

OPTIMAL DESIGN OF CONSTANT-MODULUS CHANNEL TRAINING SEQUENCES

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ABSTRACT

Unimodular sequences have been widely used in communications and radars, for which some numerical algorithms have been proposed recently to obtain good autocorrelation properties [1, 2]. Design of such "good" sequences, however, does not take into account any prior information of the channel to be estimated. Although shaping the autocorrelation of a training sequence may imply a good performance, it may be advantageous to directly optimize the performance measure of interest. In this paper, we consider the problem of optimal constant-modulus training sequence design for MMSE estimation of the channel impulse response and conditional mutual information maximization. Efficient iterative algorithms based on the majorization-minorization framework are proposed for each formulation. Numerical examples show that our proposed training sequences achieve better performances than that of low sidelobes or random phases.

Index Terms— Unimodular sequences, minimum mean square error, conditional mutual information, majorization-minorization.

1. INTRODUCTION

In this paper we consider the problem of constant-modulus training sequence design with prior information incorporated. As a special case, the unimodular sequence has been extensively studied in the literature; see [1] and many references therein. It finds a lot of applications ranging from wireless communications to radars. For instance, the digital modulation techniques like phase-shift keying requires the transmitted symbols to be of constant modulus to satisfy the practical hardware system constraints of RF amplifiers and A/D converters [3].

Recently, some studies have been conducted to construct unimodular sequences of good autocorrelation properties via numerical optimizations; the obtained sequences share impulse-like autocorrelation characteristics. With appropriately chosen metrics, the sequence design problem can be well formulated and then solved by effective optimization techniques. In [1], several cyclic algorithms are proposed for

either minimizing integrated sidelobe level (ISL) or maximizing ISL-related metric called merit factor (MF). Another work [2] further proposes computationally efficient algorithms for the minimization of ISL, and it is demonstrated that the proposed algorithms result in lower autocorrelation sidelobes with less computational complexity.

For channel estimation, sequences of such good autocorrelation properties are desirable only in the ideal and limited situation where no priors of channel impulse response are considered and channel noise is white; thus, a matched filter is employed for post-processing. On the other hand, minimum mean square error (MMSE) estimation and conditional mutual information (CMI) maximization have been widely adopted as criteria of optimal training design for frequency-selective channels [4] and MIMO channels [5], to name a few. With commonly available channel statistics, we model the optimal constant-modulus sequence design problem with respect to MMSE and CMI, respectively. Both formulated problems, however, are non-convex especially with the hard constant-modulus constraint, and even more difficult to solve if without restricting prior channel covariance to some amenable structures. To tackle those issues, we resort to the majorization-minorization (MM) technique [6] to solve the original problem by a series of simpler problems, each of which turns out to have a closed-form solution. An iterative algorithm is thus obtained and will converge to a stationary point. We compare our proposed training sequences with those of low sidelobes or random phases with respect to MSE and CMI. The accelerated algorithms of better convergence properties are also provided with numerical results demonstrated.

2. PROBLEM FORMULATIONS

We consider a quasistatic single-input single-output LTI channel with impulse response $\mathbf{h} = [h_0, \dots, h_K]^T \in \mathbb{C}^{K+1}$ of order K , which follows a circularly complex Gaussian distribution $\mathbf{h} \sim \mathcal{CN}(\mathbf{h}_0, \mathbf{R}_0)$. A length- N constant-modulus training sequence $\mathbf{u} = [u_1, \dots, u_N]^T \in \mathcal{U}$ is to be transmitted for MMSE channel estimation or CMI maximization, where $\mathcal{U} = \{\mathbf{u} \mid |u_n| = \sqrt{\frac{\alpha}{N}}, n = 1, \dots, N\}$ with power constraint $\|\mathbf{u}\|^2 = \alpha$. Assuming an additive noise \mathbf{v} with $\mathbf{v} \sim$

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$\mathcal{CN}(\mathbf{0}, \mathbf{W})$, the received sequence is then given by

$$y_n = \sum_{k=0}^K h_k u_{n-k} + v_n, \quad (1)$$

for $n = 1, \dots, N + K$, where $u_{n-k} = 0$ for $n - k \leq 0$ or $n - k > N$. This input-output relationship can be written in a matrix form as

$$\mathbf{y} = \mathbf{S}\mathbf{h} + \mathbf{v}, \quad (2)$$

where $\mathbf{S} \in \mathbb{C}^{(N+K) \times (K+1)}$ is a Toeplitz convolution matrix with $[u_1, \dots, u_N, 0, \dots, 0]^T$ as the first column, and is denoted by $\mathbf{S} = \mathcal{T}(\mathbf{u})$.

