Signal Detection of Ambient Backscatter System with Differential Modulation

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Abstract—We study the problem of signal detection for the ambient backscatter system (ABS) when data are transmitted with differential modulation. An implementation of the maximum likelihood (ML) detection algorithm is proposed. To reduce the computational complexity, we further design a suboptimal detector and derive its bit error rate (BER) closed-form expression. Moreover, both the upper and the lower bounds of the BER, which can tell more insight of how system parameters affect the detection performance, are obtained. Simulations are then provided to corroborate the studies.

Index Terms—ambient backscatter, differential modulation, signal detection, performance analysis, BER bound

I. INTRODUCTION

Ambient radio frequency (RF) signals have been widely used for energy harvesting [1], which captures and recycles environmental energy, such as broadcast TV, radio and cellular signals, to operate low-power devices. It could also be used for the simultaneous wireless information and power transfer (SWIPT) [2], [3] where the receivers not only harvest the RF signal but also decode the information carried by the signal.

Recently, a novel communication mechanism called the ambient backscatter system (ABS) that employs ambient RF signals in backscatter communication was introduced in [4]. Traditional backscatter system [5], i.e., the radio frequency identification (RFID), consists of a reader (the transmitter/receiver) and a tag (the backscatter node). The reader usually generates a carrier signal, a part of which is harvested to power the tag while the remainder signal will be backscattered by the tag to the reader. However, ABS differs from RFID in two aspects: (1) it makes use of ambient RF signals but does not require a special-purpose infrastructure (e.g., a RFID reader) to transmit signals; (2) it enables communications almost everywhere and any time.

Following [4], connecting ambient backscatter tags with the Internet via the existing Wi-Fi infrastructure was proposed in [6], while the multi-antenna interference cancellation scheme operating on the backscatter devices was proposed in [7] are proposed. Nevertheless, these works mainly focus on the hardware design and the prototype presentation but did not provide the fundamental results from theoretical aspects.

Motivated by this, we propose a theoretical implementation of the optimal maximum likelihood (ML) detector for an ABS with differential modulation. As the ML detector may suffer from high complexity, we next design a suboptimal detector



Fig. 1. A three-node ABS consisting of an RF signal source, a passive tag and a reader with differential modulation and demodulation at the tag and the reader.

with an approximate detection threshold and derive the closedform bit error rate (BER). To further analyze how the system parameters can affect the detection performance, we compute the upper and the lower bound of the BER. A practical approach to estimate the parameters required by the detectors is also presented. Finally simulation results are provided to demonstrate the effectiveness of the proposed detectors.

II. SYSTEM MODEL

Consider an ABS with differential modulation which consists of an RF source, a passive tag and a reader, as depicted in Fig. 1. Denote h_{st} , h_{sr} and h_{tr} as the coefficients of the channels from the source to the tag, the source to the reader, and the tag to the reader, respectively. All these channel information are assumed unknown in our study. In this model, the RF signal transmitted from the ambient source can be received by both the tag and the reader.

The received signal at the tag from the source is

$$x[n] = h_{st}s[n],\tag{1}$$

where s[n] is the RF signal transmitted from the sources and satisfies $s[n] \sim C\mathcal{N}(0, P_s)$. Since the on-tag integrated circuit only consists of passive components and involves little signal processing, the noise at the tag is negligible [8].

The k-th symbol of the tag d_k is differentially encoded at the tag as

$$b_k = b_{k-1} \otimes d_k, \tag{2}$$

where b_k is the k-th modulated symbol with the reference symbol $b_0 = 1$, and \otimes represents addition modulo 2. We assume

that d_k has the equiprobability of being 0 and 1. Normally the tag transmits at a much lower rate than the ambient RF signal, then b_k remains unchanged for N consecutive s[n]. The backscattered signal by the tag is then

$$x_b[n] = \alpha c[n]x[n], \tag{3}$$

where c[n] is given as

$$c[n] = b_k, n = (k-1)N + 1, \cdots, kN,$$
 (4)

and α is a real scaling term related to scattering efficiency and antenna gain at a given direction [9].

The reader receives the superposition of the RF signal transmitted from the source and the signal backscattered from the tag, then we can get the received signal at the reader as

$$y[n] = h_{sr}s[n] + h_{tr}x_b[n] + w[n], = (h_{sr} + \alpha h_{st}h_{tr}c[n])s[n] + w[n],$$
(5)

where w[n] is the zero-mean additive white Gaussian noise (AWGN) with variance N_w .

