MULTIANTENNA SPECTRUM SENSING FOR IMPROPER SIGNALS OVER FREQUENCY SELECTIVE CHANNELS

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ABSTRACT

We consider multiple antenna spectrun sensing for improper complex primary user (PU) signals. Past work on this problem is limited to PU Gaussian signals over flat fading channels. We allow non-Gaussian signals over frequency-selective channels. A binary hypothesis testing approach is formulated and a generalized likelihood ratio test (GLRT) is derived using the power spectral density estimator of an augmented sequence. An analytical solution for calculating the test threshold is provided. The results are illustrated via simulations.

***Index Terms*—** Spectrum sensing; improper signals; generalized likelihood ratio test (GLRT); multiple antennas

1. INTRODUCTION

The cognitive receiver's spectrum sensing problem is to decide if the received signal, in addition to noise, contains signals from a single or multiple primary users (PUs). This is formulated as a binary hypothesis testing problem and is a well-investigated topic [1]. A wide variety of approaches exist based on differing signal and noise models [1]. A widely used model is that of temporally white but spatially correlated proper complex Gaussian PU signal in temporally and spatially uncorrelated proper complex Gaussian noise [2]. Temporally colored, proper complex signals in spatially uncorrelated but temporally correlated Gaussian noise have been considered in [3, 4] assuming multiple independent measurement records (snapshots) and Gaussian PU signals, whereas only one data realization is needed in [5, 6] where the PU signals can be non-Gaussian but are assumed to be proper. All these approaches ([2, 3, 4, 5, 6]) exploit the generalized likelihood ratio test (GLRT) paradigm.

In statistical signal processing in general, if the underlying signals are improper, much can be gained in performance if they are treated as improper [7]. In communications, BPSK or offset-QPSK modulation based signals are improper. In [8] a variant of the Hadamard ratio test has been devised for improper signals and it is shown that detection performance improves compared to the case where improper signals are treated as proper.

Relation to Prior Work: The model of [8] is limited to temporally white but spatially correlated improper complex Gaussian PU signal in temporally and spatially uncorrelated proper complex Gaussian noise. In this paper we allow temporal correlation for both signal and noise, and also allow signal to be non-Gaussian.

Contributions: A binary hypothesis testing approach is formulated and a generalized likelihood ratio test (GLRT) is derived using the power spectral density estimator of an augmented sequence. An asymptotic analytical solution for calculating the test threshold is provided. The results are illustrated via computer simulations.

Notation: We use $\mathbf{S} \succeq 0$ and $\mathbf{S} \succ 0$ to denote that Hermitian $\mathbf{S}$ is positive semi-definite and positive definite, respectively. For a square matrix $\mathbf{A}$, $|\mathbf{A}|$ and $\mathrm{etr}(\mathbf{A})$ denote the determinant and the exponential of the trace of $\mathbf{A}$, respectively, i.e., $\mathrm{etr}(\mathbf{A}) = \exp(\mathrm{tr}(\mathbf{A}))$, $\mathbf{B}_{k;i:l,j:m}$ denotes the submatrix of the matrix $\mathbf{B}_k$ comprising its rows i through l and columns j through m, $\mathbf{B}_{k;ij}$ is its ijth element, and $\mathbf{I}$ is the identity matrix. The superscripts $*$, T and H denote the complex conjugate, transpose and the Hermitian (conjugate transpose) operations, respectively.

2. SYSTEM MODEL

Let $p \times 1$ $\mathbf{n}(t)$ denote a zero-mean staionary proper Gaussian random sequence (noise) and $p \times 1$ $\mathbf{s}(t)$ denote a zero-mean stationary improper random sequence (signal) which is independent of $\{\mathbf{n}(t)\}$ and could be non-Gaussian. We consider the following binary hypothesis testing problem for the measurement sequence $\mathbf{x}(t)$:

$$\begin{array}{ll} \mathcal{H}_0 : & \mathbf{x}(t) = \mathbf{n}(t), \ \text{ noise only} \\ \mathcal{H}_1 : & \mathbf{x}(t) = \mathbf{s}(t) + \mathbf{n}(t), \ \text{ signal and noise,} \end{array} \tag{1}$$

where $\mathcal{H}_0$ is the null hypothesis that the cognitive user is receiving just noise, and $\mathcal{H}_1$ is the alternative that signal from PU (or PUs) is also present. We assume that noise is uncorrelated across sensors (antennas) but it may be nonwhite with possibly different power spectra at different sensors. The signal $\mathbf{s}(t)$ is not necessarily an i.i.d. sequence.

