Optimal Space Signalling For Intensity Modulated MIMO Optical Wireless Communications

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***Abstract*—This paper investigates the design of optimal space signaling for an intensity modulated direct detection multi-input-multi-output optical wireless communication system with independent and non-identically log-normal distributed channel coefficients. In order to optimize both large-scale diversity and small-scale diversity gains with a maximum likelihood detector, the design problem is formulated into a max-min optimization problem with continuous-discrete mixed design variables. Two techniques are proposed to solve the problem: 1) by taking advantage of the full large scale diversity condition, all spatial codewords are properly sorted to simplify the inner minimization problem with the discrete design variables; and 2) a novel geometrically weighted inequality is established to carefully deal with a specific objective function of the power products in the outer maximization problem with the continuous variables. Among all the high dimensional nonnegative space signaling vectors, we rigorously prove that the spatial repetition signaling (RS) with an optimal power allocation is optimal under an average optical power constraint. Simulation results indicate that in a high signal to noise ratio regime, our optimally designed space signaling has better error performance than RS, which is the best space signaling available in literature for this application.**

***Index Terms*—Intensity modulation with direct detection (IM/DD), multi-input-multi-output (MIMO), optical wireless communication (OWC), log-normal fading channels, large scale and small scale diversity gains, repetition signaling (RS), maximum likelihood (ML) decoding.**

I. INTRODUCTION

Recently, intensity modulated direct detection optical wireless communication (IM/DD OWC) [1]–[6] has become an attractive research area. To attain more robust error performance in the atmospheric environments, IM/DD MIMO-OWC is usually deployed [7]–[15] with properly designed diversity transmitters. However, the diversity transmission for IM/DD MIMO-OWC cannot be designed by directly following the existing techniques for MIMO radio frequency (RF) [16]–[22] and coherent MIMO-OWC [23]–[27], since there are *two* major constraints on the signal of IM/DD MIMO-OWC [28]. The *first* major constraint is that the transmitted signals of IM/DD MIMO-OWC are nonnegative. It is for this reason that the presently well-established MIMO-RF [16] and coherent MIMO-OWC techniques [26], [27] cannot be employed for IM/DD MIMO-OWC directly. Although they can provide a full diversity gain by adding some proper direct-current components into transmitter designs [7], [10], [11], some modified orthogonal space time block codes have worse error performance than the space-only repetition signaling (RS) [8], [9], [12], [13]. The *second* major constraint is that the channel coefficients of IM/DD MIMO-OWC are also nonnegative [28]. This constraint on channels renders a properly designed space-only transmission of IM/DD MIMO-OWC to enable full diversity [14], [15]. In fact, the well known space-only scheme, RS, which transmits the same symbol across all the transmitter apertures [8], [9], [12], [13], is the best existing full diversity space signaling [8], [9], [12]–[15], when no channel state information at the transmitter (CSIT) is available.

Therefore, we consider the design of space signaling for an IM/DD MIMO-OWC system. In this scenario, when perfect CSIT is available, the selection of the transmitter apertures with the greater optical path scintillation was proposed in [29]–[32]. However, perfect CSIT is not easily obtainable in practice. Thus, we specifically consider the space signaling for an IM/DD MIMO-OWC system over commonly used log-normal fading channels [33], [34] where different channel coefficients have nonidentical variances resulted from atmospheric environments [35], [36]. More recently, we have established an error performance criterion [14], [15] for the design of a space signaling with a maximum likelihood (ML) detector. Using this criterion, we formulate the design problem of an optimal space signaling maximizing both large-scale and small-scale diversity gains into a max-min optimization problem with continuous-discrete mixed design variables. By sorting all the space signal vectors and developing a novel geometrically-weighted inequality, we will prove that for a general IM/DD MIMO-OWC system, RS with an optimal power allocation is the optimal space signaling.

