PERFORMANCE ANALYSIS OF EWF CODES WITH INTERMEDIATE FEEDBACK

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ABSTRACT

In this paper, we investigate feedback in EWF codes. The considered type of feedback is an acknowledgment, which is a single bit per source block to inform the most important bits have been decoded completely. Moreover, we propose a practical prediction model to calculate the average overheads of successful decoding for different importance classes of the source block. Asymptotic analysis and simulation results demonstrate that, compared with standard EWF codes, the proposed EWF codes with feedback can obtain a lower average overhead of successful decoding in lower prioritized data, and the proposed prediction model is effective.

Index Terms— EWF codes, feedback, prediction model.

1. INTRODUCTION

Fountain codes [1], such as LT codes [2] or Raptor codes [3], are proposed as a flexible and efficient solution for data transmission over packet erasure networks. Unlike traditional coding schemes, fountain codes are rateless codes in the sense that such codes can generate a potentially limitless number of encoded symbols from k input symbols. Theoretically, fountain codes can recover all k input symbols from any $(1+\varepsilon)k$ received encoded symbols, where ε is the reception overhead and slightly larger than 0. These codes can approach the channel capacity for the case of no feedback, regardless of the channel state information. Two classical degree distributions for fountain codes were given in [2] and [3], respectively.

Fountain codes are originally designed to provide equal error protection (EEP) for all information bits. But in some applications, e.g., video streaming, image and science data in deep space communications, the EEP scheme is undesirable. Since these cases call for the error-correcting codes with unequal error protection (UEP) capability. UEP LT codes are found in [4–6].

Traditional fountain codes do not consider any feedback message from the receiver. Recently there are many significant researches on fountain codes with the feedback. With the feedback message about the number of input symbols already decoded at the receivers, A. Hagedorn et al. [7] modify the degree distribution of LT codes to obtain a better performance. In [8], authors present an improved LT code with decreasing ripple size based on the feedback of the number of input symbols already recovered. Jesper et al. [9] study the impact of feedback in LT codes, including both EEP LT codes and weighted UEP LT codes [4]. In [10], they further analyze quantitatively the expected redundancy during the transmission. Analysis demonstrates that a rational and effective use of a small amount of feedback information will benefit the performance.

In [10], the authors consider point-to-point communication, where it is reasonable to utilize the feedback because of easy obtainment of the feedback channel, and restrict the feedback to only a single bit per k input symbols. Firstly, motivated by [10] and considering that expanding window fountain (EWF) codes [5] are more flexible than weighted LT codes, we study EWF codes in combination with intermediate feedback in this paper. We assume similar communication scenario and same-type feedback. In this work, feedback is considered in the form of acknowledgments (ACKs). Secondly, we propose a practical prediction model to calculate the average overheads of completely recovering different importance classes of the source block, which has never been investigated by previous works (including [10]). E.g., in [1] and [5] the average overhead is achieved by the simulation rather than the analytical method. Compared with the state-of-the-art technique of [10], analysis method of our scheme can be efficient for large k. However, in [10] for large k, the complexity is prohibitive, so the order of k is just 10^2.

The rest of this paper is organized as follows. Section 2 reviews EWF codes. Section 3 presents the analytical work of our scheme, followed by a numerical analysis in section 4. Section 5 gives the simulation results.

2. RELATED WORK

In this section, we mainly introduce EWF codes. More details can be found in [5].

EWF codes utilize an expanding window technique to achieve UEP property. Assume that, for video streaming