A stationary complex zero-mean process $\{\mathbf{x}(t)\}$ of dimension p is said to be proper [7] if its matrix complementary correlation (covariance) function (called pseudo-correlation in [9]) $\tilde{\mathbf{R}}_{xx}(\tau)$ vanishes, i.e.,

$$\tilde{\mathbf{R}}_{xx}(\tau) = E\{\mathbf{x}(t+\tau)\mathbf{x}^T(t)\} = 0, \ \tau = 0, \pm 1, \cdots, \tag{2}$$

where $\mathbf{x}(t) = \mathbf{x}_r(t) + j\mathbf{x}_i(t)$, with $\mathbf{x}_r(t)$ and $\mathbf{x}_i(t)$ denoting its real and imaginary components, respectively. Define $\mathbf{R}_{xx}(\tau) = E\{\mathbf{x}(t+\tau)\mathbf{x}^H(t)\}$, the conventional matrix correlation function. The PSD $\mathbf{S}_x(f)$ of $\{\mathbf{x}(t)\}$ is the Fourier transform of $\mathbf{R}_{xx}(\tau)$, $\mathbf{S}_x(f) = \sum_{\tau=-\infty}^{\infty} \mathbf{R}_{xx}(\tau)e^{-j2\pi f\tau}$, whereas the complementary PSD (C-PSD) $\tilde{\mathbf{S}}_x(f)$ of $\{\mathbf{x}(t)\}$ is $\tilde{\mathbf{S}}_x(f) = \sum_{\tau=-\infty}^{\infty} \tilde{\mathbf{R}}_{xx}(\tau)e^{-j2\pi f\tau}$. Clearly, for a proper process, the C-PSD vanishes.

We observe $\mathbf{x}(t)$ for $t = 0, 1, \cdots, N-1$ (N samples). We employ multivariate spectral analysis to discriminate between the two hypotheses. Since $\mathbf{s}(t)$ is assumed to be improper, we will exploit both PSD and C-PSD. Define the augmented complex process $\{\mathbf{y}(t)\}$ and the real-valued process $\{\mathbf{z}(t)\}$ as

$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}^*(t) \end{bmatrix}, \quad \mathbf{z}(t) = \begin{bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_i(t) \end{bmatrix}. \tag{3}$$

We assume that $\{\mathbf{z}(t)\}$ satisfies Assumption 2.6.1 of [10] so that some asymptotic results from [10] regarding PSD estimators can be invoked; the time series need not be Gaussian but its moments of all orders should be finite.

Consider the finite Fourier transform (FFT) $\mathbf{d}_z(f_n)$ of $\mathbf{z}(t)$, $t = 1, 2, \cdots, N-1$, given by

$$\mathbf{d}_z(f_n) := \sum_{t=0}^{N-1} \mathbf{z}(t)e^{-j2\pi f_n t} \tag{4}$$

where $f_n = n/N, n = 0, 1, \cdots, N-1$. Then the estimator of the PSD of $\mathbf{z}(t)$ at frequency f_n, based on the Daniell method, is given by

$$\hat{\mathbf{S}}_z(f_n) = \frac{1}{K} \sum_{l=-m_t}^{m_t} \left(N^{-1}\mathbf{d}_z(f_{n+l})\mathbf{d}_z^H(f_{n+l})\right) \tag{5}$$

