Blind Channel Estimation in OFDM-Based Amplify-and-Forward Two-Way Relay Networks

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***Abstract*— In this paper, we propose a blind channel estimation algorithm for amplify-and-forward two-way relay networks (AF-TWRN). The orthogonal frequency division multiplexing (OFDM) modulation is adopted for frequency selective channel. The channels are estimated in two steps. First the cascaded channel causing the self-interference is estimated using a proposed power reduction method. Then the other cascaded channel from source to destination is estimated by subspace method. Closed form formulas are derived and the numerical results are provided.**

***Index Terms*—blind channel estimation, orthogonal frequency division multiplexing (OFDM), two-way relay network (TWRN).**

I. Introduction

Research on wireless relay networks becomes popular recently. In particular, the two-way relay network (TWRN) has drawn a lot of attention because its overall communication rate is approximately twice of that achieved in the one-way relay network (OWRN) [1]. In this paper, we study the channel estimation problem in amplify-and-forward (AF) TWRN [2]. Many methods have been proposed for channel estimation in AF-TWRN. These methods can be divided into two groups: data-aided [3]-[6] and non data-aided (blind) [7]-[10]. This paper focuses on blind estimation. In [7], the authors propose a maximum likelihood (ML) approach to estimate the flat-fading channels blindly, but the transmitted signals are limited on constant modulus modulation. [8] finds a closed form solution and thus provides a low-complexity ML algorithm. For non-constant modulus modulation, [9] gives an iterative algorithm, which is based on the maximum a posteriori (MAP) approach and requires a large number of received blocks. In [10], the authors consider the frequency selective environment. They apply a non-unitary linear precoding at both terminals and derive a blind channel estimation algorithm from second-order statistics of the received signals. However, the use of non-unitary linear precoding leads to degradation in bit error rate (BER) performance.

In this paper, we develop a blind channel estimation algorithm for TWRN under orthogonal frequency division multiplexing (OFDM) modulation. Our method contains two steps. The first step is to estimate the cascaded channel causing the self-interference. Since the terminal knows its own transmitted signal, we propose a method based on power reduction to estimate the channel. The second step is to estimate the cascaded channel from source to destination. We utilize the subspace method [11]. Closed form formulas for these two cascaded channel estimates are derived. Simulation is provided to show the merit of the proposed method.

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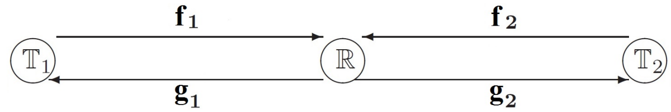

Fig. 1: System configuration for two-way relay network

The rest of this paper is organized as follows. The system model for OFDM-based AF-TWRN is introduced in Section II. Section III describes the proposed algorithm for blind channel estimation. Simulation results are presented in Section IV and concluding remarks are drawn in Section V.

Notation: In this paper, $\mathrm{E}\{x\}$ means the statistical expectation of the random variable x. The symbols $\mathbf{A}^T$, $\mathbf{A}^*$, and $\mathbf{A}^\dagger$ denote the transpose, the complex conjugate, and the conjugate-transpose of matrix $\mathbf{A}$ respectively. $\|\mathbf{A}\|_F$ is the Frobenius norm of matrix $\mathbf{A}$. $\mathbf{I}_m$ is the $m \times m$ identity matrix, whereas $\mathbf{0}$ represents an all-zero matrix with appropriate dimension. $\jmath = \sqrt{-1}$ is the imaginary unit. $\mathbf{T}_m(\mathbf{c})$ is an $m \times (m+n-1)$ Toeplitz matrix with first column $[c_n, \mathbf{0}_{1\times(m-1)}]^T$ and first row $[c_n, \ldots, c_1, \mathbf{0}_{1\times(m-1)}]$, and $\tilde{\mathbf{T}}_m(\mathbf{c})$ is an $(m+n-1) \times m$ Toeplitz matrix with first column $[c_1, \ldots, c_n, \mathbf{0}_{1\times(m-1)}]^T$ and first row $[c_1, \mathbf{0}_{1\times(m-1)}]$, where $\mathbf{c} = [c_1, c_2, \ldots, c_n]^T$ is an arbitrary vector.