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application, a source block having k input symbols is transmitted over an erasure channel. Suppose this source block is partitioned into r classes s_1, s_2,$\cdots$,s_r of length k_1, k_2,$\cdots$, k_r, such that $k_1+k_2+\cdots+k_r = k$. The division is compactly described by the generating polynomial $\Pi(x) = \sum_{i=1}^{r} \Pi_i x^i$, where $\Pi_i = k_i/k$. Further assume that the importance of classes decreases with the class index, i.e., the ith class is more important than the jth class when $i < j$. The concept of window is introduced, e.g., r windows W_1, W_2, $\cdots$, W_r. These expanding windows, where each window is contained in the next window, are defined over the source block. The tth window W_t consists of the blocks of the classes s_1, s_2, $\cdots$, s_t. Namely the length of W_t is $L_t = \sum_{j=1}^{t} k_j$. For every window $W_t (t = 1, 2, \cdots, r)$, an LT code with a degree distribution $\Omega^{(t)}(x) = \sum_i \Omega_i^{(t)} x^i$ is defined. To generate an encoded symbol, a window is firstly chosen at random with respect to a probability distribution $\Gamma(x) = \sum_{i=1}^{r} \Gamma_i x^i$, where Γ_i is the probability that the ith window is chosen. E.g., the jth window W_j is chosen, then the chosen window is encoded by using the LT code with the degree distribution $\Omega^{(j)}(x) = \sum_i \Omega_i^{(j)} x^i$. UEP is achieved by choosing suitable degree distribution for every window and appropriate values for $\Gamma_1, \cdots, \Gamma_r$.

Lemma 1. Suppose an EWF code $F_{EW}(\Pi, \Gamma, \Omega^{(1)}, \cdots, \Omega^{(r)})$ with a fixed reception overhead ε, i.e., with a total of $(1+\varepsilon)k$ encoded symbols collected at the receiver, and let $y_{l,j}(\varepsilon)$ be the probability that an input symbol in s_j is not recovered after l belief propagation (BP) decoding iterations for the overhead ε. Then, we have $y_{l,j}(\varepsilon)$ for $l > 0$ and $y_{0,j}(\varepsilon) = 1$:

$$y_{l,j}(\varepsilon) = e^{\left(-(1+\varepsilon)\sum_{i=j}^{r} \frac{\Gamma_i}{\sum_{t=1}^{i}\Pi_t}\Omega^{(i)\prime}\left(1-\frac{\sum_{m=1}^{i}\Pi_m y_{l-1,m}(\varepsilon)}{\sum_{t=1}^{i}\Pi_t}\right)\right)}. \tag{1}$$

The particularly simple scenario is that the set of input symbols is divided into two importance classes, i.e., most important bits (MIB) and least important bits (LIB). In the following, we will show a special case of **Lemma 1** for this scenario and analyze the impact of adding feedback in the form of an ACK in EWF codes with two importance classes.

3. PROPOSED WORK

In this section, we consider a special case of two importance classes of input symbols, i.e., MIB and LIB, and analyze the EWF codes with intermediate feedback. First the density evolution formulas of EWF codes with two importance classes are presented, followed by our proposed EWF codes with intermediate feedback.

3.1. EWF Codes ($r = 2$)

Let $y_{l,M}(\varepsilon)$ and $y_{l,L}(\varepsilon)$ be the asymptotic erasure probabilities (as the source block length $k \to \infty$) of MIB and LIB after l BP decoding iterations for the reception overhead ε, respectively. For an EWF code $F_{EW}(\Pi_1 x + \Pi_2 x^2, \Gamma_1 x + \Gamma_2 x^2, \Omega^{(1)}, \Omega^{(2)})$, from **Lemma 1** we obtain the erasure probabilities of MIB and LIB after l iterations, $y_{l,M}(\varepsilon)$ and $y_{l,L}(\varepsilon)$, respectively, for $l > 0$ and $y_{0,M}(\varepsilon) = y_{0,L}(\varepsilon) = 1$:

$$\begin{aligned} y_{l,M}(\varepsilon) =& e^{\left(-(1+\varepsilon)\left(\frac{\Gamma_1}{\Pi_1}\Omega^{(1)\prime}\left(1-y_{l-1,M}(\varepsilon)\right)\right.\right.} \\ & {}^{\left.\left.+\Gamma_2\Omega^{(2)\prime}\left(1-\Pi_1 y_{l-1,M}(\varepsilon)-\Pi_2 y_{l-1,L}(\varepsilon)\right)\right)\right)} \end{aligned} \tag{2}$$

$$y_{l,L}(\varepsilon) = e^{\left(-(1+\varepsilon)\Gamma_2\Omega^{(2)\prime}\left(1-\Pi_1 y_{l-1,M}(\varepsilon)-\Pi_2 y_{l-1,L}(\varepsilon)\right)\right)}. \tag{3}$$

In the limit $l \to \infty$, let us call the corresponding fixed points as $y_M(\varepsilon)$ and $y_L(\varepsilon)$, respectively.