where $K = 2m_t + 1$ is the smoothing window size. By [10, Theorem 7.3.3], $\hat{\mathbf{S}}_z(f_n)$ is asymptotically ("large" N) distributed as $W_C\left(2p, K, K^{-1}\mathbf{S}_z(f_n)\right)$ so long as the smoothing window in (5) does not include the frequency at $n = 0$ or $n = N/2$, where $W_C\left(2p, K, K^{-1}\mathbf{S}_z(f_n)\right)$ denotes the complex Wishart distribution of dimension $2p$, degrees of freedom K, and mean value $\mathbf{S}_z(f_n)$. If a random matrix $\mathbf{X} \sim W_C\left(p, K, \mathbf{S}(f)\right)$, then by [10, Sec. 4.2], $E\{\mathbf{X}\} = K\mathbf{S}(f)$, cov$\{\mathbf{X}_{jk}, \mathbf{X}_{lm}\} = K\mathbf{S}_{jl}(f)\mathbf{S}^*_{km}(f)$, and for $K \geq p$, the probability density function (pdf) of $\mathbf{X}$ is given by

$$f_{\mathbf{X}}(\mathbf{X}) = \frac{1}{\Gamma_p(K)} \frac{1}{|\mathbf{S}(f)|^K} |\mathbf{X}|^{K-p} \operatorname{etr}\{-\mathbf{S}^{-1}(f)\mathbf{X}\} \tag{6}$$

where the pdf (6) is defined for $\mathbf{S}(f) \succ 0$ and $\mathbf{X} \succeq 0$, and is otherwise zero, and $\Gamma_p(K) := \pi^{p(p-1)/2} \prod_{j=1}^{p} \Gamma(K-j+1)$ where $\Gamma(n)$ denotes the (complete) Gamma function $\Gamma(z) := \int_0^\infty t^{z-1}e^{-t}\,dt$.

We will confine our attention to the frequency points over which the spectral estimators are approximately mutually independent, which for the Daniell method are given by

$$\tilde{f}_k = \frac{(k-1)K + m_t + 1}{N}, \; 1 \leq k \leq M = \left\lfloor \frac{\frac{N}{2} - m_t - 1}{K} \right\rfloor. \tag{7}$$

Let $\mathcal{M} := \{\tilde{f}_k : 1 \leq k \leq M\}$ denote the set of M frequency bins as in (7) of interest. From (3) we have

$$\mathbf{y}(t) = \mathcal{T}\mathbf{z}(t), \quad \mathcal{T} = \begin{bmatrix} \mathbf{I} & j\mathbf{I} \\ \mathbf{I} & -j\mathbf{I} \end{bmatrix} \tag{8}$$

where $(2p) \times (2p)$ $\mathcal{T}$ is full-rank . Hence, $\mathbf{d}_y(f_n) = \mathcal{T}\mathbf{d}_z(f_n)$ and $\hat{\mathbf{S}}_y(f_n) = \mathcal{T}\hat{\mathbf{S}}_z(f_n)\mathcal{T}^H$ where $\mathbf{d}_y(f_n) := \sum_{t=0}^{N-1} \mathbf{y}(t)e^{-j2\pi f_n t}$ and

$$\hat{\mathbf{S}}_y(f_n) = \frac{1}{K} \sum_{l=-m_t}^{m_t} \left(N^{-1}\mathbf{d}_y(f_{n+l})\mathbf{d}_y^H(f_{n+l})\right). \tag{9}$$

By the complex-valued counterpart of [11, Thm. 3.2.5], for any $m \times p$ matrix $\mathbf{A}$ of rank m, if $\mathbf{X} \sim W_C\left(p, K, \mathbf{S}(f)\right)$, then $\mathbf{AXA}^H \sim W_C\left(m, K, \mathbf{AS}(f)\mathbf{A}^H\right)$. Therefore $\hat{\mathbf{S}}_y(f_n)$ is (asymptotically) distributed as $W_C\left(2p, K, K^{-1}\mathbf{S}_y(f_n)\right)$. Furthermore, $\hat{\mathbf{S}}_y(\tilde{f}_k)$'s for $\tilde{f}_k$ as in (7) are asymptotically mutually independent complex Wishart random matrices.