II. System model

Consider a TWRN with two terminal nodes $\mathbb{T}_1$ and $\mathbb{T}_2$, and one relay node $\mathbb{R}$, as shown in Fig. 1. Each node has one antenna which cannot transmit and receive simultaneously. The channel from $\mathbb{T}_i$ to $\mathbb{R}$ is denoted as $\mathbf{f}_i = [f_{i,0}, f_{i,1}, \ldots, f_{i,L}]^T$, whereas the one from $\mathbb{R}$ back to $\mathbb{T}_i$ is denoted as $\mathbf{g}_i = [g_{i,0}, g_{i,1}, \ldots, g_{i,L}]^T$ for $i = 1$ and 2. For notational simplicity, we assume that the lengths of $\mathbf{f}_1$, $\mathbf{f}_2$, $\mathbf{g}_1$, and $\mathbf{g}_2$ do not exceed $L+1$.[1] Similar to most other algorithms, we assume that the channels do not change when the channel estimation is performed.

A. OFDM modulation at terminals

Denote the kth OFDM block from $\mathbb{T}_i$ as $\mathbf{s}_k^{(i)} = [s_{k,0}^{(i)}, s_{k,1}^{(i)}, \ldots, s_{k,N-1}^{(i)}]^T$, where N is the OFDM block length.

[1]The proposed method can be applied to the more general case of different channel lengths by simply using an appropriate cyclic prefix length.

The corresponding time-domain signal block is obtained from the normalized inverse discrete Fourier transform (IDFT) as

$$\mathbf{x}_k^{(i)} = \mathbf{W}^\dagger \mathbf{s}_k^{(i)} = \left[\begin{array}{cccc} x_{k,0}^{(i)} & x_{k,1}^{(i)} & \cdots & x_{k,N-1}^{(i)} \end{array} \right]^T, \quad (1)$$

where $\mathbf{W}$ is the $N \times N$ normalized DFT matrix with the (m,n)th entry given by $\frac{1}{\sqrt{N}} e^{-\jmath 2\pi mn/N}$. To maintain the subcarrier orthogonality during the overall transmission, we propose to add a cyclic prefix (CP) of length $2L$. Define $\mathbf{x}_{k,cp}^{(i)} = [x_{k,N-2L}^{(i)}, \ldots, x_{k,N-1}^{(i)}]^T$. The signal sent out from $\mathbb{T}_i$ is expressed as $[\ \mathbf{x}_{k,cp}^{(i)T} \ \ \mathbf{x}_k^{(i)T}\ ]^T$ for $i = 1$ and 2.

B. Relay processing

The relay $\mathbb{R}$ receives the signal

$$\mathbf{r}_k = \left[\begin{array}{c} r_{k,0} \\ r_{k,1} \\ \vdots \\ r_{k,N+2L-1} \end{array} \right] = \sum_{i=1}^{2} \mathbf{T}_{N+2L}(\mathbf{f}_i) \left[\begin{array}{c} \mathbf{x}_{k-1,isi}^{(i)} \\ \mathbf{x}_{k,cp}^{(i)} \\ \mathbf{x}_k^{(i)} \end{array} \right] + \mathbf{n}_{k,r}, \quad (2)$$

where $\mathbf{x}_{k-1,isi}^{(i)} = [x_{k-1,N-L}^{(i)}, \ldots, x_{k-1,N-1}^{(i)}]^T$ is the term which causes the inter-symbol interference (ISI). Moreover, each element in the noise vector $\mathbf{n}_{k,r}$ is assumed to be independent and identically distributed (i.i.d.) zero-mean complex white Gaussian, with variance $\sigma_{n_r}^2$.

We assume that the relay $\mathbb{R}$ employs the amplify-and-forward scheme. It scales $\mathbf{r}_k$ by the factor of

$$\alpha = \sqrt{\frac{P_r}{\mathrm{E}\{\|\mathbf{r}_k\|_F^2\}}} = \sqrt{\frac{P_r}{\|\mathbf{f}_1\|_F^2 \sigma_1^2 + \|\mathbf{f}_2\|_F^2 \sigma_2^2 + \sigma_{n_r}^2}}, \quad (3)$$

where P_r is the average transmission power of $\mathbb{R}$. In the second equality, we have made the assumptions that the transmitted signals $\mathbf{x}_k^{(1)}$ and $\mathbf{x}_k^{(2)}$ are uncorrelated with variances σ_1^2 and σ_2^2 respectively. Then the relay broadcasts $\alpha \mathbf{r}_k$ to both terminals.