2.1. Optimal Sequence Design by Minimizing MMSE Criterion

Given the linear channel model (2), the MMSE estimator of the channel impulse response is

$$\hat{\mathbf{h}} = \mathbf{R}_0 \mathbf{S}^H (\mathbf{S} \mathbf{R}_0 \mathbf{S}^H + \mathbf{W})^{-1} (\mathbf{y} - \mathbf{S} \mathbf{h}_0) + \mathbf{h}_0, \quad (3)$$

with the error covariance matrix

$$\begin{aligned} \mathbf{R} &= (\mathbf{R}_0^{-1} + \mathbf{S}^H \mathbf{W}^{-1} \mathbf{S})^{-1} \\ &= \mathbf{R}_0 - \mathbf{R}_0 \mathbf{S}^H (\mathbf{S} \mathbf{R}_0 \mathbf{S}^H + \mathbf{W})^{-1} \mathbf{S} \mathbf{R}_0, \end{aligned} \quad (4)$$

where the second equality is by matrix inversion lemma. In this case, the MMSE estimator is also a MAP estimator with conditional distribution $\mathbf{h} | \mathbf{y}, \mathbf{S} \sim \mathcal{CN}(\hat{\mathbf{h}}, \mathbf{R})$. The MMSE given \mathbf{S} is thus

$$\text{MMSE}(\mathbf{S}) = \text{Tr} \left((\mathbf{R}_0^{-1} + \mathbf{S}^H \mathbf{W}^{-1} \mathbf{S})^{-1} \right). \quad (5)$$

The optimal constant-modulus sequence for MMSE minimization can be obtained by solving the problem

$$\underset{\mathbf{u}, \mathbf{S}}{\text{minimize}} \quad \text{MMSE}(\mathbf{S}) \quad \text{subject to} \quad \mathbf{S} = \mathcal{T}(\mathbf{u}), \mathbf{u} \in \mathcal{U}. \quad (6)$$

2.2. Optimal Sequence Design by Maximizing CMI Criterion

Conditional mutual information (CMI) has been used as a criterion in training sequence design problem, e.g., [5], defined as

$$\begin{aligned} \text{CMI}(\mathbf{S}) &= I(\mathbf{h}; \mathbf{y} | \mathbf{S}) \\ &= H(\mathbf{h}) - H(\mathbf{h} | \mathbf{y}, \mathbf{S}) \\ &= \frac{1}{2} \log \det(\mathbf{R}_0 \mathbf{R}^{-1}). \end{aligned} \quad (7)$$

where $H(\cdot)$ is the differential entropy of a distribution [7]. Naturally, the optimal sequence design problem can be formulated by maximizing the CMI as

$$\underset{\mathbf{u}, \mathbf{S}}{\text{maximize}} \quad \text{CMI}(\mathbf{S}) \quad \text{subject to} \quad \mathbf{S} = \mathcal{T}(\mathbf{u}), \mathbf{u} \in \mathcal{U}. \quad (8)$$

3. ALGORITHMS FOR OPTIMAL SEQUENCE DESIGN

Actually there are a lot of works dealing with the same objectives as that of (6) and (8) in regard to \mathbf{S} with only power constraint imposed. Assuming various special structures for the prior channel covariance matrix, the problems are reformulated as power allocation using the majorization theory and the waterfilling solutions are obtained [8, 5]. For our problems, however, the difficulty lies not only in the structure constraint on matrix \mathbf{S} (Toeplitz in this case), but also the bothersome modulus constraint. In addition, it is preferable to avoid those particular structure assumptions on the channel covariance matrix when seeking the solutions, as it finds wider applicability in various channel environments.