Under $\mathcal{H}_0$, $\mathbf{x}(t)$ is proper with $\mathbf{S}_x(f) = \operatorname{diag}\{\mathbf{S}_{x;ii}(f),\, i = 1, 2, \cdots, p\}$ and

$$\mathbf{S}_y(f) = \begin{bmatrix} \mathbf{S}_x(f) & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_x^*(-f) \end{bmatrix}, \tag{10}$$

whereas under $\mathcal{H}_1$, $\mathbf{x}(t)$ is improper with $\mathbf{S}_y(f) \succeq 0$ with no specific structure. Testing for the presence of an improper PU signal in spatially uncorrelated proper Gaussian noise then reformulated as the problem:

$$\begin{aligned} \mathcal{H}_0 &: \quad \mathbf{S}_y(\tilde{f}_k) = \operatorname{diag}\{\mathbf{S}_{x;ii}(\tilde{f}_k),\, i = 1, 2, \cdots, p, \\ &\qquad \tilde{\mathbf{S}}_{x;\ell\ell}(-\tilde{f}_k),\, \ell = 1, 2, \cdots, p\}\ \forall \tilde{f}_k \in \mathcal{M} \\ \mathcal{H}_1 &: \quad \mathcal{H}_0^c = \text{complement of } \mathcal{H}_0. \end{aligned} \tag{11}$$

We assume that $\mathbf{S}_y(f) \succ 0$ for any f. Otherwise, one can add artificial proper white Gaussian noise to $\mathbf{x}(t)$ to achieve $\mathbf{S}_y(f) \succ 0$.

3. PSD-BASED GLRT

In this section we derive the GLRT. We will denote the spectral estimator at the k-th frequency bin $\tilde{f}_k$ (see (7), acquired from $\{\mathbf{y}(t)\}_{t=0}^{N-1}$, via (9), as $\mathbf{Y}_k$. We have

$$\mathbf{Y}_k \overset{a}{\sim} W_C\left(2p, K, K^{-1}\mathbf{S}_y(\tilde{f}_k)\right) \tag{12}$$

and $\mathbf{Y}_k$s are mutually independent for $k \in [1, M]$. The joint probability density function (pdf) of $\mathbf{Y}_k$ for $\tilde{f}_k \in \mathcal{M}$ under $\mathcal{H}_0$ is maximized w.r.t. $\mathbf{S}_{x;ii}(\tilde{f}_k)$ for $\hat{\mathbf{S}}_{x;ii}(\tilde{f}_k) = \mathbf{Y}_{k;ii}$, and w.r.t. $\mathbf{S}_{x;\ell\ell}(-\tilde{f}_k)$ for $\hat{\mathbf{S}}_{x;\ell\ell}(-\tilde{f}_k) = \mathbf{Y}_{k;(\ell+p)(\ell+p)}$. Under $\mathcal{H}_1$, the joint pdf of $\mathbf{Y}_k$ for $k \in [1, M]$ is maximized w.r.t. the Hermitian matrix $\mathbf{S}_y(\tilde{f}_k)$ for $\hat{\mathbf{S}}_y(\tilde{f}_k) = \mathbf{Y}_k$. Define $\mathcal{Y} = \{\mathbf{Y}_k, k \in \mathcal{M}\}$. Then one gets the GLRT $\mathcal{L} :=$

$$\frac{f_{\mathcal{Y}}(\mathbf{Y}_k, k \in \mathcal{M}|\mathcal{H}_1, \hat{\mathbf{S}}_y(\tilde{f}_k))}{f_{\mathcal{Y}}(\mathbf{Y}_k, k \in \mathcal{M}|\mathcal{H}_0, \hat{\mathbf{S}}_{x;ii}(\tilde{f}_k), \hat{\mathbf{S}}^*_{x;ii}(-\tilde{f}_k), i \in [1, p])} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau_1 \tag{13}$$

where the threshold τ_1 is picked to achieve a pre-specified probability of false alarm $P_{fa} = P\{\mathcal{L} \geq \tau_1 \,|\, \mathcal{H}_0\}$. This requires pdf of $\mathcal{L}$ under $\mathcal{H}_0$ which is discussed in Sec. 4. Simplifying, one obtains

$$\mathcal{L} = \prod_{k=1}^{M} \mathcal{L}_k, \ \mathcal{L}_k := \frac{\left[\prod_{\ell=1}^{p} \left(\mathbf{Y}_{k;\ell\ell}\,\mathbf{Y}_{k;(\ell+p)(\ell+p)}\right)\right]^K}{|\mathbf{Y}_k|^K} \tag{14}$$