C. Signal reformulation at terminals

Due to symmetry, we only illustrate the processing at $\mathbb{T}_1$. The $(N+2L) \times 1$ vector received at $\mathbb{T}_1$ can be expressed as

$$\mathbf{y}_k = \left[\begin{array}{c} y_{k,0} \\ y_{k,1} \\ \vdots \\ y_{k,N+2L-1} \end{array} \right] = \mathbf{T}_{N+2L}(\mathbf{g}_1) \left[\begin{array}{c} \alpha \mathbf{r}_{k-1,isi} \\ \alpha \mathbf{r}_k \end{array} \right] + \mathbf{n}_{k,t}, \quad (4)$$

where $\mathbf{r}_{k-1,isi} = [r_{k-1,N+L}, \ldots, r_{k-1,N+2L-1}]^T$, and each element in the noise vector $\mathbf{n}_{k,t}$ is assumed to be i.i.d. zero-mean complex white Gaussian, with variance $\sigma_{n_t}^2$. Substituting (2) into (4), we have

$$\mathbf{y}_k = \mathbf{T}_{N+2L}(\mathbf{h}_1) \left[\begin{array}{c} \mathbf{x}_{k-1,cp}^{(1)} \\ \mathbf{x}_{k,cp}^{(1)} \\ \mathbf{x}_k^{(1)} \end{array} \right] + \mathbf{T}_{N+2L}(\mathbf{h}_2) \left[\begin{array}{c} \mathbf{x}_{k-1,cp}^{(2)} \\ \mathbf{x}_{k,cp}^{(2)} \\ \mathbf{x}_k^{(2)} \end{array} \right] + \mathbf{n}_{k,e}, \quad (5)$$

where $\mathbf{h}_1 = \alpha(\mathbf{g}_1 * \mathbf{f}_1)$ and $\mathbf{h}_2 = \alpha(\mathbf{g}_1 * \mathbf{f}_2)$ with $*$ being the linear convolution between two vectors, and $\mathbf{n}_{k,e}$ denotes the equivalent noise. Note that $\mathbf{n}_{k,e}$ is not white. However, when $N \gg L$, it can be approximated as white noise.

D. Data detection at terminals

After removing the first $2L$ elements of $\mathbf{y}_k$ in (5), we obtain a vector of size N:

$$\bar{\mathbf{y}}_k = \mathbf{T}_N(\mathbf{h}_1) \left[\begin{array}{c} \mathbf{x}_{k,cp}^{(1)} \\ \mathbf{x}_k^{(1)} \end{array} \right] + \mathbf{T}_N(\mathbf{h}_2) \left[\begin{array}{c} \mathbf{x}_{k,cp}^{(2)} \\ \mathbf{x}_k^{(2)} \end{array} \right] + \bar{\mathbf{n}}_{k,e}, \quad (6)$$

where $\bar{\mathbf{n}}_{k,e}$ is the last N elements of $\mathbf{n}_{k,e}$. If the cascaded channel $\mathbf{h}_1$ is known to $\mathbb{T}_1$, then the first term on the right-hand side of (6) can be removed since $\mathbb{T}_1$ knows its own signal $\mathbf{x}_k^{(1)}$. If $\mathbf{h}_2$ is known, the regular OFDM detection can be efficiently performed using fast Fourier transform. So $\mathbb{T}_1$ can recover the data from $\mathbb{T}_2$ if both $\mathbf{h}_1$ and $\mathbf{h}_2$ are available. Hence, our goal is to estimate $\mathbf{h}_1$ and $\mathbf{h}_2$. Below we will show how to blindly estimate these two cascaded channels from the received signal $\mathbf{y}_k$.

III. Proposed method for channel estimation

In this paper, we assume that $\mathbf{x}_k^{(1)}$ and $\mathbf{x}_k^{(2)}$ are uncorrelated. Moreover, the transmitted signals and the noises are uncorrelated as well. Under these two assumptions, we propose an algorithm to estimate $\mathbf{h}_1$ and $\mathbf{h}_2$ blindly.