Remark 1: The time delay between the arriving of s[n] and $x_b[n]$ at the reader can be ignored because [4]: (1) the transmission speed of the electrical signal inside tag is as fast as light speed; (2) the communication range of the RF-powered devices is limited.

III. SIGNAL DETECTION

For notation simplicity, let us denote $\boldsymbol{y}_k = [y[(k-1)N + 1], \cdots, y[kN]]^T$ as the k-th received signal vector at the reader corresponding to the k-th modulated symbol b_k .

A. Optimal Detector

The optimal ML detector can be derived from the joint probability density function (PDF) of y_k and y_{k-1} conditioned on b_k and b_{k-1} . Define $r = [y_{k-1}, y_k]^T$, then we have

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{w},\tag{6}$$

where $\boldsymbol{s} = [s[(k-2)N+1], \cdots, s[kN]]^T$, $\boldsymbol{w} = [w[(k-2)N+1], \cdots, w[kN]]^T$, and

$$\boldsymbol{H} = \begin{bmatrix} t_{k-1}\boldsymbol{I}_N & \boldsymbol{0} \\ \boldsymbol{0} & t_k\boldsymbol{I}_N \end{bmatrix}, \qquad (7)$$

where $t_k = h_{sr} + \alpha b_k h_{st} h_{tr}$ and I_N is the N-order unit matrix.

Clearly, r is a complex Gaussian vector given b_{k-1} and b_k , i.e.,

$$p(\boldsymbol{r}|b_{k-1}, b_k) = \frac{1}{(2\pi)^{2N} \det(\boldsymbol{C})} \exp\left\{-\boldsymbol{r}^H \boldsymbol{C}^{-1} \boldsymbol{r}\right\}, \quad (8)$$

where

$$\boldsymbol{C} = \begin{bmatrix} \xi_{k-1} \boldsymbol{I}_N & \boldsymbol{0} \\ \boldsymbol{0} & \xi_k \boldsymbol{I}_N \end{bmatrix}, \qquad (9)$$

with $\xi_k = |t_k|^2 P_s + N_w$. There exists an one-to-one correspondence between b_k and ξ_k : if $b_k = 0$, then $\xi_k = \sigma_0^2$; if $b_k = 1$, then $\xi_k = \sigma_1^2$, where

$$\sigma_0^2 \triangleq |h_0|^2 P_s + N_w, \ \ \sigma_1^2 \triangleq |h_1|^2 P_s + N_w,$$
 (10)

with $h_0 = h_{sr}$ and $h_1 = h_{sr} + \alpha h_{st} h_{tr}$. Though the reader does not have the channel knowledge, the values of σ_i^2 can be estimated as will be presented in Section III-D.

Let $Z_k = \|\boldsymbol{y}_k\|^2$, then (8) can be further expanded as

$$p(\boldsymbol{r}|\xi_{k-1},\xi_k) = \frac{(2\pi)^{-2N}}{(\xi_{k-1}\xi_k)^N} \exp\left\{-\frac{Z_{k-1}}{\xi_{k-1}} - \frac{Z_k}{\xi_k}\right\}.$$
 (11)

Then the optimal detector can be formulated as

$$[\hat{\xi}_{k-1}, \hat{\xi}_k] = \arg \max_{\xi_{k-1}, \xi_k \in \{\sigma_0^2, \sigma_1^2\}} p(\boldsymbol{r}|\xi_{k-1}, \xi_k), \quad (12)$$

and

$$\begin{cases} \hat{d}_k = 0, & \text{if } \hat{\xi}_{k-1} = \hat{\xi}_k, \\ \hat{d}_k = 1, & \text{if } \hat{\xi}_{k-1} \neq \hat{\xi}_k. \end{cases}$$

The detection rule here is similar to the traditional differential demodulation, but unfortunately we have to do the joint detection based on two consecutive received signal vectors.

From the total probability theorem, the PDFs of r under the two hypotheses are

$$p(\mathbf{r}|d_{k}=0) = \frac{p(\mathbf{r}|\xi_{k-1}=\xi_{k}=\sigma_{0}^{2}) + p(\mathbf{r}|\xi_{k-1}=\xi_{k}=\sigma_{1}^{2})}{2}$$

$$p(\mathbf{r}|d_{k}=1) = \frac{p(\mathbf{r}|\xi_{k-1}=\sigma_{0}^{2},\xi_{k}=\sigma_{1}^{2}) + p(\mathbf{r}|\xi_{k-1}=\sigma_{1}^{2},\xi_{k}=\sigma_{0}^{2})}{2}$$
(13)

where $p(\mathbf{r}|\xi_{k-1},\xi_k)$ is given by (11).