To develop efficient algorithms to solve the problems (6) and (8), we refer to the majorization-minorization (MM) framework [6] that has been shown to give a stationary solution with monotonic convergence. Instead of approaching the original problems, a series of problems are solved, each of which is indexed by t hereafter. Let us introduce $\Sigma = \mathbf{S} \mathbf{R}_0 \mathbf{S}^H + \mathbf{W}$. According to equation (4), we have an equivalent form for error covariance matrix $\mathbf{R} = \mathbf{R}_0 - \mathbf{R}_0 \mathbf{S}^H \Sigma^{-1} \mathbf{S} \mathbf{R}_0$, and $\text{Tr}(\mathbf{R})$ is a quadratic-over-linear function. Then $\text{MMSE}(\mathbf{S})$ is jointly concave in \mathbf{S} and Σ [9]. Since a concave function can be majorized by its first order Taylor expansion, we have

$$\begin{aligned} \text{MMSE}(\mathbf{S}) &\leq \text{const} + \text{Tr} \left((\mathbf{A}^{(t)})^H \mathbf{S} \mathbf{R}_0 \mathbf{S}^H \mathbf{A}^{(t)} \right) \\ &\quad - 2 \text{Re} \left\{ \text{Tr} \left(\mathbf{R}_0 (\mathbf{A}^{(t)})^H \mathbf{S} \right) \right\}, \end{aligned} \quad (9)$$

where $\mathbf{A}^{(t)} = (\Sigma^{(t)})^{-1} \mathbf{S}^{(t)} \mathbf{R}_0$, and $\mathbf{S}^{(t)} = \mathcal{T}(\mathbf{u}^{(t)})$. Following the MM framework, the MMSE minimization problem can be solved by a series of problems with (9) as objective functions.

In order to solve the maximization problem (8), we need to minorize the objective function (7). Observe that $\mathbf{R} = \mathbf{R}_0 - \mathbf{R}_0 \mathbf{S}^H \Sigma^{-1} \mathbf{S} \mathbf{R}_0$ is a concave matrix fractional function over the positive semidefinite cone, on which $-\log \det(\cdot)$ is convex and decreasing. Therefore, $\text{CMI}(\mathbf{S})$ is convex in $\{\mathbf{S}, \Sigma\}$, and thus can be minorized by its first order Taylor expansion

$$\begin{aligned} \text{CMI}(\mathbf{S}) &\geq -\frac{1}{2} \left(\text{Tr} \left((\mathbf{R}^{(t)})^{-1} (\mathbf{A}^{(t)})^H \mathbf{S} \mathbf{R}_0 \mathbf{S}^H \mathbf{A}^{(t)} \right) \right. \\ &\quad \left. - 2 \text{Re} \left\{ \text{Tr} \left(\mathbf{R}_0 (\mathbf{R}^{(t)})^{-1} (\mathbf{A}^{(t)})^H \mathbf{S} \right) \right\} \right) + \text{const}, \end{aligned} \quad (10)$$

where $(\mathbf{R}^{(t)})^{-1} = \mathbf{R}_0^{-1} + (\mathbf{S}^{(t)})^H \mathbf{W}^{-1} \mathbf{S}^{(t)}$ by (4), and a series of maximization problems are obtained with the objective (10).

Ignoring the constants, the objectives (9) and (10) share a similar form. And by reversing the sign of the objectives (10), both problems fall into a series of minimization prob-

lems each with the objective function

$$g(\mathbf{S}; \mathbf{S}^{(t)}, \mathbf{V}^{(t)}) = \text{Tr} \left(\mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \mathbf{S} \mathbf{R}_0 \mathbf{S}^H \mathbf{A}^{(t)} \right) - 2\text{Re} \left\{ \text{Tr} \left(\mathbf{R}_0 \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \mathbf{S} \right) \right\} \quad (11)$$

and then we arrive at the following problems

$$\begin{aligned} & \underset{\mathbf{u}, \mathbf{S}}{\text{minimize}} && g(\mathbf{S}; \mathbf{S}^{(t)}, \mathbf{V}^{(t)}) \\ & \text{subject to} && \mathbf{S} = \mathcal{T}(\mathbf{u}), \mathbf{u} \in \mathcal{U}, \end{aligned} \quad (12)$$

where $\mathbf{V}^{(t)} = \mathbf{I}_{K+1}$ for MMSE minimization problem and $\mathbf{V}^{(t)} = (\mathbf{R}_0)^{-1}$ for CMI maximization problem. Notice that the function (11) is quadratic in \mathbf{S} , with

$$\begin{aligned} & \text{Tr} \left(\mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \mathbf{S} \mathbf{R}_0 \mathbf{S}^H \mathbf{A}^{(t)} \right) \\ &= \text{vec}^H(\mathbf{S}) \left(\mathbf{R}_0^T \otimes \mathbf{A}^{(t)} \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \right) \text{vec}(\mathbf{S}) \quad (13) \\ &= \mathbf{x}^H \mathbf{H}^{(t)} \mathbf{x}, \end{aligned}$$