II. SYSTEM MODEL

Let us consider an $M \times N$ IM/DD MIMO-OWC system equipped with M receiver apertures and N transmitter apertures, which transmit a space signal $\mathbf{x} = [x_1, x_2, \cdots, x_N]^T$ from a constellation $\mathcal{X} = \{\mathbf{x}_k, k = 0, 1, \cdots, 2^K - 1\}$ to be designed. To satisfy the unipolarity requirement of intensity modulation, the symbol x_n for $n = 1, 2, \cdots, N$ to be transmitted from the n-th transmitter aperture is non-negative. These symbols are then transmitted to the receivers through an $M \times N$ channel matrix $\mathbf{H}$, the elements of which are flat-fading path coefficients. Therefore, the received signal, denoted by an

$M \times 1$ vector $\mathbf{y}$, can be written in the following form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where the entries of $\mathbf{H}$ are assumed to be independent and log-normal distributed, i.e., $h_{ij} = e^{z_{ij}}$, where $z_{ij} \sim \mathcal{N}\left(\mu_{ij}, \sigma_{ij}^2\right), i = 1, \cdots, M, j = 1, \cdots, N$. The probability density function (PDF) of h_{ij} is

$$f_H\left(h_{ij}\right) = \frac{1}{\sqrt{2\pi} h_{ij} \sigma_{ij}} e^{-\frac{\left(\ln h_{ij} - \mu_{ij}\right)^2}{2\sigma_{ij}^2}}$$

where σ_{ij}^2 is altitude-dependent and determined by the light wavelength, the link distance and the root mean square wind speed [35], [36], implying that σ_{ij}^2 may be different in practical scenarios. The PDF of $\mathbf{H}$ is $f\left(\mathbf{H}\right) = \prod_{i=1}^{M} \prod_{j=1}^{N} f_H\left(h_{ij}\right)$. From [28], [35], [37], the noise vector $\mathbf{n}$ is modelled as signal-independent AWGN with zero mean and co-variance matrix $\frac{\sigma_{\mathbf{n}}^2}{M} \mathbf{I}_{M \times M}$.

The basic assumptions throughout this paper are made as follows:

1) *Power constraint*. In most practical optical sources with intensity modulation and direct detection, a power constraint is usually on the amplitude sum average of the nonnegative optical signals [28], [38], i.e., $\frac{1}{2^K} \sum_{k=0}^{2^K-1} \sum_{n=1}^{N} x_{nk} \le 1$.

2) *Channel information*. The receiver exactly knows CSI and the transmitter only knows the following vector in terms of the channel variances, $\frac{1}{\Omega}\left[\Omega_1, \cdots, \Omega_N\right]^T$, where $\Omega_j = \sum_{i=1}^{M} \sigma_{ij}^{-2}$ and $\Omega = \sum_{j=1}^{N} \Omega_j$.

Under the above assumptions, we specifically aim at designing the optimal space constellation $\mathcal{X} = \left\{\mathbf{x}_0, \cdots, \mathbf{x}_{2^K-1}\right\} \subseteq \mathbb{R}_+^N$ that maximizes both the large-scale diversity gain and the small-scale diversity gain, where notation $\mathbb{R}_+^N$ denotes the set of all the nonnegative $N \times 1$ vectors.

III. Diversity Gains of Space Signalings for IM/DD MIMO-OWC

In this section, we first briefly review our recently established error performance criterion for the space signaling design and then, introduce the concept of large-scale diversity gain and small-scale diversity gain for the IM/DD MIMO-OWC system over log-normal fading channels [14], [15].

Proposition 1: [14], [15] For $\forall \mathbf{x}, \hat{\mathbf{x}} \in \mathcal{X}$ with $\mathbf{x} \neq \hat{\mathbf{x}}$, if $\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x}$ is positive up to a scale, then, the average pair-wise error probability for $M \times N$ IM/DD MIMO-OWC with a space signaling is bounded by

$$\begin{aligned} & C_L \left(\ln \rho\right)^{-MN} e^{-\frac{\mathcal{D}_l}{8}\left(\ln \rho + \ln \Omega - \ln\left(M \sum_{k=1}^{N} e_k^2\right)\right)^2} \\ & \le P\left(\mathbf{x} \to \hat{\mathbf{x}}\right) \le P_U\left(\mathbf{x} \to \hat{\mathbf{x}}\right) + \mathcal{O}\left(e^{-\frac{\mathcal{D}_l}{8} \ln^2 \rho}\right) \end{aligned} \tag{2}$$