Therefore, we can summarize the algorithm for the optimal ML detector in Algorithm 1 as

Algorithm 1

Require:

The k-1-th and k-th received signal vectors at the reader, y_{k-1} and y_k .

Ensure:

The k-th transmitted symbol by the tag, d_k ;

- 1: Calculate the two consecutive signal energies as $Z_{k-1} = \|\boldsymbol{y}_{k-1}\|^2$ and $Z_k = \|\boldsymbol{y}_k\|^2$;
- 2: Substitute Z_{k-1} and Z_k into (13) to obtain $p(\mathbf{r}|d_k = 0)$ and $p(\mathbf{r}|d_k = 1)$;

3: If $p(\mathbf{r}|d_k = 0) > p(\mathbf{r}|d_k = 1)$, $\hat{d}_k = 0$; otherwise, $\hat{d}_k = 1$; 4: return \hat{d}_k ;

B. Suboptimal Detector

From Algorithm 1, we have to do two PDF calculations before every symbol detection. We next propose a suboptimal detector with simple implementation.

From (11), we see that Z_k is the key statistics in the detection. Clearly, Z_k is a central chi-square random variable with 2N degrees of freedom. From the central limit theorem, when N is large, Z_k asymptotically becomes a Gaussian random variable, denoted by \tilde{Z}_k ,

The decision rule is based on the difference between two adjacent Z_k 's. The detector must identify whether there is a significant change in $|V_k| = |Z_k - Z_{k-1}|$, i.e., if $|V_k| \ge T_h$,

then $\hat{d}_k = 1$; otherwise, $\hat{d}_k = 0$. For clarity, we further denote $\tilde{Z}_{k|i}$ as the \tilde{Z}_k under the assumption of $b_k = i$, and define $V_{k|ij}$ as $V_{k|ij} = \tilde{Z}_{k|j} - \tilde{Z}_{k-1|i}$. Then we can obtain

$$\tilde{Z}_{k|i} \sim \mathcal{N}(N\sigma_i^2, N\sigma_i^4), \quad i = 0, 1.$$
(14)

It can be readily checked that

$$V_{k|00} \sim \mathcal{N}(0,\varsigma_1^2), \quad V_{k|11} \sim \mathcal{N}(0,\varsigma_2^2),$$

 $V_{k|01} \sim \mathcal{N}(\mu,\varsigma_3^2), \quad V_{k|10} \sim \mathcal{N}(-\mu,\varsigma_3^2),$ (15)

where $\varsigma_1^2 = 2N\sigma_0^4$, $\varsigma_2^2 = 2N\sigma_1^4$, $\varsigma_3^2 = N(\sigma_0^4 + \sigma_1^4)$, and $\mu = N(\sigma_1^2 - \sigma_0^2)$.

Denote \mathcal{H}_0 and \mathcal{H}_1 as the hypotheses that $d_k = 0$ and $d_k = 1$, respectively. The PDFs of $|V_k|$ under the two hypotheses are given as

$$p_{|V_k|}(v|\mathcal{H}_0) = p_{|V_k|00|}(v) + p_{|V_k|11|}(v)$$

= $\frac{1}{\sqrt{2\pi\varsigma_1^2}} e^{-\frac{v^2}{2\varsigma_1^2}} + \frac{1}{\sqrt{2\pi\varsigma_2^2}} e^{-\frac{v^2}{2\varsigma_2^2}},$ (16)

$$p_{|V_k|}(v|\mathcal{H}_1) = p_{|V_k|01|}(v) + p_{|V_k|10|}(v)$$

= $\frac{1}{\sqrt{2\pi\varsigma_3^2}} e^{-\frac{(v-\mu)^2}{2\varsigma_3^2}} + \frac{1}{\sqrt{2\pi\varsigma_3^2}} e^{-\frac{(v+\mu)^2}{2\varsigma_3^2}}.$ (17)

Thus, the decision can be alternatively made through

$$p_{|V_k|}(v|\mathcal{H}_0) \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} p_{|V_k|}(v|\mathcal{H}_1) \Leftrightarrow |V_k| \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\lessgtr}} T_h.$$
(18)

Compared the with optimal detector, the suboptimal one is obtained with reduced computational complexity, by transforming the calculation and comparison of two PDFs to the comparison with threshold.

