ON THE IMPACT OF RESIDUAL CFO IN UL MU-MIMO

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ABSTRACT

Uplink multiuser MIMO (UL MU-MIMO) is a new feature introduced in 802.11ax (currently under development). This paper investigates the residual carrier frequency offset (CFO) impact on the performance of UL MU-MIMO OFDM systems. We show that if a station (STA) has zero residual CFO, then its packet error rate (PER) will not suffer from other STAs that have residual CFO when a zero-forcing (ZF) or linear minimum mean squared error (LMMSE) MIMO receiver is used at the access point (AP). To reduce the impacts of residual CFO on system PER performance, two CFO correction methods are designed to compensate for the phase shift and cancel the inter-user interference caused by the residual CFO. Several simulation examples are provided to verify the results.

Index Terms— MU-MIMO; zero-forcing MIMO receiver; residual CFO; CFO correction

1. INTRODUCTION

OFDM is widely used in the present generation of wireless communication systems (e.g., WLAN and LTE/LTE-A). By dividing the whole bandwidth into smaller subbands and adding a cyclic prefix, OFDM can effectively cancel the inter-symbol interference due to multipath delay. However, the symbol error rate (SER) performance of an OFDM system is vulnerable to carrier frequency offset (CFO), which will cause inter-carrier interference and phase shift to the transmitted data symbols. How to estimate and compensate the CFO is a critical problem in the design of an OFDM system, and this problem has attracted a large amount of research interest in recent years [1-5]. In [1], the SNR degradation due to CFO is analyzed, and the approximate average SNR is derived. The results indicate that the SNR degradation increases monotonically with an increase in CFO. To reduce the impact of CFO, different CFO estimation and compensation methods are investigated for single-user OFDM or uplink OFDMA systems in [2-5], and the results show that after CFO compensation, a system's SER can be significantly improved.

Uplink multiuser MIMO-OFDM (UL MU-MIMO-OFDM) is a new feature introduced in 802.11ax [6]. Channel estimation for this new feature uses the same P matrix structure as is used for MIMO transmission in the current 802.11 standards [7]. In this paper, we consider the uplink transmissions from single-antenna stations (STAs) to a multi-antenna access point (AP). To compensate for CFO, we assume that STAs can estimate CFO during the downlink trigger frame and pre-rotate the phase of data before uplink transmissions. If residual CFO exists during the channel estimation phase, STA channel estimations will be degraded. In general, using a CFO-corrupted channel estimation for a MIMO receiver introduces inter-user interference and a phase shift for data payloads. Our main result is: if a zero-forcing (ZF) or linear minimum mean squared error (LMMSE) MIMO receiver is used at the AP and a STA can do a perfect job in estimating CFO based on the downlink trigger frame (zero residual CFO), then the STA's packet error rate (PER) will not suffer from other STAs that don't do a good job in estimating CFO (nonzero residual CFO). This fundamental result provides an incentive to estimate CFO at the STA as accurately as possible. To reduce the impact of residual CFO, two CFO-correction approaches are designed to cancel the inter-user interference and compensate for the phase shift.

In section II, the impact of CFO is first analyzed for a two-user case and then the result is extended to the general case. Based on the analysis results, CFO correction methods are proposed in section III. Several simulation examples are provided in section IV, and conclusions are summarized in section V.

2. ANALYSIS OF PERFORMANCE DEGRADATION DUE TO RESIDUAL CFO

It is well known that good CFO estimates improve the performance of single-user (SU) transmissions. Furthermore, in uplink (UL) SU OFDMA, a STA's CFO estimation quality impacts its performance because users are separated in the frequency domain. In this section, we will investigate whether this concept holds true for UL MU-MIMO, where different users' signals mix with each other, and thus potentially impact each other's performance. The investigation seeks to determine whether the performance of a STA that perfectly estimates CFO (no residual CFO) is affected by other STAs with residual CFO.