A. The estimation of $\mathbf{h}_1$

To estimate the $(2L+1) \times 1$ vector $\mathbf{h}_1$, our method is based on power reduction. Define a cost function

$$J(\hat{\mathbf{h}}_1) = \mathrm{E}\{ \left\| \bar{\mathbf{y}}_k - \mathbf{T}_N(\hat{\mathbf{h}}_1) \left[\begin{array}{c} \mathbf{x}_{k,cp}^{(1)} \\ \mathbf{x}_k^{(1)} \end{array} \right] \right\|_F^2 \}, \quad (7)$$

where $\bar{\mathbf{y}}_k$ is the $N \times 1$ vector in (6) and $\hat{\mathbf{h}}_1$ is an estimate of $\mathbf{h}_1$. Substituting (6) into (7) and simplifying the expression, we have

$$\begin{aligned} J(\hat{\mathbf{h}}_1) &= N(\sigma_1^2 \|\mathbf{h}_1 - \hat{\mathbf{h}}_1\|_F^2 + \sigma_2^2 \|\mathbf{h}_2\|_F^2 + |\alpha|^2 \sigma_{n_r}^2 \|\mathbf{g}_1\|_F^2 + \sigma_{n_t}^2) \\ &\geq N\left(\sigma_2^2 \|\mathbf{h}_2\|_F^2 + |\alpha|^2 \sigma_{n_r}^2 \|\mathbf{g}_1\|_F^2 + \sigma_{n_t}^2\right). \end{aligned} \quad (8)$$

Obviously, the cost function has the minimum if and only if $\|\mathbf{h}_1 - \hat{\mathbf{h}}_1\|_F^2 = 0$, or equivalently, $\hat{\mathbf{h}}_1 = \mathbf{h}_1$. Assume that $\mathbb{T}_1$ has collected K blocks. Then (7) can be approximated as

$$\begin{aligned} \bar{J}(\hat{\mathbf{h}}_1) &= \frac{1}{K} \sum_{k=0}^{K-1} \left\| \bar{\mathbf{y}}_k - \mathbf{T}_N(\hat{\mathbf{h}}_1) \left[\begin{array}{c} \mathbf{x}_{k,cp}^{(1)} \\ \mathbf{x}_k^{(1)} \end{array} \right] \right\|_F^2 \\ &= \frac{1}{K} \sum_{k=0}^{K-1} \left\| \bar{\mathbf{y}}_k - \sqrt{N} \mathbf{W}^\dagger \mathbf{D}(\mathbf{s}_k^{(1)}) \mathbf{W}_{2L+1} \hat{\mathbf{h}}_1 \right\|_F^2, \end{aligned} \quad (9)$$

where $\mathbf{D}(\mathbf{s}_k^{(1)})$ is a diagonal matrix with the elements of $\mathbf{s}_k^{(1)}$ on the main diagonal, and $\mathbf{W}_{2L+1}$ is the first $2L+1$ columns of the DFT matrix $\mathbf{W}$. Define that

$$\mathbf{y} = \left[\begin{array}{cccc} \bar{\mathbf{y}}_0^T & \bar{\mathbf{y}}_1^T & \cdots & \bar{\mathbf{y}}_{K-1}^T \end{array} \right]^T \quad (10)$$

and

$$\mathbf{S} = \left[\begin{array}{cccc} \mathbf{D}(\mathbf{s}_0^{(1)}) & \mathbf{D}(\mathbf{s}_1^{(1)}) & \cdots & \mathbf{D}(\mathbf{s}_{K-1}^{(1)}) \end{array} \right]^T. \quad (11)$$