where ρ is the signal-to-noise ration (SNR), $\mathcal{D}_l = \Omega$, $C_L = \frac{\prod_{i=1}^{M} \prod_{j=1}^{N} \sigma_{ij}}{(4\pi)^{MN} e^{-\frac{MN}{2}}} Q\left(\frac{1}{2}\left(\sum_{k=1}^{N} e_k^2\right)^{-\frac{1}{2}}\right)$,

$$\begin{aligned} P_U\left(\mathbf{x} \to \hat{\mathbf{x}}\right) &= C_U \mathcal{G}\left(\mathbf{e}\right) \left(\frac{\rho}{\ln^2 \rho}\right)^{\frac{\Omega}{4} \ln\left(\frac{N\Omega}{M}\right) - \frac{3}{4} \ln \mathcal{D}_s(\mathbf{e})} \\ &\times \left(\ln \rho\right)^{-MN} e^{-\frac{\mathcal{D}_l}{8} \ln^2 \frac{\rho}{\ln^2 \rho}} \end{aligned}$$

where $C_U = \frac{N^{MN}}{2 \prod_{i=1}^{M} \prod_{j=1}^{N} \sigma_{ij}} e^{-\frac{\Omega}{8} \ln^2\left(\frac{N\Omega}{M}\right)}$, $\mathcal{D}_s\left(\mathbf{e}\right) = \prod_{j=1}^{N} \left|e_j\right|^{\Omega_j}$ and $\mathcal{G}\left(\mathbf{e}\right) = \exp\left(\frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(\ln \left|e_j\right|^{\sigma_{ij}}\right)^2\right) \left(\frac{N\Omega}{M}\right)^{\frac{\ln \ln \mathcal{D}_s(\mathbf{e})}{2}}$. ■

To make $P_U\left(\mathbf{x} \to \hat{\mathbf{x}}\right)$ decay as fast as possible against increasing SNRs, we should optimize the following three factors of $P_U\left(\mathbf{x} \to \hat{\mathbf{x}}\right)$:

1) *Large-scale diversity gain*. When SNR is sufficiently high, the exponentially decaying speed of $P_U\left(\mathbf{x} \to \hat{\mathbf{x}}\right)$ is mainly determined by $\mathcal{D}_l$. For this reason, we call $\mathcal{D}_l$ the *large-scale diversity gain*. When we design the optimal space signaling, a full large-scale diversity achievement should be satisfied first by utilizing up all the MN terms in $\mathcal{D}_l$.
2) *Small-scale diversity gain*. By Proposition 1, $\mathcal{D}_s\left(\mathbf{e}\right) = \prod_{j=1}^{N} \left|e_j\right|^{\Omega_j}$ dictates the polynomial decaying of $\frac{\rho}{\ln^2 \rho}$, which is relatively small-scale compared with the large-scale exponential decaying speed. Therefore, we name $\mathcal{D}_s\left(\mathbf{e}\right)$ *small-scale diversity gain*. Under the condition that full large-scale diversity gain is assured, $\min_{\mathbf{e}} \mathcal{D}_s\left(\mathbf{e}\right)$ should be maximized.
3) *Coding gain*. Since $\mathcal{G}\left(\mathbf{e}\right)$ only affects the horizontal shift of the error curve, we define this term as *coding gain*. When both large-scale diversity gain and small-scale diversity gain are optimized and if there remain design freedoms, $\max_{\mathbf{e}} \mathcal{G}\left(\mathbf{e}\right)$ should be further minimized.

IV. Design of Optimal Space Signaling

Our primary task of this section is to design the optimal constellation $\mathcal{X}$ that maximizes both the small-scale diversity gain and the large-scale diversity gain.

A. Problem and Simplification

Let us consider an $M \times N$ IM/DD MIMO-OWC with a space constellation $\mathcal{X}$. For such system, with the design criterion established in Proposition 1, our main target in this paper is to solve the following optimization problem:

Original Main Problem: Find a space constellation $\mathcal{X} = \left\{\mathbf{x}_k, k = 0, 1, \cdots, 2^K - 1\right\} \subseteq \mathbb{R}^N$ such that