2.1. Two-User Case

For analysis simplicity, we first consider a two-user case in the UL MU-MIMO transmission, where each user has a single antenna and the AP has four antennas. To assist the AP to estimate the MIMO channel, in the long training field (LTF) of the preamble, each user spreads each LTF symbol over multiple time instances by multiplying it by the entries belonging to a P matrix which has orthogonal row vectors. The detailed definition of the P matrix can be found in Section. 22.3 [7]. At the AP side, after receiving all the LTF symbols, the AP can multiply the received signal with the Hermitian of the P matrix to estimate the channel:

$$\widehat{\boldsymbol{H}} = \frac{1}{2}(\boldsymbol{H}\boldsymbol{P}_{2\text{x}2} + \boldsymbol{n}_t)\boldsymbol{P}_{2\text{x}2}{}^{H} \approx \boldsymbol{H}\,, \tag{1}$$

where the channel estimation at AP is $\widehat{\boldsymbol{H}} = [\widehat{\boldsymbol{h}}_1,\ \widehat{\boldsymbol{h}}_2]$, $\boldsymbol{H} = [\boldsymbol{h}_1,\ \boldsymbol{h}_2]$ and $\boldsymbol{h}_1 \in \mathbb{C}^{4\times 1}$, $\boldsymbol{h}_2 \in \mathbb{C}^{4\times 1}$ are the channel vectors between the STA and AP, $\boldsymbol{n}_t$ is the additive noise on the AP and the 2x2 P matrix is defined as $\boldsymbol{P}_{2\text{x}2} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

Assume user 1 has zero CFO and user 2 has residual CFO, then $\boldsymbol{P}_{2\text{x}2}$ in the right hand side of equation (1) becomes

$$\boldsymbol{P}_{2\text{x}2} = \begin{pmatrix} 1 & -1 \\ 1 & e^{j\theta_2} \end{pmatrix}, \tag{2}$$

thus, the channel estimation is calculated as:

$$\widehat{\boldsymbol{h}}_1 = \boldsymbol{h}_1 + \frac{1}{2}\left(1 - e^{j\theta_2}\right)\boldsymbol{h}_2 \tag{3}$$

$$\widehat{\boldsymbol{h}}_2 = \frac{1}{2}\left(1 + e^{j\theta_2}\right)\boldsymbol{h}_2\,, \tag{4}$$

where θ_2 is the phase shift due to user 2's CFO and for simplicity, the SNR is assumed to be high enough such that the additive noise terms are neglected in the above equations. For a single subcarrier, the estimated channel can be written as:

$$\widehat{\boldsymbol{H}} = \boldsymbol{H}\boldsymbol{M}\,, \tag{5}$$

where $\boldsymbol{H}$ and $\boldsymbol{M}$ are defined as:

$$\boldsymbol{H} \triangleq (\boldsymbol{h}_1, \boldsymbol{h}_2) = \begin{pmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \\ h_{13} & h_{23} \\ h_{14} & h_{24} \end{pmatrix}, \text{ and } \boldsymbol{M} \triangleq \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix},$$

with $\alpha = \frac{1}{2}\left(1 - e^{j\theta_2}\right)$ and $\beta = \frac{1}{2}\left(1 + e^{j\theta_2}\right)$.

The received signal on a single subcarrier is:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}\,, \tag{6}$$

where $\boldsymbol{x}$ is the data symbol to be detected and $\boldsymbol{n}$ is the additive noise at the AP with covariance matrix $\sigma_n^2\boldsymbol{I}$. After the ZF receiver [8], the equalized signal is given by:

$$\begin{aligned} \widehat{\boldsymbol{x}} &= \left(\widehat{\boldsymbol{H}}^H\,\widehat{\boldsymbol{H}}\right)^{-1}\widehat{\boldsymbol{H}}^H(\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}) \\ &= (\boldsymbol{M}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{M})^{-1}\boldsymbol{M}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{x} + (\boldsymbol{M}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{M})^{-1}\boldsymbol{M}^H\boldsymbol{H}^H\boldsymbol{n} \\ &= \boldsymbol{M}^{-1}\boldsymbol{x} + \boldsymbol{M}^{-1}(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{n} \\ &= \boldsymbol{M}^{-1}(\boldsymbol{x} + \widetilde{\boldsymbol{n}})\,, \end{aligned} \tag{7}$$

where $\widetilde{\boldsymbol{n}} \triangleq (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\boldsymbol{n}$ and $\boldsymbol{M}^{-1} = \begin{pmatrix} 1 & 0 \\ -\alpha/\beta & 1/\beta \end{pmatrix}$.

Equation (7) indicates that user 1 is free of inter-user interference. Please note that the above analysis can be directly generalized to an LMMSE receiver in the high SNR region.