Unfortunately, there does not exist a closed-form solution for the inequality (18). Since the PDF in (16) can be approximated as the PDF of the absolute value of a single Gaussian random variable with zero-mean and the variance of ς_3^2 , i.e.,

$$p_{|V_k|}(v|\mathcal{H}_0) \approx \frac{2}{\sqrt{2\pi\varsigma_3^2}} e^{-\frac{v^2}{2\varsigma_3^2}},$$
 (19)

we can obtain the threshold T_h in (18) as

$$T_h = \frac{|\mu|}{2} + \frac{\varsigma_3^2 \ln\left(1 + \sqrt{1 - e^{-\mu^2/\varsigma_3^2}}\right)}{|\mu|}.$$
 (20)

It can be seen that as N becomes large, T_h can be approximated as

$$T_h \approx T_h^{apx} = \frac{|\mu|}{2}.$$
 (21)

Remark 3: From (15), we can derive that $E\{|V_k|\} = |\mu|/2$. Thus, $E\{|V_k|\}$ can be a practical alternative to the detection threshold which could be directly obtained at the reader as the number of k is large.

C. BER Performance

According to (18), the BER of the suboptimal detector is given as

$$\begin{split} P_{b} &= \frac{1}{2} \Pr(|V_{k}| > T_{h}^{apx} | \mathcal{H}_{0}) + \frac{1}{2} \Pr(|V_{k}| < T_{h}^{apx} | \mathcal{H}_{1}) \\ &= \frac{1}{2} \int_{T_{h}^{apx}}^{\infty} p_{|V_{k}|}(v | \mathcal{H}_{0}) \mathrm{d}v + \frac{1}{2} \int_{0}^{T_{h}^{apx}} p_{|V_{k}|}(v | \mathcal{H}_{1}) \mathrm{d}v \\ &= \frac{1}{2} \left[Q \left(\frac{|\mu|}{2\sqrt{\zeta_{1}^{2}}} \right) + Q \left(\frac{|\mu|}{2\sqrt{\zeta_{2}^{2}}} \right) + Q \left(\frac{|\mu|}{2\sqrt{\zeta_{3}^{2}}} \right) - Q \left(\frac{3|\mu|}{2\sqrt{\zeta_{3}^{2}}} \right) \right] \\ &= \frac{1}{2} \left[Q \left(\frac{\sqrt{N}\Delta}{2\sqrt{2}(|h_{0}|^{2} + 1/\gamma)} \right) + Q \left(\frac{\sqrt{N}\Delta}{2\sqrt{2}(|h_{1}|^{2} + 1/\gamma)} \right) \right. \\ &\left. + Q \left(\frac{\sqrt{N}\Delta}{2\sqrt{\Sigma_{1} + \Sigma_{2}/\gamma + 2/\gamma^{2}}} \right) - Q \left(\frac{3\sqrt{N}\Delta}{2\sqrt{\Sigma_{1} + \Sigma_{2}/\gamma + 2/\gamma^{2}}} \right) \right] \end{split}$$

$$(22)$$

where Q(x) is the tail probability of the standard normal distribution, and

$$\gamma = \frac{P_s}{N_w}, \qquad \Delta = \left| |h_1|^2 - |h_0|^2 \right|,$$

$$\Sigma_1 = |h_0|^4 + |h_1|^4, \quad \Sigma_2 = 2(|h_0|^2 + |h_1|^2). \tag{23}$$

Furthermore, we define $\varsigma_{\max}^2 = \max\{\varsigma_1^2, \varsigma_2^2\}$ and $\varsigma_{\min}^2 = \min\{\varsigma_1^2, \varsigma_2^2\}$. there is $\varsigma_{\min}^2 \leq \varsigma_3^2 \leq \varsigma_{\max}^2$. Since Q(x) is a decreasing function of x, we can obtain

$$P_{b} \geq \frac{3}{2}Q\left(\frac{|\mu|}{2\sqrt{\zeta_{\min}^{2}}}\right) - \frac{1}{2}Q\left(\frac{3|\mu|}{2\sqrt{\zeta_{\max}^{2}}}\right) \triangleq P_{bl}$$
$$P_{b} \leq \frac{3}{2}Q\left(\frac{|\mu|}{2\sqrt{\zeta_{\max}^{2}}}\right) \triangleq P_{bu}$$
(24)

where P_{bl} and P_{bu} are the lower and the upper bounds of P_b , respectively.