1) the large-scale diversity gain is maximized first;
2) when a full large-scale diversity achievement is fulfilled, we then optimize the small-scale diversity gain, i.e.,

$$\max_{\mathbf{x}_k} \min_{0 \le k_2 \neq k_1 \le 2^K-1} \prod_{n=1}^{N} \left|x_{nk_2} - x_{nk_1}\right|^{\Omega_n} \tag{3a}$$

$$s.t. \forall k_2 > k_1, \mathbf{x}_{k_2} - \mathbf{x}_{k_1} \in \mathbb{R}_+^N. \tag{3b}$$

subject to $\frac{1}{2^K} \sum_{k=0}^{2^K-1} \sum_{n=1}^{N} x_{nk} = 1$. ■

By Proposition 1, $\mathcal{X}$ assures full large-scale diversity gain if and only if $\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x} \neq \mathbf{0}$ is positive up to a scale. This necessary and sufficient condition enables us to sort all the entries of $\mathcal{X}$, $\mathbf{x}_0, \cdots, \mathbf{x}_{2^K-1} \in \mathbb{R}_+^N$, such that

$$\mathbf{x}_0 < \cdots < \mathbf{x}_{2^K-1} \tag{4}$$

where notation "$\mathbf{a} < \mathbf{b}$" means that the entries of $\mathbf{b} - \mathbf{a}$ are all positive. Note that the objective of (3) is a function with continuous and discrete variables. Here, one of the main challenges is how to deal with the inner minimization problem involving the discrete variables, i.e., $\min_{k_1,k_2\in\{0,\ \cdots,\ 2^K-1\},k_2>k_1}\prod_{n=1}^{N}|x_{nk_2}-x_{nk_1}|^{\Omega_n}$. Using (4), $\forall k_1,k_2\in\{0,1,\cdots,2^K-1\}$ and $k_2\geq k_1+1$, we have $x_{nk_2}\geq x_{n(k_1+1)}>x_{nk_1}$, therefore, resulting in $\prod_{n=1}^{N}|x_{nk_2}-x_{nk_1}|^{\Omega_n}=\prod_{n=1}^{N}\left(x_{nk_2}-x_{n(k_1+1)}+x_{n(k_1+1)}-x_{nk_1}\right)^{\Omega_n}\geq\prod_{n=1}^{N}\left(x_{n(k_1+1)}-x_{nk_1}\right)^{\Omega_n}$, where the equality holds if and only if $k_2=k_1+1$. Hence, we obtain $\min_{k_1,k_2\in\{0,\ \cdots,\ 2^K-1\},k_2>k_1}\prod_{n=1}^{N}|x_{nk_2}-x_{nk_1}|^{\Omega_n}=\min_{k\in\{0,\ \cdots,\ 2^K-2\}}\prod_{n=1}^{N}\left(x_{n(k+1)}-x_{nk}\right)^{\Omega_n}$. This shows that the original main problem is transformed into the following equivalent optimization problem:

Problem 1: For any given positive integers N and K, find a constellation $\mathcal{X}=\{\mathbf{x}_k,k=0,1,\ \cdots,\ 2^K-1\}\subseteq\mathbb{R}_+^N$ such that

$$\max_{\mathbf{x}_k}\min_{k\in\{0,\ \cdots,\ 2^K-2\}}\prod_{n=1}^{N}\left(x_{n(k+1)}-x_{nk}\right)^{\Omega_n}\tag{5a}$$

$$s.t.\begin{cases}\mathbf{x}_{k+1}-\mathbf{x}_k>\mathbf{0}\\ \frac{1}{2^K}\sum_{k=0}^{2^K-1}\sum_{n=1}^{N}x_{nk}=1.\end{cases}\tag{5b}$$

■

Here, we would like to make an important emphasis on the fact that it is the positivity of the error vectors or the vector version of the strict monotonicity of all the space signal vectors assuring the full-scale diversity gain that results successfully in significantly simplifying the inner discrete minimization problem in the original main problem, i.e., the optimization problem (3).

B. Main Lemma

To deal with the power product in the objective function of (5), let us first develop an inequality resulted from Jensen's inequality, which will play another key role in allowing us to attain an explicit expression for the optimal space signaling.