2.2. Extension to a General Number of Users

To extend the above analysis to the general case, we assume there are N single-antenna users in the uplink transmission, and at the AP the received signal for channel estimation is:

$$\boldsymbol{y_t} = \boldsymbol{H}\widetilde{\boldsymbol{P}}s + \boldsymbol{n}_t\,, \tag{8}$$

where $\boldsymbol{H} \in \mathbb{C}^{M\times N}$ is the channel state matrix between the users and AP, $\boldsymbol{y_t} \in \mathbb{C}^{M\times N}$ and each column of $\boldsymbol{y_t}$ correspond to the received signal during a single LTF symbol, s is the LTF symbol in the LTF field, and $\widetilde{\mathbf{P}}$ is the CFO-corrupted P matrix. An example 4x4 $\widetilde{\mathbf{P}}$ matrix is:

$$\widetilde{\mathbf{P}} = \begin{pmatrix} 1 & -1e^{j\theta_1} & 1e^{j2\theta_1} & 1e^{j3\theta_1} \\ 1 & 1e^{j\theta_2} & -1e^{j2\theta_2} & 1e^{j3\theta_2} \\ 1 & 1e^{j\theta_3} & 1e^{j2\theta_3} & -1e^{j3\theta_3} \\ -1 & 1e^{j\theta_4} & 1e^{j2\theta_4} & 1e^{j3\theta_4} \end{pmatrix}, \tag{9}$$

where θ_i is the i th user's phase shift during an LTF symbol. The channel estimation is calculated as:

$$\widehat{\boldsymbol{H}} = \frac{1}{N}\boldsymbol{y_t}\boldsymbol{P}^H s^* \approx \frac{1}{N}\boldsymbol{H}\widetilde{\boldsymbol{P}}\boldsymbol{P}^H = \boldsymbol{H}\boldsymbol{M}\,, \tag{10}$$

where $\boldsymbol{M} = \frac{1}{N}\widetilde{\boldsymbol{P}}\boldsymbol{P}^H$, $\boldsymbol{P}$ is the P-matrix ($\boldsymbol{P}\boldsymbol{P}^H = N\boldsymbol{I}$), and the approximation is due to the high SNR assumption. In (10), it can be observed that each element of the channel estimation $\widehat{\boldsymbol{H}}$ is a combination of the true channel and the residual CFO of all users. Motivated by the result of the two-use case, an interesting problem to investigate is whether a user with zero CFO will see interference from users that have a residual CFO. For analysis purposes, we can assume the first k users have no CFO and the last l users have residual CFOs (total $N = k + l$ users), and we can write:

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{pmatrix} \text{ and } \widetilde{\boldsymbol{P}} = \begin{pmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \widetilde{\boldsymbol{P}}_{21} & \widetilde{\boldsymbol{P}}_{22} \end{pmatrix}, \tag{11}$$

where $\boldsymbol{P}_{11}$ has size $(k \times k)$, and $\boldsymbol{P}_{22}$ and $\widetilde{\boldsymbol{P}}_{22}$ have size $(l \times l)$. (The other sub-matrices have corresponding sizes.) Note the first k rows of $\widetilde{\boldsymbol{P}}$ are exactly the same as those for $\boldsymbol{P}$ because the first k users do not have CFO. Since $\boldsymbol{P}\boldsymbol{P}^H = N\boldsymbol{I}$, we have:

$$\begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \end{bmatrix}\boldsymbol{P}^H = [N\boldsymbol{I}_{k\times k} \quad \boldsymbol{O}_{k\times l}]\,. \tag{12}$$

Therefore,

$$\boldsymbol{M} = \frac{1}{N}\widetilde{\boldsymbol{P}}\boldsymbol{P}^H = \frac{1}{N}\begin{pmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \widetilde{\boldsymbol{P}}_{21} & \widetilde{\boldsymbol{P}}_{22} \end{pmatrix}\boldsymbol{P}^T = \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} & \boldsymbol{B}_{l\times l} \end{pmatrix}, \tag{13}$$

where $[\boldsymbol{A}_{l\times k} \quad \boldsymbol{B}_{l\times l}] = \frac{1}{N}\begin{bmatrix} \widetilde{\boldsymbol{P}}_{21} & \widetilde{\boldsymbol{P}}_{22} \end{bmatrix}\boldsymbol{P}^H$, $\boldsymbol{I}_{k\times k}$ is the $k \times k$ identity matrix, and $\boldsymbol{O}_{k\times l}$ is the $k \times l$ all-zero matrix.