From (22) and (24), we can see that the BER performance is a decreasing function of SNR γ , the length of the received vector N, and the relative channel difference (RCD) $\frac{\Delta}{\sqrt{\Sigma_{T}}}$.

D. Estimation of σ_0^2 and σ_1^2

Note that σ_0^2 and σ_1^2 are required for both detectors (13) and (21). We then propose an approach to estimate them. Assume that the channel coherent time spans M transmitted symbols of the tag. Then the estimation method is described as follows: Step 1: Compute the normalized energy of M symbols as $A_k = ||\boldsymbol{y}_k||^2/N$ for $k = 1, \dots, M$.

Step 2: Arrange
$$A_k$$
 in ascending order, denoted as A_k^{\uparrow} .

Step 3: Considering the equiprobability of 0 and 1, the reader computes

$$A_{\min} = \frac{2}{M} \sum_{k=1}^{M/2} A_k^{\uparrow}, A_{\max} = \frac{2}{M} \sum_{k=M/2+1}^{M} A_k^{\uparrow}, \quad (25)$$

which are the two estimated results.

Remark 2: Seen from (13) and (21), we do not need to judge which of the two values $(A_{\min} \text{ and } A_{\max})$ should be assigned which to σ_i^2 , which eliminates the need of training.



Fig. 2. The BER versus SNR for the two detectors with N=100 and RCD = 0.5.



Fig. 3. BER versus RCD for the two detectors with SNR = 10dB and N = 100.

IV. NUMERICAL RESULTS

In this section, we resort to numerical examples to evaluate the proposed studies. The channels and the AGWN are assumed to follow $\mathcal{CN}(0,1)$, and the channels hold unchanged during 100 transmitted symbols of the tag, i.e., M = 100. The tag coefficient α is set to 0.5. The detection thresholds of the suboptimal detector are set as T_h^{apx} and $E\{|V_k|\}$, respectively. Totally 10^6 Monte-Carlo runs are adopted for average.

We first demonstrate the BER performance of the optimal ML detector and the suboptimal detector in Fig. 2. The theoretical BER in (22) is also displayed for comparison. We set N = 100 and RCD = 0.5. We can see that the suboptimal detector performs worse than the optimal ML detector due to the Gaussian approximation. For the suboptimal detector, the theoretical BER is perfect consistent with the simulated results and the threshold $E\{|V_k|\}$ outperforms the threshold



Fig. 4. BER, the upper and the lower bounds of BER versus N for the suboptimal detector with SNR = 10dB and RCD = 0.5.

 T_h^{apx} especially for large SNR. It can also be found that the higher SNR leads to the reduced BER. However, for the suboptimal detector, the performance improvement flattens as SNR is relatively large, say above 15 dB, which verifies (22) that as γ turns infinity, P_b is nearly uncontrolled by SNR.

Fig. 3 depicts the curves of BER versus RCD of the two detectors. We set SNR = 10dB and N = 100. The BER approaches to 0.5 at small RCD and there exists little gap between the BER curves. We can infer that when RCD is too small, all the detectors will fail to work with the poorest detection performance. It can be also seen that larger RCD will totally improve the detection performance regardless of detectors.

Fig. 4 depicts the curves of BER versus N for the suboptimal detector when SNR = 10dB and RCD = 0.5. For comparison, the upper and the lower bound of the BER (24) are also plotted. It can be seen that the two simulated BERs both approach the theoretical BER very well and $E\{|V_k|\}$ outperforms the threshold T_h^{apx} as N becomes large. Besides, the two bounds are both accurate.

V. CONCLUSION

This paper mainly considered the signal detection of the ABS with differential modulation. We proposed a theoretical algorithm implementation of the optimal ML detector. To reduce the computational complexity, we designed a suboptimal detector with the approximate detection threshold and derived the corresponding closed-form BER expression. The upper and the lower bound of the BER were also obtained. Moreover, we presented a method to estimate the parameters required by the two detectors. Finally simulations were provided to verify the theoretical results, where the larger the SNR, the N, and the RCD are, the better performance the detectors can achieve.

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