Lemma 1: If $w_i>0$ and $x_i>0$ for $i=1,\ \cdots,\ N$, then, we have $\prod_{i=1}^{N}x_i^{w_i}\leq\left(\frac{\sum_{i=1}^{N}x_i}{\sum_{i=1}^{N}w_i}\right)^{\sum_{i=1}^{N}w_i}\prod_{i=1}^{N}w_i^{w_i}$, where the equality holds if and only if $x_i=\frac{w_i\sum_{i=1}^{N}x_i}{\sum_{i=1}^{N}w_i}$. ■

Proof: First, the Jensen's inequality [39] states that for $\lambda_i\geq 0$, $\sum_{i=1}^{N}\lambda_i=1$, any concave function $f(t)$ with respect to t satisfies that $\sum_{i=1}^{N}\lambda_i f(t_i)\leq f\left(\sum_{i=1}^{N}\lambda_i t_i\right)$, where the equality holds if and only if $t_1=\ \cdots\ =t_N$. Now, by specifically taking concave function $f(t)=\ln t$ for $t>0$, $t_i=\frac{x_i}{w_i}$, and $\lambda_i=\frac{w_i}{\sum_{i=1}w_i}$ for $w_i>0$ and $x_i>0$, we have

$$\sum_{i=1}^{N}\lambda_i\ln t_i\leq\ln\sum_{i=1}^{N}\lambda_i t_i=\ln\frac{\sum_{i=1}x_i}{\sum_{i=1}w_i}\tag{6}$$

where the equality holds if and only if $\frac{x_1}{w_1}=\ \cdots\ =\frac{x_N}{w_N}$. In addition, notice that the left side of (6) can be rewritten as $\sum_{i=1}^{N}\lambda_i\ln t_i=\sum_{i=1}^{N}\frac{w_i}{\sum_{i=1}w_i}\ln x_i-\sum_{i=1}^{N}\frac{w_i}{\sum_{i=1}w_i}\ln w_i=\frac{1}{\sum_{i=1}w_i}\left(\ln\prod_{i=1}^{N}x_i^{w_i}-\ln\prod_{i=1}^{N}w_i^{w_i}\right)$. Substituting this equality into (6) leads to the following inequality $\frac{1}{\sum_{i=1}w_i}\left(\ln\prod_{i=1}^{N}x_i^{w_i}-\ln\prod_{i=1}^{N}w_i^{w_i}\right)\leq\ln\frac{\sum_{i=1}x_i}{\sum_{i=1}w_i}$. After some manipulations, we arrive at $\ln\prod_{i=1}^{N}x_i^{w_i}\leq\ln\prod_{i=1}^{N}w_i^{w_i}+\ln\left(\frac{\sum_{i=1}x_i}{\sum_{i=1}w_i}\right)^{\sum_{i=1}w_i}$, which is equivalent to $\prod_{i=1}^{N}x_i^{w_i}\leq\left(\frac{\sum_{i=1}^{N}x_i}{\sum_{i=1}^{N}w_i}\right)^{\sum_{i=1}^{N}w_i}\prod_{i=1}^{N}w_i^{w_i}$, where the equality holds if and only if $x_i=\frac{w_i\sum_{i=1}^{N}x_i}{\sum_{w_i}}$. This completes the proof of Lemma 1. □

C. Optimal Space Signaling

In this subsection, we aim at rigorously solving Problem 1, whose solution, in fact, is given by theorem below:

Theorem 1: The optimal solution to (5) is determined by

$$\widetilde{\mathcal{X}}=\{k\mathbf{p},k=0,1,\ \cdots,\ 2^K-1\}\tag{7}$$

where $\mathbf{p}=\frac{2[\Omega_1,\ \cdots,\ \Omega_N]^T}{\Omega(2^K-1)}$. ■

Proof: On one hand, we notice that when $\mathcal{X}=\widetilde{\mathcal{X}}$, given by (7), we can have

$$\min_{0\leq k\leq 2^K-2}\prod_{n=1}^{N}\left(x_{n(k+1)}-x_{nk}\right)^{\Omega_n}=\frac{2^{\Omega}\prod_{n=1}^{N}\Omega_n^{\Omega_n}}{\left(\left(2^K-1\right)\Omega\right)^{\Omega}}\tag{8}$$

On the other hand, we claim that for any constellation $\mathcal{X}$ satisfying the power constraint, the following inequality is true,

$$\min_{0\leq k\leq 2^K-2}\prod_{n=1}^{N}\left(x_{n(k+1)}-x_{nk}\right)^{\Omega_n}\leq\frac{2^{\Omega}\prod_{n=1}^{N}\Omega_n^{\Omega_n}}{\left(\left(2^K-1\right)\Omega\right)^{\Omega}}\tag{9}$$

