X}) - D_n^2\right) - \operatorname{Tr}(\mathbf{S}\mathbf{X}). \tag{15}$$

It can be checked that (P1−SDR) satisfies the Slater's condition and thus the strong duality holds for this problem. Let $\mathbf{X}^\star, \lambda^\star, \boldsymbol{\rho}^\star, \boldsymbol{\mu}^\star$, and $\mathbf{S}^\star$ be the optimal primal and dual variables, respectively. Moreover, the Karush−Kuhn−Tucker (KKT) conditions [11] of (P1−SDR) are given by

$$\nabla_{\mathbf{X}} L(\mathbf{X}^\star, \lambda^\star, \boldsymbol{\rho}^\star, \boldsymbol{\mu}^\star, \mathbf{S}^\star) = \frac{1}{2}\overline{\mathbf{B}} - \lambda^\star\mathbf{M} + \sum_{n=1}^{N}\rho_n^\star\mathbf{B}_n + \sum_{n=1}^{N}\mu_n^\star\mathbf{Q}_n - \mathbf{S}^\star = \mathbf{0}. \tag{16}$$

$$\mathbf{S}^\star\mathbf{X}^\star = \mathbf{0}. \tag{17}$$

Next, by multiplying (16) by $\mathbf{X}^\star$ on both sides and substituting (17) into the obtained equation, we have

$$\frac{1}{2}\overline{\mathbf{B}}\mathbf{X}^\star - \lambda^\star\mathbf{M}\mathbf{X}^\star + \sum_{n=1}^{N}\rho_n^\star\mathbf{B}_n\mathbf{X}^\star + \sum_{n=1}^{N}\mu_n^\star\mathbf{Q}_n\mathbf{X}^\star = \mathbf{0}. \tag{18}$$

We thus have

$$\operatorname{rank}\left(\left(\frac{1}{2}\overline{\mathbf{B}} + \sum_{n=1}^{N}\rho_n^\star\mathbf{B}_n + \sum_{n=1}^{N}\mu_n^\star\mathbf{Q}_n\right)\mathbf{X}^\star\right) = \operatorname{rank}(\mathbf{M}\mathbf{X}^\star) \leq \operatorname{rank}(\mathbf{M}) = 1. \tag{19}$$

Since $\overline{\mathbf{B}}$ is PSD, the matrix $\left(\frac{1}{2}\overline{\mathbf{B}} + \sum_{n=1}^{N}\rho_n^\star\mathbf{B}_n + \sum_{n=1}^{N}\mu_n^\star\mathbf{Q}_n\right)$ must have full rank. Hence, (19) implies that

$$\operatorname{rank}(\mathbf{X}^\star) = \operatorname{rank}\left(\left(\frac{1}{2}\overline{\mathbf{B}} + \sum_{n=1}^{N}\rho_n^\star\mathbf{B}_n + \sum_{n=1}^{N}\mu_n^\star\mathbf{Q}_n\right)\mathbf{X}^\star\right) = 1. \tag{20}$$

Since $\mathbf{X}^\star$ is rank-one, it can be written as $\mathbf{X}^\star = \mathbf{i}^\star(\mathbf{i}^\star)^H$. The vector $\mathbf{i}^\star$ is thus the solution to (P1−SDP) and hence (P1). □

Remark 3. The optimal solution to (P1−SDR) can be efficiently solved via e.g., the interior-point method [11], and the solution obtained numerically always has rank-one in accordance to Theorem 2.

5. NUMERICAL RESULTS

As shown in Fig. 2, we consider an MISO MRC-WPC system with $N = 5$ TX coils and one RX coil, each of which has 100 turns and a radius of 0.1 meter. We use cooper wire with radius of 0.1 millimeter for all coils. All the TX coils are in the xy-plane, while the RX coil is in the plane of $z = \frac{3\sqrt{2}}{4}$. The resistance of all TXs is set identically as 0.336Ω. For the RX, its parasitic resistance and load resistance are $r_{\mathrm{p},0} = 0.336\Omega$ and $r_{\mathrm{l},0} = 50\Omega$, respectively. The self and mutual inductances are given in Table 1. All the capacitances are set such that the resonance angular frequency is $w = 6.78 \times 2\pi$ rad/second [8]. We assume that the peak voltage/current constraints are $V_n = 30\sqrt{2}$ V and $A_n = 5\sqrt{2}$ A.