According to (7), after the ZF equalizer, the received signal becomes:

$$\widehat{\boldsymbol{x}} = \boldsymbol{M}^{-1}(\boldsymbol{x} + \widetilde{\boldsymbol{n}}) = \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} & \boldsymbol{B}_{l\times l} \end{pmatrix}^{-1}(\boldsymbol{x} + \widetilde{\boldsymbol{n}})\,. \tag{14}$$

In the above equation, it can be observed that the inter-user interference depends on the inverse matrix of $\begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} & \boldsymbol{B}_{l\times l} \end{pmatrix}$. According to Lemma 1 in the appendix, we have:

$$\begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} & \boldsymbol{B}_{l\times l} \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{C}_{l\times k} & \boldsymbol{B}_{l\times l}^{-1} \end{pmatrix}. \tag{15}$$

After plugging (15) into (14), we find that the first k users do not suffer from the residual CFOs of the other l users, but that the l users experience degraded performance from inter-user interference.

3. RESIDUAL CFO COMPENSATION

Based on the analysis in the last section, although the uplink performance of a user with zero CFO will not degrade, the performance of users with residual CFO will suffer from inter-user interference. In this section, we will investigate how to compensate for residual CFO at the AP.

3.1. Modified ZF (LMMSE) Receiver

Equation (7) shows that $\boldsymbol{M}^{-1}$ causes inter user interference and degrades the PER performance of users experiencing residual CFO. A straightforward way to cancel inter-user interference is to multiply $\hat{\boldsymbol{x}}$ by $\boldsymbol{M}$. Thus, we propose the following modified ZF (LMMSE) receiver:

$$\widehat{\boldsymbol{W}}_{ZF} = \boldsymbol{M}\left(\widehat{\boldsymbol{H}}^H\,\widehat{\boldsymbol{H}}\right)^{-1}\widehat{\boldsymbol{H}}^H\,, \tag{16}$$

$$\widehat{\boldsymbol{W}}_{LMMSE} = \boldsymbol{M}\left(\widehat{\boldsymbol{H}}^H\,\widehat{\boldsymbol{H}} + \sigma_n^2\boldsymbol{I}\right)^{-1}\widehat{\boldsymbol{H}}^H\,. \tag{17}$$

The equalized signal after the modified ZF receiver is:

$$\begin{aligned}\bar{\boldsymbol{x}} &= \boldsymbol{M}\left(\widehat{\boldsymbol{H}}^H\,\widehat{\boldsymbol{H}}\right)^{-1}\widehat{\boldsymbol{H}}^H\boldsymbol{y} \\ &= \boldsymbol{x} + \widetilde{\boldsymbol{n}}\,, \end{aligned} \tag{18}$$

and we will get the same signal as ZF with perfect channel estimation. The modified ZF (LMMSE) requires an additional matrix multiplication with $\boldsymbol{M}$ at the traditional ZF (LMMSE) receiver. If a method with lower complexity is preferable, the diagonal normalization method in the next section can be used.

3.2. Diagonal Normalization

To reduce the impact of multiplying the matrix $\boldsymbol{M}$ with $\widehat{\boldsymbol{H}}$, we propose a diagonal approximation of the matrix $\boldsymbol{M}$. We start with normalizing the diagonal values:

$$\widetilde{\boldsymbol{h}}_i = \frac{\widehat{\boldsymbol{h}}_i}{\boldsymbol{M}_{i,i}}\,, \tag{19}$$

where $\widetilde{\boldsymbol{h}}_i$ is the i-th column of $\widetilde{\boldsymbol{H}}$, $\widehat{\boldsymbol{h}}_i$ is the i-th column of the channel estimation $\widehat{\boldsymbol{H}}$, and $\boldsymbol{M}_{i,i}$ is the i-th diagonal element of matrix $\boldsymbol{M}$. We further assume that the residual CFOs are constant during different LTF symbols. Thus $\boldsymbol{M}_{i,i}$ is calculated as:

$$\begin{aligned}\boldsymbol{M}_{i,i} &= \frac{1}{N}\left(1 + e^{j\theta_i} + e^{j2\theta_i} + \cdots + e^{j(N-1)\theta_i}\right) \\ &= \frac{1}{N}\frac{sin\left(\frac{N}{2}\theta_i\right)}{sin\left(\frac{1}{2}\theta_i\right)}e^{j\frac{(N-1)}{2}\theta_i} \\ &\approx e^{j\frac{(N-1)}{2}\theta_i}\,, \end{aligned} \tag{20}$$

where the approximation in (20) is due to $\theta_i \ll 1$. We use $\widetilde{\boldsymbol{H}}$ to build the ZF (LMMSE) receiver:

$$\widetilde{\boldsymbol{W}}_{ZF} = \left(\widetilde{\boldsymbol{H}}^H\widetilde{\boldsymbol{H}}\right)^{-1}\widetilde{\boldsymbol{H}}^H, \tag{21}$$

$$\widetilde{\boldsymbol{W}}_{LMMSE} = \left(\widetilde{\boldsymbol{H}}^H\widetilde{\boldsymbol{H}} + \sigma_n^2\boldsymbol{I}\right)^{-1}\widetilde{\boldsymbol{H}}^H. \tag{22}$$