Otherwise, if there exists a constellation $\mathcal{X}=\breve{\mathcal{X}}=\{\breve{\mathbf{x}}_k,k=0,1,\ \cdots,\ 2^K-1\}$ such that $\min_{0\leq k\leq 2^K-2}\prod_{n=1}^{N}\left(\breve{x}_{n(k+1)}-\breve{x}_{nk}\right)^{\Omega_n}>\frac{2^{\Omega}\prod_{n=1}^{N}\Omega_n^{\Omega_n}}{((2^K-1)\Omega)^{\Omega}}$, then, $\forall k\in\{0,1,\ \cdots,\ 2^K-2\}$, we attain

$$\prod_{n=1}^{N}\left(\breve{x}_{n(k+1)}-\breve{x}_{nk}\right)^{\Omega_n}>\frac{2^{\Omega}\prod_{n=1}^{N}\Omega_n^{\Omega_n}}{\left(\left(2^K-1\right)\Omega\right)^{\Omega}}\tag{10}$$

By Lemma 1, we have $\prod_{n=1}^{N}\left(\breve{x}_{n(k+1)}-\breve{x}_{nk}\right)^{\Omega_n}\leq\sum_{n=1}^{N}\left(\breve{x}_{n(k+1)}-\breve{x}_{nk}\right)^{\Omega}\prod_{n=1}^{N}\left(\frac{\Omega_n}{\Omega}\right)^{\Omega_n}$. Substituting this inequality into (10) yields that $\forall k\in\{0,1,\ldots,2^K-2\}$, $\sum_{n=1}^{N}\breve{x}_{n(k+1)}-\sum_{n=1}^{N}\breve{x}_{nk}>\frac{2}{2^K-1}$. It follows that

$$\sum_{n=1}^{N}\breve{x}_{nk}>\frac{2k}{2^K-1}+k\sum_{n=1}^{N}\breve{x}_{n0}\tag{11}$$

for $k=1,2,\cdots,2^K-1$. Now, summing all the inequalities in (11) yields that $\sum_{k=0}^{2^K-1}\sum_{n=1}^{N}\breve{x}_{nk}>\frac{2\sum_{k=0}^{2^K-1}k}{2^K-1}+\left(1+\frac{2^K\left(2^K-1\right)}{2}\right)\sum_{n=1}^{N}\breve{x}_{n0}=2^K+\left(1+\frac{2^K\left(2^K-1\right)}{2}\right)\sum_{n=1}^{N}\breve{x}_{n0}\geq 2^K$, which contradicts with our power constraint $\frac{1}{2^K}\sum_{k=0}^{2^K-1}\sum_{n=1}^{N}x_{nk}=1$. Thus, inequality (9) is indeed true. Now, combining (8) with (9), the solution (7) is indeed an optimal solution to (5). This completes the proof of Theorem 1. □

Now, subject to the average optical power constraint, the optimal space constellations have been successfully designed for any IM/DD MIMO-OWC with any bit rates for the ML detector using Theorem 1. To further appreciate the optimal design, we would like to make the following three remarks:

1) *Optimal High-Dimensional Space Constellation*. Based on our design criterion, designing the optimal space signaling is essentially finding a high-dimensional space constellation that maximizes the both the large scale and small-scale diversity gains. It turns out that such an optimal space constellation is actually the constellation that allows each aperture to transmit different scaled versions of the same point from an equally-spaced 2^K-ary unipolar pulse amplitude modulation (PAM) constellation. Here, it should be emphasized that the optimal space signaling is attained among all the space signaling schemes. Hence, this optimality of our design is for all space signaling in the sense of maximizing both the small-scale and large-scale diversity gains.
2) *Optimality of RS*. As a spatial diversity transmission scheme, RS is conjectured to be optimal in the sense of error performance for IM/DD MIMO-OWC over log-normal fading channels with equal variances. Despite the fact that all the experimental evidences [9], [12], thus far, have strongly demonstrated that this hypothesis is indeed true, its rigorous mathematical proof remains a long-standing open problem mainly due to the lack of an explicit signal design criterion like MIMO RF communications. Particularly when $\Omega_1 = \cdots = \Omega_N$, Theorem 1 reveals that the corresponding optimal space signaling is exactly RS. Hence, we actually solve this long-standing open problem on the RS optimality in the sense of optimizing both the large scale and small scale diversity gains.
3) *Knowledge of Channel Information From Transmitters*. From Theorem 1, we know that the optimal solution requires that the transmitters only need the knowledge of Ω_n for $n = 1, 2, \cdots, N$ rather than each variance $\sigma_{ij}^2, i = 1, \ \cdots, \ M, j = 1, \ \cdots, \ N$ itself.