3.2. EWF Codes ($r = 2$) with Feedback

In our proposed scheme, an ACK is transmitted over the feedback channel to inform the transmitter when MIB have been completely recovered. Thus, only LIB are considered in the future encoding. This just needs a single feedback message, which makes this scheme practical. We assume that the cost of transmitting the ACK is not taken into account and lossless feedback channel is considered [10].

Algorithm 1 summarizes our proposed EWF coding with intermediate feedback.

Algorithm 1 EWF Coding with Intermediate Feedback

Initialization: ACK=0, *Source1* consists of both MIB and LIB while *Source2* just contains LIB. *Encoder1* is the EWF encoder for the case of $r = 2$, and *Encoder2* is the normal LT encoder

Encode *Source1* with *Encoder1*
Send the encoded symbols over the transmission channel
Collect a number of encoded symbols and perform BP decoding
while ACK=0 **do**
    if MIB are completely recovered **then**
        Set ACK=1 and send ACK to the transmitter over the feedback channel
    else
        Keep collecting extra encoded symbols and performing BP decoding
    end if
end while
Encode *Source2* with *Encoder2*
Send the encoded symbols over the transmission channel
Collect encoded symbols and perform BP decoding

Lemma 2. For an EWF code $F_{EW}(\Pi_1 x + \Pi_2 x^2, \Gamma_1 x + \Gamma_2 x^2, \Omega^{(1)}, \Omega^{(2)})$, assume that MIB are completely decoded while collecting $N' = (1+\varepsilon')k$ encoded symbols at the receiver, and set $y_L(\varepsilon')$ be the erasure probability of LIB for ε'.

After transmitter successfully receives the ACK, let $y_{l,MF}(\varepsilon)$ and $y_{l,LF}(\varepsilon)$ denote the erasure probabilities of MIB and LIB for the overhead ε ($\geq \varepsilon'$) at the lth decoding iteration, respectively. $y_{l,MF}(\varepsilon) = 0$, and $y_{l,LF}(\varepsilon)$ is obtained for $l > 0$ and $y_{0,LF}(\varepsilon) = y_L(\varepsilon')$.

$$\begin{aligned} y_{l,LF}(\varepsilon) &\\ = e^{\left(-(1+\varepsilon')\Gamma_2\Omega^{(2)\prime}\left(1-\Pi_1 y_{l-1,MF}(\varepsilon)-\Pi_2 y_{l-1,LF}(\varepsilon)\right)\right)} &\\ \cdot e^{\left(-\frac{\varepsilon-\varepsilon'}{\Pi_2}\Omega^{(2)\prime}\left(1-y_{l-1,LF}(\varepsilon)\right)\right)}. & \end{aligned} \quad (4)$$

Similarly, in the limit $l \to \infty$, we call the corresponding fixed points of $y_{l,MF}(\varepsilon)$ and $y_{l,LF}(\varepsilon)$ as $y_{MF}(\varepsilon)$ and $y_{LF}(\varepsilon)$, respectively.

4. NUMERICAL ANALYSIS

In this section, we provide a numerical example for an EWF code $F_{EW}(\Pi_1 x + \Pi_2 x^2, \Gamma_1 x + \Gamma_2 x^2, \Omega^{(1)}, \Omega^{(2)})$ with intermediate feedback, and show that the simple use of a feedback message can significantly decrease the asymptotic average overhead of completely recovering LIB.

4.1. EWF codes without Feedback

Using $y_M(\varepsilon)$ and $y_L(\varepsilon)$ for a given overhead ε, and under the asymptotic assumption that the symbol erasure probabilities for different input symbols of same importance are independent, we can calculate the asymptotic frame erasure rate (FER), which is defined as the probability that one importance class of the source block is not completely recovered, i.e., $FER_M(\varepsilon)$ $\left(FER_L(\varepsilon)\right)$ is probability that MIB (LIB) are not recovered completely at ε. Then

$$FER_M(\varepsilon) = 1 - (1 - y_M(\varepsilon))^{k_1} \quad (5)$$

$$FER_L(\varepsilon) = 1 - (1 - y_L(\varepsilon))^{k_2} \quad (6)$$

where k_1 and k_2 are the number of MIB and LIB, respectively.