The channel normalization method only requires the phase rotation of each column of the channel estimation $\widehat{\boldsymbol{H}}$, and the complexity is lower than the modified ZF method. On the other hand, as shown in the simulation results, a mild PER performance degradation will be observed for the channel normalization method.

4. SIMULATION RESULTS

To verify the correctness and effectiveness of the theoretical analysis and the CFO compensation methods, several simulation examples are provided. In the simulation, each STA has a single antenna and the AP is configured with multiple antennas. System bandwidth is assumed to be 20 MHz, and the FFT size is 256. For the data payload and the LTF, the useful symbol length is 12.8 us and the cyclic prefix (CP) is 1.6 us. The 11nD channel model is used to model the wireless fading channel between the STA and the AP. To make the simulation results more realistic, RF impairments are considered in the simulation, including phase noise and power amplifier nonlinearity. For simplicity, we assume the time domain synchronization is perfect. In the simulation, each packet is assumed to include 1000 bytes, and the modulation and coding scheme used is MCS 7 (64-QAM with 3/4 rate binary convolution code). To evaluate the impact of residual CFO on PER performance, we assume that the users have a fixed CFO value, for example, 1000 Hz or 400 Hz. To assist the implementation of the CFO correction methods, we further assume that the AP has perfect knowledge of each STA's residual CFO.

Fig. 1 shows PER degradation for different CFO values. There are three users in the uplink transmission, and the AP has four antennas. To verify the analytical result that the user with zero CFO will has no performance loss and meanwhile to investigate the impact of the increasing CFO on the PER performance, in the simulation we assume the first and third users have fixed CFO (+/-1000 Hz) and the second user's CFO varies from 0 to 750 Hz. A 3x3 P matrix is used for channel estimation and the PER of the second user is plotted. In the results, it can be observed that when the three users have CFO [-1000 0 1000] Hz and the SNR is higher than 28 dB, the second user's PER is the same as when all three users have zero CFO. This indicates that the second user will not suffer from the CFO of the first and third users. This observation essentially verifies our result that if a STA has zero CFO, then its performance will not be degraded by users that have residual CFO. With the second user's CFO increasing, the PER performance degrades monotonically due to the larger phase shift introduced by the CFO and the higher inter-user interference from the first and the third users.

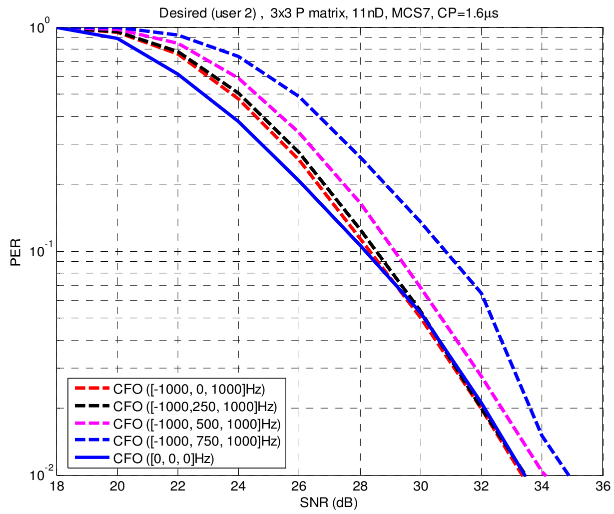

Fig. 1. Comparison of residual CFO impacts at MCS 7.

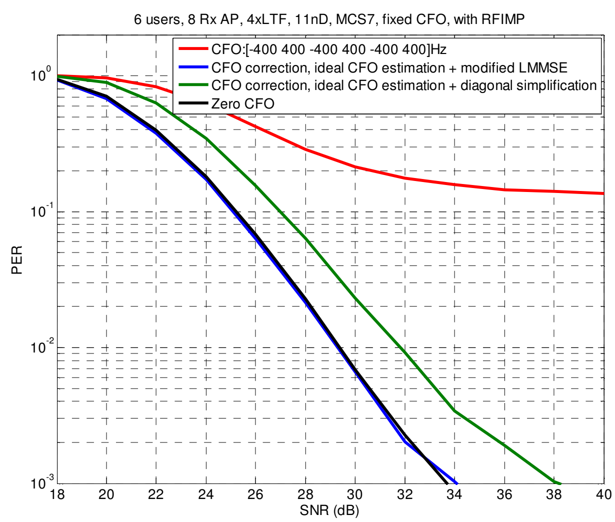

Fig. 2. Performance of CFO correction methods for fixed CFO (+/-400 Hz) at MCS 7. The modified LMMSE and diagonal simplification methods are defined in (17) and (19) respectively.