Invariance of GLRT: Note that $\mathcal{L}_k$ is invariant to transformation $\mathbf{Y}_k \to \mathbf{A}_k\mathbf{Y}_k\mathbf{A}_k^H$ for any nonsingular diagonal $\mathbf{A}_k \in \mathbb{C}^{2p\times 2p}$. This observation allows us to transform any $\mathbf{Y}_k$ to $\tilde{\mathbf{Y}}_k \sim W_C\,(2p, K, \mathbf{I})$ under $\mathcal{H}_0$ by choosing $\mathbf{A}_{k;\ell\ell} = \sqrt{K}\mathbf{S}_{x;\ell\ell}^{-1/2}(\tilde{f}_k)$ for $\ell \in [1,p]$, and $\mathbf{A}_{k;\ell\ell} = \sqrt{K}\mathbf{S}_{x;\ell\ell}^{-1/2*}(-\tilde{f}_k)$ for $\ell \in [1+p, 2p]$. Then $\mathcal{L}$ is invariant and transformed $\tilde{\mathbf{Y}}_k$s now correspond to proper i.i.d. (white) sequence $\mathbf{x}(t)$ which can be used to compute the test threshold via Monte Carlo simulations. This threshold is valid for any other PSD.

4. THRESHOLD SELECTION

We now turn to determination of an asymptotic expansion of the distribution of $\mathcal{L}$ under $\mathcal{H}_0$ following [11, 12, 13]. First we need the following result:

Lemma 1 : Under $\mathcal{H}_0$, $E\{\frac{1}{\mathcal{L}^h} \,|\, \mathcal{H}_0\}$

$$= \frac{\Gamma^{2Mp}(K)}{\left[\prod_{\ell=1}^{2p}\Gamma(K-\ell+1)\right]^M}\frac{\left[\prod_{k=1}^{2p}\Gamma(K(1+h)-k+1)\right]^M}{[\Gamma(K(1+h))]^{2Mp}} \ \bullet \tag{15}$$

Proof : Using the transformation specified in Sec. 3 to obtain $\tilde{\mathbf{Y}}_k \sim W_C\,(2p, K, \mathbf{I})$ under $\mathcal{H}_0$, we have

$$E\{1/\mathcal{L}_k^h \,|\, \mathcal{H}_0\} = \int \frac{|\tilde{\mathbf{Y}}_k|^{Kh+K-2p}}{\prod_{\ell=1}^{p}\left(\mathbf{Y}_{k;\ell\ell}^{Kh}\,\mathbf{Y}_{k;(\ell+p)(\ell+p)}^{Kh}\right)} \times \frac{1}{\Gamma_{2p}(K)}\,\mathrm{etr}\{-\tilde{\mathbf{Y}}_k\}\,d\tilde{\mathbf{Y}}_k$$
$$= \frac{\Gamma_{2p}(K+Kh)}{\Gamma_{2p}(K)} E\left\{\prod_{\ell=1}^{p}(\mathbf{Y}'_{k;\ell\ell}\,\mathbf{Y}'_{k;(\ell+p)(\ell+p)})^{-Kh}\right\}, \tag{16}$$

where $\tilde{\mathbf{Y}}'_k \sim W_C\,(2p, K(1+h), \mathbf{I})$. Hence $\tilde{\mathbf{Y}}'_{k;ii}$s are independent for $i \in [1,2p]$ and $\tilde{\mathbf{Y}}'_{k;ii} \sim \frac{1}{2}\chi^2_{2K(1+h)}$. Since (see [11, p. 101])

$$E\{W^r\} = \frac{2^r\Gamma((n/2)+r)}{\Gamma((n/2))} \quad \text{for} \quad W \sim \chi_n^2, \tag{17}$$

we obtain

$$E\left\{(\tilde{\mathbf{Y}}'_{k;ii})^{-Kh}\right\} = \frac{\Gamma(K)}{\Gamma(K(1+h))} \quad \forall i \in [1,2p]. \tag{18}$$