Then (9) can be rewritten as

$$\bar{J}(\hat{\mathbf{h}}_1) = \frac{1}{K} \left\| \mathbf{y} - \sqrt{N} (\mathbf{I}_K \otimes \mathbf{W}^\dagger) \mathbf{S} \mathbf{W}_{2L+1} \hat{\mathbf{h}}_1 \right\|_F^2, \quad (12)$$

where the symbol $\otimes$ denotes the Kronecker product. The least squares solution of (12) can be calculated as

$$\hat{\mathbf{h}}_1 = \frac{1}{\sqrt{N}} \left(\mathbf{W}_{2L+1}^\dagger \mathbf{S}^\dagger \mathbf{S} \mathbf{W}_{2L+1} \right)^{-1} \mathbf{W}_{2L+1}^\dagger \mathbf{S}^\dagger (\mathbf{I}_K \otimes \mathbf{W}) \mathbf{y}. \tag{13}$$

When K is large enough, we have $\mathbf{S}^\dagger \mathbf{S} \approx K\sigma_1^2 \mathbf{I}_N$. In this case, (13) can be approximated as

$$\hat{\mathbf{h}}_1 \approx \frac{1}{\sqrt{N} K \sigma_1^2} \mathbf{W}_{2L+1}^\dagger \sum_{k=0}^{K-1} (\mathbf{s}_k^{(1)})^* \odot (\mathbf{W} \bar{\mathbf{y}}_k), \tag{14}$$

where the symbol $\odot$ denotes the Hadamard product. Notice that there is no scalar ambiguity in the estimation of $\mathbf{h}_1$ since $\mathbf{s}_k^{(1)}$ and $\bar{\mathbf{y}}_k$ are known at $\mathbb{T}_1$.

B. The estimation of $\mathbf{h}_2$

In order to estimate $\mathbf{h}_2$, we first remove the self-interfering signal from the received vector. Define

$$\mathbf{z}_k = \mathbf{y}_k - \mathbf{T}_{N+2L}(\hat{\mathbf{h}}_1) \begin{bmatrix} \mathbf{x}_{k-1,cp}^{(1)} \\ \mathbf{x}_{k,cp}^{(1)} \\ \mathbf{x}_k^{(1)} \end{bmatrix}. \tag{15}$$

Assuming that the estimation of $\mathbf{h}_1$ is perfect (i.e. $\hat{\mathbf{h}}_1 = \mathbf{h}_1$), from (5) and (15) we have

$$\mathbf{z}_k = \mathbf{T}_{N+2L}(\mathbf{h}_2) \begin{bmatrix} \mathbf{x}_{k-1,cp}^{(2)} \\ \mathbf{x}_{k,cp}^{(2)} \\ \mathbf{x}_k^{(2)} \end{bmatrix} + \mathbf{n}_{k,e}. \tag{16}$$

Note that the vector $\mathbf{z}_k$ is simply the received vector in an usual OFDM system with channel $\mathbf{h}_2$ and transmitted vector $\mathbf{x}_k^{(2)}$. Many blind estimation methods have been proposed for the estimation of $\mathbf{h}_2$ from $\mathbf{z}_k$. Below, we will adopt the subspace based algorithm in [11]. Define the re-modulated vector $\tilde{\mathbf{z}}_k = [z_{k-1,2L}, \ldots, z_{k-1,N+2L-1}, z_{k,0}, \ldots, z_{k,2L-1}]^T$, where $z_{k,i}$ is the ith entry of $\mathbf{z}_k$. Then we construct the vector

$$\mathbf{v}_k = \mathbf{z}_k - \tilde{\mathbf{z}}_k. \tag{17}$$

Substituting (5) and (15) into (17), we have

$$\mathbf{v}_k = \tilde{\mathbf{T}}_N(\mathbf{h}_2)(\begin{bmatrix} \mathbf{x}_{k,cp}^{(2)} \\ \mathbf{x}_{k,up}^{(2)} \end{bmatrix} - \mathbf{x}_{k-1}^{(2)}) + \boldsymbol{\eta}_k \triangleq \tilde{\mathbf{T}}_N(\mathbf{h}_2)\mathbf{d}_k + \boldsymbol{\eta}_k, \tag{18}$$

where $\mathbf{x}_{k,up}^{(2)}$ is the first $N-2L$ elements of $\mathbf{x}_k^{(2)}$ and $\boldsymbol{\eta}_k$ is color noise. Therefore, we carry out the whitening process [11] to get $\mathbf{R}_w^{-1/2}\mathbf{v}_k$, where