The performance of the CFO correction methods are evaluated in Fig. 2. In the simulation, we assume there are six single-antenna STAs and each STA's residual CFO is set to +/-400 Hz. The AP is configured with eight antennas, and to evaluate the effectiveness of the CFO correction methods, the AP is assumed to have a perfect knowledge of the residual CFOs. After implementing the CFO correction methods, the average PER of all the STAs is used as the performance metric. The results show that the modified LMMSE method achieves a PER performance that is almost the same as the case with zero CFO, which confirms that multiplying the matrix $\boldsymbol{M}$ with the traditional LMMSE receiver can completely cancel the inter-user interference caused by the residual CFO. Meanwhile, the low complexity diagonal simplification method has a PER that is slightly worse than the modified LMMSE, but there is still significant improvement compared with the one without CFO corrections.

5. CONCLUSIONS

In this paper, we studied the residual CFO problem in the uplink MU-MIMO OFDM transmission in the framework of IEEE 802.11ax. Our analysis indicates that, for uplink MU-MIMO transmissions, the PER after the LMMSE or ZF MIMO receiver for a user with zero CFO will not degrade when other users experience residual CFO. However, the PER performance of users that experience residual CFO will degrade due to phase shift and inter-user interference. To reduce the performance degradation caused by residual CFO, two CFO correction methods (modified ZF/LMMSE and diagonal normalization) were designed to completely or partially cancel the impact of residual CFO. Simulation results have verified the correctness of the mathematical analysis and the effectiveness of the CFO compensation methods. Future work will include the investigation of the residual CFO estimation method at the AP such that the CFO estimations can be used as input for the CFO correction methods.

6. APPENDIX

Lemma 1: For a matrix $\boldsymbol{M} = \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} & \boldsymbol{B}_{l\times l} \end{pmatrix}$ and $\boldsymbol{B}_{l\times l}$ of full rank, we have

$$\boldsymbol{M}^{-1} = \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{C}_{l\times k} & \boldsymbol{B}_{l\times l}^{-1} \end{pmatrix},$$

where $\boldsymbol{C}_{l\times k} \triangleq -\boldsymbol{B}_{l\times l}^{-1}\boldsymbol{A}_{l\times k}$.

Proof: By the definition of the matrix inversion, it is sufficient to verify that $\boldsymbol{M}(\boldsymbol{M}^{-1}) = (\boldsymbol{M}^{-1})\boldsymbol{M} = \boldsymbol{I}_{N\times N}$, where $N \triangleq k + l$. The following equations are true since $\boldsymbol{C}_{l\times k} = -\boldsymbol{B}_{l\times l}^{-1}\boldsymbol{A}_{l\times k}$,

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{M}^{-1}) &= \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} & \boldsymbol{B}_{l\times l} \end{pmatrix} \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{C}_{l\times k} & \boldsymbol{B}_{l\times l}^{-1} \end{pmatrix} \\ &= \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} + \boldsymbol{B}_{l\times l}\boldsymbol{C}_{l\times k} & \boldsymbol{I}_{l\times l} \end{pmatrix} \\ &= \boldsymbol{I}_{N\times N}, \end{aligned} \tag{23}$$

$$\begin{aligned} (\boldsymbol{M}^{-1})\boldsymbol{M} &= \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{C}_{l\times k} & \boldsymbol{B}_{l\times l}^{-1} \end{pmatrix} \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{A}_{l\times k} & \boldsymbol{B}_{l\times l} \end{pmatrix} \\ &= \begin{pmatrix} \boldsymbol{I}_{k\times k} & \boldsymbol{O}_{k\times l} \\ \boldsymbol{C}_{l\times k} + \boldsymbol{B}_{l\times l}^{-1}\boldsymbol{A}_{l\times k} & \boldsymbol{I}_{l\times l} \end{pmatrix} \\ &= \boldsymbol{I}_{N\times N}. \end{aligned} \tag{24}$$

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