Here we will give an example of the calculations of both $FER_M(\varepsilon)$ and $FER_L(\varepsilon)$ for various overheads, as shown in Fig. 1. For both $\Omega^{(1)}$ and $\Omega^{(2)}$, we consider the same degree distribution adopted by [4](originally proposed in [3]):

$$\begin{aligned} \Omega(x) =& 0.007969x + 0.49357x^2 \\ &+ 0.16622x^3 + 0.072646x^4 \\ &+ 0.082558x^5 + 0.056058x^8 + 0.037229x^9 \\ &+ 0.05559x^{19} + 0.025023x^{65} + 0.003135x^{66}. \end{aligned} \quad (7)$$

Fig. 1 shows $FER_M(\varepsilon_i) > FER_M(\varepsilon_j)$ if $\varepsilon_i < \varepsilon_j$. We denote by $P_M(\varepsilon_j = \text{“}S\text{”}, \varepsilon_i = \text{“}F\text{”})$ the probability that complete recovery of MIB fails at ε_i but succeeds at ε_j. Similarly, $P_M(\varepsilon_j)$ ($P_M(\varepsilon_i)$) denotes the probability of completely recovering MIB at ε_j (ε_i). We have the equation as follows:

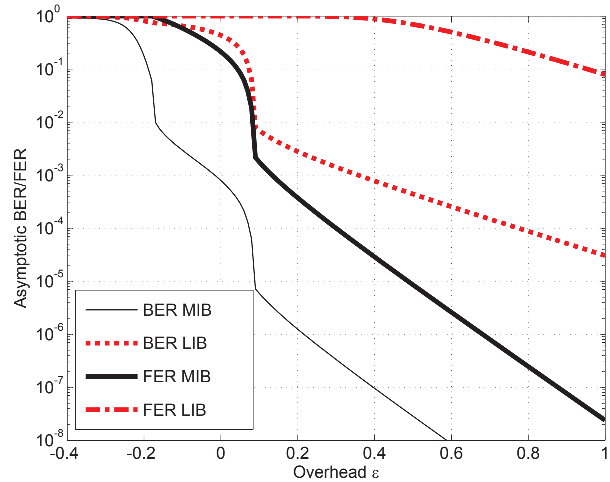

Fig. 1. Asymptotic bit erasure rate (BER) and FER versus the overhead ε with $k = 3000$, $\Pi_1 = 0.1$ and $\Gamma_1 = 0.11$.

$$P_M(\varepsilon_j) = P_M(\varepsilon_i) + P_M(\varepsilon_j = \text{“}S\text{”}, \varepsilon_i = \text{“}F\text{”}) \quad (8)$$

where $P_M(\varepsilon_j) = 1 - FER_M(\varepsilon_j)$ and $P_M(\varepsilon_i) = 1 - FER_M(\varepsilon_i)$. Then

$$\begin{aligned} & P_M(\varepsilon_j = \text{“}S\text{”}, \varepsilon_i = \text{“}F\text{”}) \\ &= \Big(1 - FER_M(\varepsilon_j)\Big) - \Big(1 - FER_M(\varepsilon_i)\Big) \\ &= FER_M(\varepsilon_i) - FER_M(\varepsilon_j) \end{aligned} \quad (9)$$

where “S” (“F”) means that MIB are (are not) recovered completely at the corresponding overhead. For the convenience in the rest of this paper, we will use $P_M(\varepsilon_j, \bar{\varepsilon}_i)$ to indicate $P_M(\varepsilon_j = \text{“}S\text{”}, \varepsilon_i = \text{“}F\text{”})$.

The asymptotic average overhead ε_M^{avg} necessary to completely recover MIB is obtained as follows:

$$\varepsilon_M^{avg} = \varepsilon_1\big(1 - FER_M(\varepsilon_1)\big) + \sum_{i=2}^{\infty} \varepsilon_i P_M(\varepsilon_i, \bar{\varepsilon}_{i-1}). \quad (10)$$

The definitions and calculations of probability $P_L(\varepsilon_j, \bar{\varepsilon}_i)$ and average overhead ε_L^{avg} for LIB are similar to that of $P_M(\varepsilon_j, \bar{\varepsilon}_i)$ and ε_M^{avg}.