in which $\mathbf{x} = \text{vec}(\mathbf{S})$ and $\mathbf{H}^{(t)} = \mathbf{R}_0^T \otimes \mathbf{A}^{(t)} \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \succeq 0$. Given any constant $\lambda^{(t)} \geq \lambda_{\max}(\mathbf{H}^{(t)})$, we have the following inequality

$$\begin{aligned} \mathbf{x}^H \mathbf{H}^{(t)} \mathbf{x} &\leq -2\text{Re} \left\{ \mathbf{x}^H (\lambda^{(t)} \mathbf{I} - \mathbf{H}^{(t)}) \mathbf{x}^{(t)} \right\} \\ &= +2\lambda^{(t)} \|\mathbf{x}\|^2 - (\mathbf{x}^{(t)})^H \mathbf{H}^{(t)} \mathbf{x}^{(t)}. \end{aligned} \quad (14)$$

Considering the constraint in (12) with $\|\mathbf{x}\|^2 = (K+1)\alpha$ a constant, the following majorization can be applied

$$\begin{aligned} & g(\mathbf{S}; \mathbf{S}^{(t)}, \mathbf{V}^{(t)}) \\ &\leq -2\text{Re} \left\{ \text{vec}^H(\mathbf{S}) (\lambda^{(t)} \mathbf{I} - \mathbf{H}^{(t)}) \text{vec}(\mathbf{S}^{(t)}) \right\} \\ &\quad - 2\text{Re} \left\{ \text{Tr} \left(\mathbf{R}_0 \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \mathbf{S} \right) \right\} \\ &= -2\text{Re} \left\{ \text{Tr} \left((\lambda^{(t)} \mathbf{S}^{(t)} - \mathbf{A}^{(t)} \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \mathbf{S}^{(t)} \mathbf{R}_0 \right. \right. \\ &\quad \left. \left. + \mathbf{A}^{(t)} \mathbf{V}^{(t)} \mathbf{R}_0 \right)^H \mathbf{S} \right\}. \end{aligned} \quad (15)$$

Let $\mathbf{B}(\mathbf{S}^{(t)}, \mathbf{V}^{(t)}) = \lambda^{(t)} \mathbf{S}^{(t)} - \mathbf{A}^{(t)} \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \mathbf{S}^{(t)} \mathbf{R}_0 + \mathbf{A}^{(t)} \mathbf{V}^{(t)} \mathbf{R}_0$. Due to the structure constraint $\mathbf{S} = \mathcal{T}(\mathbf{u})$ in (12), the following problems are obtained

$$\begin{aligned} & \underset{\mathbf{u}, \mathbf{S}}{\text{minimize}} && \text{Re} \left\{ \text{Tr} \left(\left(\sum_{i=1, \dots, K+1} \mathbf{B}_i(\mathbf{S}^{(t)}, \mathbf{V}^{(t)}) \right)^H \mathbf{S} \right) \right\} \\ & \text{subject to} && |u_n| = \sqrt{\frac{\alpha}{N}}, n = 1, \dots, N \end{aligned} \quad (16)$$

where $\mathbf{B}_i(\mathbf{S}^{(t)}, \mathbf{V}^{(t)})$ is a vector consisting of column i and rows $\{i, \dots, i+N-1\}$ of $\mathbf{B}(\mathbf{S}^{(t)}, \mathbf{V}^{(t)})$, for $i = 1, \dots, K+1$. It is obvious that the solution to the above optimization problem is simply

$$\mathbf{u}^{(t+1)} = \sqrt{\frac{\alpha}{N}} e^{j \arg(\sum_{i=1, \dots, K+1} \mathbf{B}_i(\mathbf{S}^{(t)}, \mathbf{V}^{(t)}))}, \quad (17)$$

by projection onto the circle on the complex plane, where $\arg(\cdot)$ is taken in an element-wise way. As an option, we choose $\lambda^{(t)} = \text{Tr}(\mathbf{H}^{(t)})$, and the algorithm is summarized in Algorithm 1.

Algorithm 1 Optimal constant-modulus sequence design.