$$\mathbf{R}_w^{-1/2} = \begin{bmatrix} c_1 \mathbf{I}_{2L} & \mathbf{0} & c_2 \mathbf{I}_{2L} \\ \mathbf{0} & \frac{1}{\sqrt{2}} \mathbf{I}_{N-2L} & \mathbf{0} \\ c_2 \mathbf{I}_{2L} & \mathbf{0} & c_1 \mathbf{I}_{2L} \end{bmatrix} \tag{19}$$

with $c_1 = \sqrt{\frac{2/3+\sqrt{1/3}}{2}}$ and $c_2 = \sqrt{\frac{2/3-\sqrt{1/3}}{2}}$. After whitening, the covariance matrix of $\mathbf{R}_w^{-1/2}\mathbf{v}_k$ can be represented as

$$\begin{aligned} \mathrm{E}\{\mathbf{R}_w^{-1/2}\mathbf{v}_k\mathbf{v}_k^\dagger\mathbf{R}_w^{-1/2}\} =& \mathbf{R}_w^{-1/2}\tilde{\mathbf{T}}_N(\mathbf{h}_2)\mathbf{R}_d\tilde{\mathbf{T}}_N^\dagger(\mathbf{h}_2)\mathbf{R}_w^{-1/2} \\ &+ \sigma_{n_e}^2 \mathbf{I}_{N+2L}, \end{aligned} \tag{20}$$

where $\mathbf{R}_d = \mathrm{E}\{\mathbf{d}_k\mathbf{d}_k^\dagger\}$ is the covariance matrix of $\mathbf{d}_k$ defined in (18) and $\sigma_{n_e}^2$ is the average power of $\mathbf{n}_{k,e}$. Utilizing eigenvalue decomposition, (20) can be computed as

$$\mathrm{E}\{\mathbf{R}_w^{-1/2}\mathbf{v}_k\mathbf{v}_k^\dagger\mathbf{R}_w^{-1/2}\} = \mathbf{U}_s\boldsymbol{\Sigma}\mathbf{U}_s^\dagger + \sigma_{n_e}^2\mathbf{U}_o\mathbf{U}_o^\dagger, \tag{21}$$

where $\boldsymbol{\Sigma}$ is an $N \times N$ diagonal matrix and the $(N+2L)\times N$ matrix $\mathbf{U}_s$ spans the signal subspace. On the other hand, the $(N+2L)\times 2L$ matrix $\mathbf{U}_o$ spans the noise subspace. That is,

$$\mathbf{U}_o^\dagger \mathbf{R}_w^{-1/2} \tilde{\mathbf{T}}_N(\mathbf{h}_2) = \mathbf{0}. \tag{22}$$

Define that $\mathbf{J}_i = [\ \mathbf{0}_{(2L+1)\times i} \quad \mathbf{I}_{2L+1} \quad \mathbf{0}_{(2L+1)\times(N-1-i)}\ ]^T$ for $i = 0, 1, \ldots, N-1$. Then (22) can be rewritten as

$$\mathbf{U}\mathbf{h}_2 = \mathbf{0}, \tag{23}$$

where $\mathbf{U} \triangleq [\mathbf{J}_0^T\mathbf{R}_w^{-1/2}\mathbf{U}_o^*, \ldots, \mathbf{J}_{N-1}^T\mathbf{R}_w^{-1/2}\mathbf{U}_o^*]^T$. Hence, we can estimate $\mathbf{h}_2$ (up to a scalar ambiguity) by calculating the eigenvector corresponding to the smallest eigenvalue of $\mathbf{U}^\dagger\mathbf{U}$.

In summary, our algorithm is as follows.

1) Estimate $\mathbf{h}_1$ by (14).
2) Eliminate the interference from $\mathbb{T}_1$ by (15).
3) Calculate $\mathbf{R}_w^{-1/2}\mathbf{v}_k$ by (17) and (19) and obtain the $(N+2L)\times 2L$ matrix $\mathbf{U}_o$ spanning the noise subspace by eigenvalue decomposition.
4) Estimate $\mathbf{h}_2$ (up to a scalar ambiguity) by (23).