4.2. EWF codes with Feedback

While considering the utilization of feedback, we have the average overhead ε_{LF}^{avg} necessary to successfully decode LIB:

$$\begin{aligned} \varepsilon_{LF}^{avg} =& \varepsilon_{LF}^{con}(\varepsilon_1)\big(1 - FER_M(\varepsilon_1)\big) \\ &+ \sum_{i=2}^{\infty} \varepsilon_{LF}^{con}(\varepsilon_i) P_M(\varepsilon_i, \bar{\varepsilon}_{i-1}) \end{aligned} \quad (11)$$

Table 1. Average Overheads while MIB and LIB are Recovered completely, respectively. The results are obtained from Asymptotic Numerical Analysis (ANA) and Finite-Length Simulation (FLS), respectively.

			$\Pi_1 = 0.1$				$\Pi_1 = 0.3$			
			$\Gamma_1 = 0.084$		$\Gamma_1 = 0.11$		$\Gamma_1 = 0.084$		$\Gamma_1 = 0.11$	
			ANA	FLS	ANA	FLS	ANA	FLS	ANA	FLS
$k = 2000$	no feedback	MIB	0.004	0.015	-0.092	-0.067	0.103	0.121	0.085	0.105
		LIB	0.518	0.504	0.569	0.551	0.478	0.463	0.521	0.505
	w. feedback	MIB	0.004	0.015	-0.092	-0.067	0.103	0.120	0.085	0.105
		LIB	0.428	0.419	0.438	0.425	0.348	0.343	0.359	0.354
$k = 3000$	no feedback	MIB	0.015	0.016	-0.069	-0.066	0.136	0.141	0.110	0.118
		LIB	0.595	0.573	0.642	0.624	0.549	0.538	0.594	0.583
	w. feedback	MIB	0.015	0.016	-0.069	-0.066	0.136	0.138	0.110	0.118
		LIB	0.494	0.479	0.501	0.487	0.405	0.395	0.414	0.403

where $\varepsilon_{LF}^{con}(\varepsilon_i)$ is the average overhead necessary to completely decode LIB, on condition that MIB have been completely recovered at ε_i:

$$\varepsilon_{LF}^{con}(\varepsilon_i) = \varepsilon_i\big(1 - FER_{LF}(\varepsilon_i)\big) + \sum_{j=i+1}^{\infty} \varepsilon_j P_{LF}(\varepsilon_j, \bar{\varepsilon}_{j-1}) \tag{12}$$

where $FER_{LF}(\varepsilon_i) = 1-(1-y_{LF}(\varepsilon_i))^{k_2}$, and the calculation of $P_{LF}(\varepsilon_j, \bar{\varepsilon}_{j-1})$ resembles that of $P_M(\varepsilon_j, \bar{\varepsilon}_{j-1})$. Notably, the selection of ε_1 in (11) is on the basis of such a fact that $FER_M(\varepsilon_1)$ enables $1 - FER_M(\varepsilon_1)$ to approach 0.

By using relevant formulas above-mentioned, we track the asymptotic average overheads of successful decoding for both MIB and LIB. Some parameter selections of k, Γ_1 and Π_1 are listed in Table 1. Besides, the overhead is in step of 0.001. The calculated asymptotic average overheads are shown in Table 1. Note that for same parameters (i.e., k, Π_1 and Γ_1), the average overheads of MIB are same in the schemes of both standard EWF codes without feedback and our proposed EWF codes with feedback. The analysis reveals that using a single feedback message significantly decreases the successful decoding overhead in LIB, e.g., for $k = 3000$, $\Pi_1 = 0.1$ and $\Gamma_1 = 0.11$, the decrement reaches 0.141.