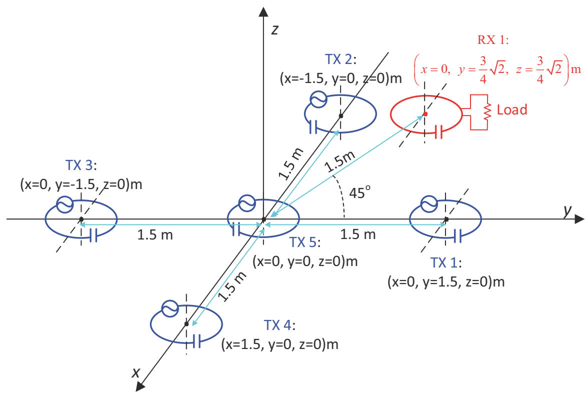

Fig. 2. Simulation setup

Table 1. Mutual/Self inductances (μH)

	TX 1	TX 2	TX 3	TX 4	TX 5
TX 1	5886.8	0.1050	0.0370	0.1050	0.2984
TX 2	0.1050	5886.8	0.1050	0.1253	0.2984
TX 3	0.0370	0.1050	5886.8	0.1050	0.2984
TX 4	0.1050	0.0370	0.1050	5886.8	0.2984
TX 5	0.2984	0.2984	0.2984	0.2984	5886.8
RX 1	-1.0172	0.02570	0.02622	0.02570	-0.1508

For performance benchmark, we consider an uncoordinated W-PT system with all TXs set to have equal current. We compare this system with our proposed coordinated WPT with optimal magnetic beamforming without or with the peak voltage and current constraints at all TXs. We define the efficiency of WPT as the ratio of the delivered load power β_0 to the total TX power p, i.e., $\eta \triangleq \frac{\beta_0}{p}$.

Fig. 3 plots the total TX power p and the efficiency η versus the delivered load power β_0. For the case without TX voltage/current constraints, it is observed that the WPT efficiencies of magnetic beamforming and benchmark system are 73.73% and 39.1%, respectively.

For the case with TX voltage/current constraints, it is observed that magnetic beamforming can deliver power up to 35 W to the RX with the efficiency of 46.62%; while the benchmark system can deliver at most 14 W to the RX with the efficiency of 39.1%. Thus, besides the WPT efficiency improvement, magnetic beamforming also significantly enhances the maximum power deliverable to the load, under practical circuit constraints.

Fig. 3 also shows that the WPT efficiency decreases over $24 < \beta_0 < 35$. To explain this observation and obtain insights for magnetic beamforming, we investigate the two cases of $\beta_0 = 24$ W and 35 W in the following. The optimal currents and consumed power

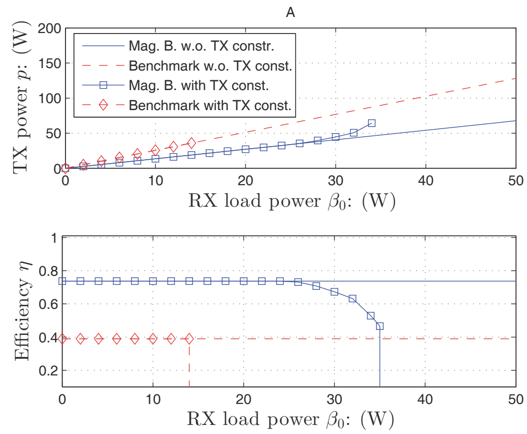

Fig. 3. TX power and efficiency v.s. RX load power

of all TXs are given in Table 2. For $\beta_0 = 24$ W, TX 1 carries a current less than the peak current, and most energy is consumed by TX 1 that has the largest mutual inductance with the RX. This implies that the TX with larger mutual inductance with the RX should carry higher current, and thus consume more power to maximize the efficiency of WPT. In this case, all TX current or voltage constraints are inactive, and it can be further verified that the current of each TX is proportional to its mutual inductance with the RX. This is in accordance with Theorem 1. In contrast, to support higher RX load power of 35 W, TXs 1 and 5 need to carry the peak current, and TXs 2, 3 and 4 increase their carried currents. The cost is the decreased efficiency, due to smaller mutual inductance between TXs 2, 3, 4, 5 and the RX, than that between TX 1 and the RX.

Table 2. Comparison under different load power

	$\beta_0 = 24$	$\beta_0 = 35$
$(i_1^\star, p_1^\star)$	$(-6.9840, 31.79)$	$(-7.071, 37.25)$
$(i_2^\star, p_2^\star)$	$(0.1765, 0.0203)$	$(6.716, 8.27)$
$(i_3^\star, p_3^\star)$	$(0.1800, 0.0211)$	$(6.852, 8.61)$
$(i_4^\star, p_4^\star)$	$(0.1765, 0.0203)$	$(6.716, 8.27)$
$(i_5^\star, p_5^\star)$	$(-1.0359, 0.6994)$	$(-7.071, 12.68)$

6. CONCLUSION

This paper studied the optimal magnetic beamforming for the MISO MRC-WPT system. We formulate an optimization problem to minimize the total power drawn from all TXs by jointly designing the currents in all TXs, subject to the RX load power constraint and the practical constraints on the peak voltage and current values at all TXs. For the special case of identical TX resistances and without the TX voltage/current constraints, the optimal current of each TX is shown to be proportional to the mutual inductance between its TX coil and the RX coil. In general, the formulated QCQP problem is non-convex while we efficiently solve it optimally by applying the S-DR technique and proving the uniqueness of rank-one solution. Numerical results show that the optimal magnetic beamforming significantly enhances both the deliverable power and the WPT efficiency, compared to the uncoordinated WPT with equal current allocation over all TXs. Furthermore, it is shown that the practical voltage and current constraints at TXs can degrade the performance of magnetic beamforming, especially when the load power is large.

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