- 1: Set $t = 0$, and $|u_n| = \sqrt{\frac{\alpha}{N}}, n = 1, \dots, N$.
 - 2: **repeat:**
 - 3: $\mathbf{S}^{(t)} = \mathcal{T}(\mathbf{u}^{(t)})$
 - 4: $\mathbf{A}^{(t)} = (\mathbf{S}^{(t)} \mathbf{R}_0 (\mathbf{S}^{(t)})^H + \mathbf{W})^{-1} \mathbf{S}^{(t)} \mathbf{R}_0$
 - 5: $\mathbf{V}^{(t)} = \mathbf{I}_{K+1}$ for MMSE minimization problem and $\mathbf{V}^{(t)} = \mathbf{R}_0^{-1} + (\mathbf{S}^{(t)})^H \mathbf{W}^{-1} \mathbf{S}^{(t)}$ for CMI maximization problem
 - 6: $\lambda^{(t)} = \text{Tr}(\mathbf{R}_0) \text{Tr}(\mathbf{A}^{(t)} \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H)$
 - 7: $\mathbf{B}(\mathbf{S}^{(t)}, \mathbf{V}^{(t)}) = \lambda^{(t)} \mathbf{S}^{(t)} - \mathbf{A}^{(t)} \mathbf{V}^{(t)} (\mathbf{A}^{(t)})^H \mathbf{S}^{(t)} \mathbf{R}_0 + \mathbf{A}^{(t)} \mathbf{V}^{(t)} \mathbf{R}_0$
 - 8: $\mathbf{u}^{(t+1)} = \sqrt{\frac{\alpha}{N}} e^{j \arg(\sum_{i=1, \dots, K+1} \mathbf{B}_i(\mathbf{S}^{(t)}, \mathbf{V}^{(t)}))}$
 - 9: $t \leftarrow t + 1$
 - 10: **until** convergence
-

3.1. Acceleration via SQUAREM

When developing the Alg. 1, majorization or minorization have been applied twice on the original objective functions, which results in the slow convergence of the update. To accelerate the computational procedures, the easy-to-use SQUAREM method [10] is employed, without the loss of the monotonic convergence property of the MM framework. For simplicity, we will consider the case for MMSE estimation, and it is easy to be extended to the CMI maximization as well. We call the steps 1 to 8 in Alg. 1 one MM update, denoted by $\text{MMupdate}(\mathbf{u}^{(t)})$ given the current iterate $\mathbf{u}^{(t)}$. The accelerated scheme is described in the Alg. 2.

Algorithm 2 Accelerated scheme for optimal constant-modulus sequence design for MMSE estimation.

- 1: Set $t = 0$, and $|u_n| = \sqrt{\frac{\alpha}{N}}, n = 1, \dots, N$.
 - 2: **repeat:**
 - 3: $\mathbf{u}_1 = \text{MMupdate}(\mathbf{u}^{(t)})$
 - 4: $\mathbf{u}_2 = \text{MMupdate}(\mathbf{u}_1)$
 - 5: $\mathbf{r} = \mathbf{u}_1 - \mathbf{u}^{(t)}$
 - 6: $\mathbf{v} = \mathbf{u}_2 - \mathbf{u}_1 - \mathbf{r}$
 - 7: Step length $\alpha = -\frac{\|\mathbf{r}\|}{\|\mathbf{v}\|}$
 - 8: $\mathbf{u} = \sqrt{\frac{\alpha}{N}} e^{j \arg(\mathbf{u}^{(t)} - 2\alpha \mathbf{r} + \alpha^2 \mathbf{v})}$
 - 9: Back-tracking: $\alpha \leftarrow \frac{\alpha-1}{2}$ when $\text{MMSE}(\mathbf{S}) >$
 - $\text{MMSE}(\mathbf{S}^{(t)}) : \mathbf{u} = \sqrt{\frac{\alpha}{N}} e^{j \arg(\mathbf{u}^{(t)} - 2\alpha \mathbf{r} + \alpha^2 \mathbf{v})}$
 - 10: $\mathbf{u}^{(t+1)} = \mathbf{u}$
 - 11: $t \leftarrow t + 1$
 - 12: **until** convergence
-

4. NUMERICAL RESULTS

Numerical results are presented in this section and we will show that by considering the prior information in design-