C. Multiple relay nodes

The extension to the case of multiple relay nodes is straight forward. Suppose that we have M relay nodes $\mathbb{R}_1, \mathbb{R}_2, \ldots, \mathbb{R}_M$. Let the channels from $\mathbb{T}_i$ to $\mathbb{R}_m$ be denoted as $\mathbf{f}_i^{(m)}$ and the channels from $\mathbb{R}_m$ to $\mathbb{T}_i$ be denoted as $\mathbf{g}_i^{(m)}$. Then (2) becomes

$$\mathbf{r}_k^{(m)} = \sum_{i=1}^{2} \mathbf{T}_{N+2L}(\mathbf{f}_i^{(m)}) \begin{bmatrix} \mathbf{x}_{k-1,isi}^{(i)} \\ \mathbf{x}_{k,cp}^{(i)} \\ \mathbf{x}_k^{(i)} \end{bmatrix} + \mathbf{n}_{k,r}^{(m)}, \tag{24}$$

where $\mathbf{r}_k^{(m)}$ is the signal received by relay node $\mathbb{R}_m$ and $\mathbf{n}_{k,r}^{(m)}$ is the noise at $\mathbb{R}_m$. When $\mathbb{T}_1$ receives the signal, (4) becomes

$$\mathbf{y}_k = \sum_{m=1}^{M} \mathbf{T}_{N+2L}(\mathbf{g}_1^{(m)}) \begin{bmatrix} \alpha_m \mathbf{r}_{k-1,isi}^{(m)} \\ \alpha_m \mathbf{r}_k^{(m)} \end{bmatrix} + \mathbf{n}_{k,t}, \tag{25}$$

where α_m is the scalar multiplied by relay node $\mathbb{R}_m$ for power normalization. Combining (24) with (25), the received vector at $\mathbb{T}_1$ continues to have the form given in (5), but now the cascaded channels are $\mathbf{h}_1 = \sum_{m=1}^{M} \alpha_m(\mathbf{g}_1^{(m)} * \mathbf{f}_1^{(m)})$ and $\mathbf{h}_2 = \sum_{m=1}^{M} \alpha_m(\mathbf{g}_1^{(m)} * \mathbf{f}_2^{(m)})$. Hence, the above methods can be applied to the case of multiple relay nodes.

D. Comparison with an existing work

A blind channel estimation algorithm in OFDM-based TWRN was proposed in [10]. Comparing our method with

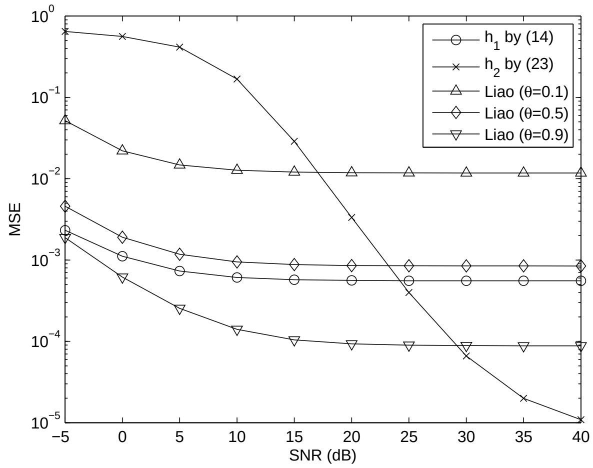

Fig. 2: Comparison of the MSE

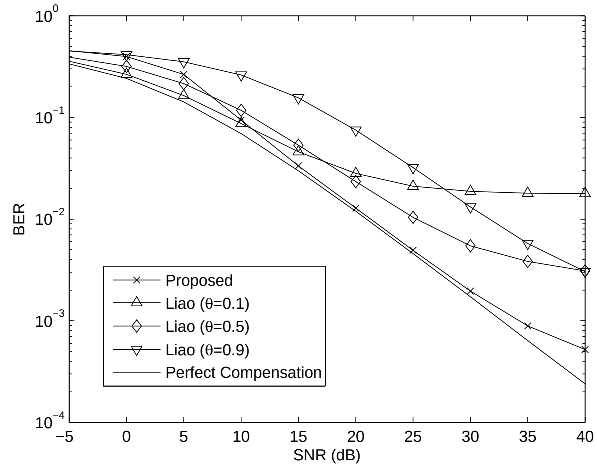

Fig. 3: Comparison of the BER

that in [10], there are two major differences. One is that [10] requires a precoding matrix $\mathbf{P}$, where

$$\mathbf{P}\mathbf{P}^{\dagger} = \begin{bmatrix} 1 & \theta & \cdots & \theta \\ \theta & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \theta \\ \theta & \cdots & \theta & 1 \end{bmatrix}.$$