V. Numerical Results

In this section, we examine the performance of the optimal space signaling for an IM/DD MIMO-OWC system. The SNR is defined by $\frac{1}{\sigma_{\mathbf{n}}^2}$ with normalized average optical power. All the schemes we would like to compare are described as follows:

1) *Repetition Signaling*. RS [8], [9], [12]–[15] is the best space signaling available in the literature for this application, and the space signal vector of RS for $M \times N$ IM/DD MIMO-OWC with K bits pcu is given by $\mathbf{x}_k = \frac{2k}{N(2^K-1)} \mathbf{1}_{N\times 1}$, where $k \in \{0, 1, \ \cdots, \ 2^K - 1\}$.
2) *Optimal Space Signaling*. This design is proposed in Theorem 1 and the space signal vector is of the form $\mathbf{x}_k = \frac{2k}{\Omega(2^K-1)} \left[\ \Omega_1, \ \cdots, \ \Omega_N \ \right]^T$, where $k \in \{0, 1, \ \cdots, \ 2^K - 1\}$.

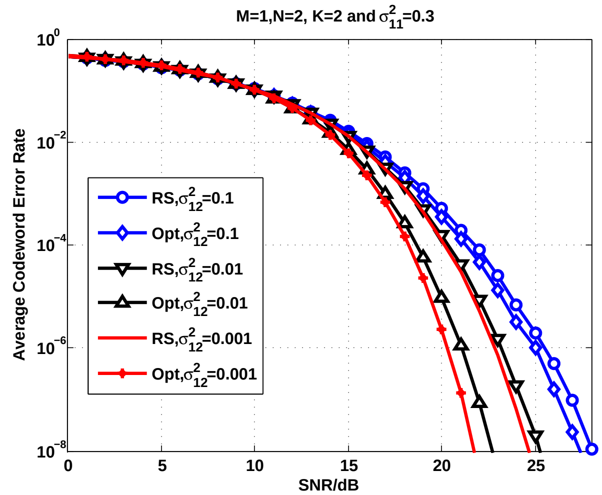

Fig. 1. Average space signal vector ($\mathbf{x}$) error rate comparison of optimal space signaling (Opt) and RS with $M = 1$, $N = 2$ and $K = 2$.

It can be seen that the above two schemes have the same bit rate, i.e., K bits pcu and normalized average optical power. To make a fair comparison, we demodulate the above signalings using ML demodulator. As illustrated in Fig. 1, we find that if $\max_{i\neq j, \Omega_i \geq \Omega_j} \frac{\Omega_i}{\Omega_j}$ becomes larger, then, the more SNR gain of our optimal design over RS is attained. In addition, it can be observed clearly that as SNR increases, the gap between two error curves become larger and larger. This is, again, due to the fact that the optimal space signaling provides larger small-scale diversity gain than RS, and the small-scale diversity gain governs the polynomial decaying speed of the error curves. When $\frac{\Omega_2}{\Omega_1} = \frac{0.3}{0.001} = 300$, as illustrated in Fig. 1, substantial SNR gains can be observed. Specifically when looking at the target codeword error rate of 10^{-5}, we can see that our optimal space signaling obtains about the 2.5 dB SNR gain over RS, which will be expected to become larger if SNR is sufficiently high.

VI. Conclusion and Discussions

In this paper, we have considered the design of the optimal space signaling that maximizes both the large-scale and small-scale diversity gains for a general IM/DD MIMO-OWC system over independent and non-identically log-normal distributed fading channels. By establishing a novel geometrically weighted inequality, we have attained closed-form optimal space signaling. We have shown that the optimal space signaling is RS with an optimal power allocation. Simulations have indicated that our optimal space signaling substantially outperforms RS, which is the best signaling available in literature for this application.

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