ing the training sequences, the MMSE estimate and CMI both outperform the random and low-ISL training sequences. For that purpose we compare our training sequences with unimodular sequences obtained by CAN [1] and MISL [2]. We choose channel coefficients $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}_{K+1}, \mathbf{R}_0)$ with $K = 15$, and $(\mathbf{R}_0)_{i,j} = 0.9^{|i-j|} 0.9^{\frac{i-1}{2}} 0.9^{\frac{j-1}{2}}$ for $i, j = 1, \dots, K+1$. The channel is thus correlated with exponentially decreasing power in time delay, which corresponds to the correlated scattering environment with multipath fading in wireless communications [11]. The channel noise is set to be $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}_{N+K}, \mathbf{W})$ with $(\mathbf{W})_{i,j} = 0.2^{|i-j|}$ for $i, j = 1, \dots, N+K$. Then the MSE of the channel estimator is $\text{MSE}(\hat{\mathbf{h}}) = \|\hat{\mathbf{h}} - \mathbf{h}\|_2^2$, and $\text{CMI}(\mathbf{S}) = \frac{1}{2} \log \det(\mathbf{I} + \mathbf{R}_0 \mathbf{S}^H \mathbf{W}^{-1} \mathbf{S})$. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = 10 \log_{10} \frac{\|\mathbf{u}\|^2 / N}{\text{Tr}(\mathbf{W}) / (N + K)} \text{ (dB)}. \quad (18)$$

For different values of SNR, we run 200 times Monte Carlo simulations and the resulting MSE and CMI are averaged.

Figs. 1 and 2 show respectively the averaged MSE and CMI with different training sequences as SNR increases from -5 dB to 10 dB. First, we can see that our proposed training sequences obtained by Alg. 1 and its accelerated algorithm, e.g., MMSE-optimal and MMSE-optimal accel. in MMSE estimation, achieves almost the same performance, and also improves upon low-ISL sequences and sequences of random phases. Fig. 3 demonstrates the convergence rates of Alg. 1 and its accelerations for both MMSE minimization and CMI maximization with SNR = 0 dB. It can be seen that the accelerated algorithms converge very fast.

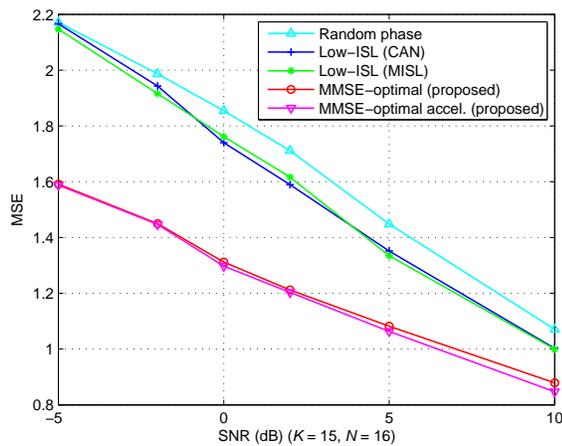


Fig. 1. Averaged MSE of channel estimators for different constant-modulus training sequences.

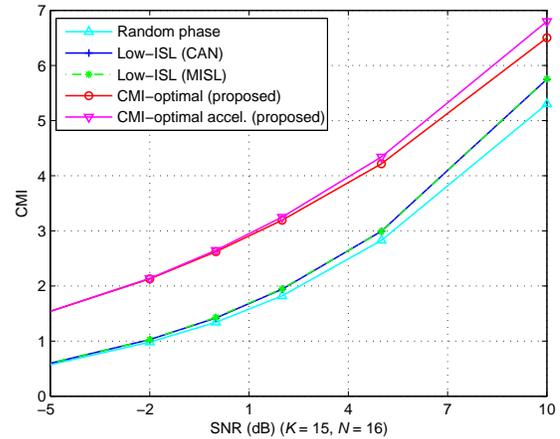


Fig. 2. Averaged CMI for different constant-modulus training sequences.

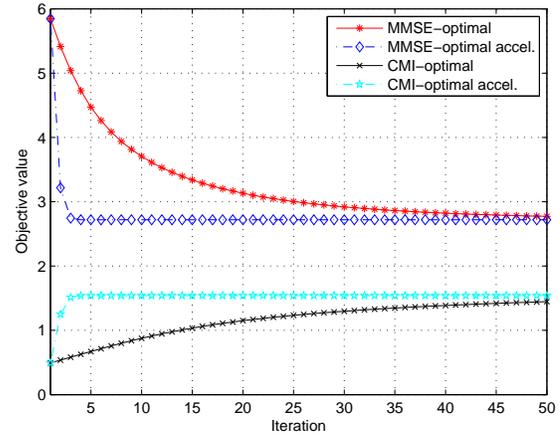


Fig. 3. Convergences of proposed algorithms for different criteria, SNR = 0 dB.

5. CONCLUSION

In this paper the optimization of constant-modulus training sequence for channel estimation is studied. We have formulated the problem for MMSE minimization and CMI maximization and then efficient algorithms based on the majorization-minimization framework have been proposed. The numerical results are also provided showing the advantages of incorporation of prior knowledge into the design of training sequences.

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