5. SIMULATION RESULTS AND DISCUSSION

In order to verify the results of asymptotic analysis developed in Section 4, we present simulations to determine the average overheads of MIB and LIB, respectively. For normal EWF codes and proposed codes with feedback, simulation parameters correspond to that of asymptotic analysis in Table 1. We consider the same degree distribution (7) for both $\Omega^{(1)}$ and $\Omega^{(2)}$. Every average overhead is computed over 10000 simulations. The simulation results are also shown in Table 1.

During the procedure of simulation, fountain codes use incremental redundancy decoding mechanism, which means that the decoder will receive redundant 10 encoded bits in case of every decoding failure and then continues to decode until the corresponding information bits are completely recovered.

By comparing the results in Table 1, we find that the simulation results are consistent with that of the asymptotic analysis and the average relative errors are less than 0.025 . In other words, the proposed prediction model for the calculation of average decoding overhead turns out to be effective.

Similarly, the impact of feedback in the form of an ACK in UEP LT codes, termed weighted UEP LT codes, is studied in [10]. This scheme analyzes the expected amount of redundant symbols in the transmission and dramatically decreases the redundancy in LIB, but can not predict the average overhead necessary to completely recover the input bits. Correspondingly, we consider same-type feedback in EWF codes and apply And-Or analysis [11] to investigate the impact of the feedback on the successful decoding of LIB. Our scheme also can be easily extended to the weighted UEP LT codes.

6. CONCLUSION

In this paper, we study the impact of feedback in the form of an ACK, which is a single bit per source block, in EWF codes. Additionally, we propose a practical prediction model to calculate the successful decoding overhead in corresponding prioritized data. This model applies to the scenario without/with feedback. Asymptotic analysis and simulation results show that, compared with standard EWF codes, our proposed EWF codes with feedback can obtain a lower average overhead of successful decoding in LIB, and the proposed prediction model is effective. Further works will focus on the research of our proposed scheme in the real-world scalable video unicast.

7. REFERENCES

[1] D. J. C. MacKay, "Fountain codes," *IEE Proc. Commun.*, vol. 152, no. 6, pp. 1062-1068, Dec. 2005.

[2] M. Luby, "LT codes," in *Proc. 43rd Annu. IEEE Symp. on Foundations of Computer Science(FOCS)*, Vancouver, BC, Canda, Nov. 2002, pp. 271-280.

[3] A. Shokrollahi, "Raptor codes," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2551-2567, Jun. 2006.

[4] N. Rahnavard, B. N. Vellambi, and F. Fekri, "Rateless codes with unequal error protection property," *IEEE Trans. Inf. Theory*, vol. 53, no.4, pp. 1521-1532, Apr. 2007.

[5] D. Sejdinović, D. Vukobratović, A. Doufexi, V. Šenk, and R. J. Piechocki, "Expanding Window Fountain Codes for Unequal Error Protection," *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2510-2516, Sep. 2009.

[6] S. Ahmad, R. Hamzaoui, and M. Al-Akaidi, "Unequal Error Protection using Fountain Codes with Applications to Video Communication," *IEEE Trans. Multimedia*, vol. 13, no. 1, pp. 92-101, Feb. 2011.

[7] A. Hagedorn, S. Agarwal, D. Starobinski, and A. Trachtenberg, "Rateless coding with feedback," in *IEEE INFOCOM*, 2009, pp. 1791-1799.

[8] L. Zhang, J. Liao, J. Wang, T. Li, and Q. Qi, "Design of improved Luby transform codes with decreasing ripple size and feedback," in *IET Commun.*, 2014, Vol. 8, Iss. 8, pp. 1409-1416.

[9] J. Sørensen, P. Popovski, and J. Østergaard, "Feedback in LT codes for prioritized and non-prioritized data," in *Proc. 2012 IEEE Vehicular Technology Conference - Fall*, pp. 1-5.

[10] J. Sørensen, P. Popovski, and J. Østergaard, "UEP LT Codes with Intermediate Feedback," *IEEE Communications Letters*, vol. 17, no. 8, pp. 1636-1639, Agu. 2013.

[11] M. Luby, M. Mitzenmacher, and A. Shokrollahi, "Analysis of random processes via and-or tree evaluation," in *Proc. 9th SIAM Symp. Discrete Algorithms(SODA)*, Jan. 1998, pp. 364-373