A necessary condition on θ is $-\frac{1}{N-1} \leq \theta \leq 1$. In other words, the kth transmitted vector from $\mathbb{T}_i$ is the precodded vector $\mathbf{P}\mathbf{s}_k^{(i)}$ instead of $\mathbf{s}_k^{(i)}$. Notice that for $\theta \neq 0$, $\mathbf{P}$ is not an unitary matrix. The channel noise can be amplified when the receiver performs the operation $\mathbf{P}^{-1}$. It was shown in [10] that when θ increases from 0 to 1, the mean square error (MSE) of channel estimate decreases. Due to noise amplification, larger θ does not necessarily yield smaller BER, so there exists a compromise between channel estimation error and BER. Another difference between our method and [10] is that there is a 2×2 ambiguity matrix in [10], or equivalently, there are four ambiguity scalars. On the other hand, there is only one ambiguity scalar in our algorithm.

IV. Simulation Results

In the simulation, we consider a TWRN with one relay node. The channel taps $f_{i,l}$ and $g_{i,l}$ are generated as independent and identically distributed zero-mean complex Gaussian random variables. The order of these channels is $L = 8$. The channels are normalized so that $\|\mathbf{f}_1\|_F^2 = \|\mathbf{f}_2\|_F^2 = \|\mathbf{g}_1\|_F^2 = \|\mathbf{g}_2\|_F^2 = 1$. The channel does not change while the channel estimation is performed. The channel noise is additive white Gaussian noise (AWGN), and the transmission symbols are QPSK. The size of the DFT matrix is $N = 64$, and the length of CP is $2L = 16$. The total number of Monte-Carlo trials is $M_c = 2000$, and the number of received blocks is $K = 500$. In all plots, we set $\sigma_1^2 = \sigma_2^2$ and $\sigma_{n_r}^2 = \sigma_{n_t}^2$. The signal-to-noise ratio (SNR) is defined as $\sigma_2^2/(\alpha^2\sigma_{n_r}^2 + \sigma_{n_t}^2)$.

We compare the performances of our method with the method proposed by Liao et al. in [10]. As mentioned in Section III-D, Liao's algorithm has a compromise between channel estimation error and BER. The parameter θ in Liao's algorithm is set to 0.1, 0.5, and 0.9. From [10], it is founded that $\theta = 0.5$ yields a good BER performance. Fig. 2 shows the MSE performances. Since the MSEs of $\mathbf{h}_1$ and $\mathbf{h}_2$ by Liao's algorithm are the same, we plot one MSE curve only. From the figure, we see that as θ increases from 0.1 to 0.9, the MSE of Liao's algorithm decreases. For the estimation of $\mathbf{h}_1$, our method is better than Liao's methods for $\theta = 0.1$ and 0.5, but worse than one for $\theta = 0.9$. As we will see in Fig. 3, the BER performance for $\theta = 0.9$ is not good due to severe noise amplification. For $\mathbf{h}_2$, Liao's method is better at low SNR whereas our method is better at high SNR.

In Fig. 3, we show BER performances. Zero-forcing equalizers are used at the receiver. The "Perfect Compensation" represents the case that the channel taps are perfectly known at the receiver. It is seen that Liao's method has the best BER performance when θ is set as 0.5. Though the MSE is the smallest when $\theta = 0.9$, its BER performance is not good due to the noise amplification problem of the precoding matrix $\mathbf{P}$. From Fig. 3, we see that the proposed algorithm outperforms Liao's methods when SNR ≥ 10 dB, and the performance of our method is close to the perfect compensation.

V. Conclusions

In this paper, we propose a blind channel estimation in OFDM-based AF-TWRN. The first cascaded channel $\mathbf{h}_1$ is estimated by the power reduction method whereas the second cascaded channel $\mathbf{h}_2$ is estimated by the subspace method. Simulation results show that our method yields a satisfactory performance.

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