Now using $\Gamma_p(K) := \pi^{p(p-1)/2}\prod_{j=1}^{p}\Gamma(K-j+1)$, (14), (16) and (18), we get the desired result. □

In order to exploit Lemma 2 (stated next), we need to establish that $0 \leq \mathcal{L}^{-1} \leq 1$. Since $\mathbf{Y}_k \succ 0$ (hence $\mathbf{Y}_{k;ii} > 0$ $\forall i$), $\mathcal{L}^{-1} \geq 0$ follows immediately. By Hadamard's inequality [14, p. 477], we have $|\mathbf{Y}_k| \leq \prod_{i=1}^{2p}\mathbf{Y}_{k;ii}$ which implies $\mathcal{L}^{-1} \leq 1$. The following result follows from [11, Sec. 8.2.4], [12, Sec. 8.5.1]:

Lemma 2 : Consider a random variable W ($0 \leq W \leq 1$) with the hth moment ($h = 0, 1, 2, \cdots$)

$$E\{W^h\} = C\left(\frac{\prod_{j=1}^{b} y_j^{y_j}}{\prod_{k=1}^{a} x_k^{x_k}}\right)^h \frac{\prod_{k=1}^{a}\Gamma(x_k(1+h)+\xi_k)}{\prod_{j=1}^{b}\Gamma(y_j(1+h)+\eta_j)}, \tag{19}$$

where a and b are integers, C is a constant such that $E\{W^0\} = 1$ and $\sum_{k=1}^{a} x_k = \sum_{j=1}^{b} y_j$. Let $B_r(h)$ denote the Bernoulli polynomial of degree r and order unity. Define $\nu = -2\left[\sum_{k=1}^{a}\xi_k - \sum_{j=1}^{b}\eta_j - \frac{1}{2}(a-b)\right]$, $\rho = 1 - \frac{1}{\nu}\left[\sum_{k=1}^{a}x_k^{-1}\left(\xi_k^2-\xi_k+\frac{1}{6}\right) - \sum_{j=1}^{b}y_j^{-1}\left(\eta_j^2-\eta_j+\frac{1}{6}\right)\right]$, $\beta_k = (1-\rho)x_k$, $\quad \epsilon_j = (1-\rho)y_j$ and $\omega_r = \frac{(-1)^{r+1}}{r(r+1)}\left\{\sum_{k=1}^{a}\frac{B_{r+1}(\beta_k+\xi_k)}{(\rho x_k)^r} - \sum_{j=1}^{b}\frac{B_{r+1}(\epsilon_j+\eta_j)}{(\rho y_j)^r}\right\}$. Then with χ_n^2 denoting a random variable with central chi-square distribution with n degrees of freedom (as well as the distribution itself),

$$P\{-2\rho\ln(W) \leq z\} = P\{\chi_\nu^2 \leq z\} + \omega_2\left[P\{\chi_{\nu+4}^2 \leq z\} - P\{\chi_\nu^2 \leq z\}\right] + \omega_3\left[P\{\chi_{\nu+6}^2 \leq z\} - P\{\chi_\nu^2 \leq z\}\right]$$
$$+\Big\{\omega_4\left[P\{\chi_{\nu+8}^2 \leq z\} - P\{\chi_\nu^2 \leq z\}\right] + \frac{1}{2}\omega_2^2\left[P\{\chi_{\nu+8}^2 \leq z\} - 2P\{\chi_{\nu+4}^2 \leq z\} + P\{\chi_\nu^2 \leq z\}\right]\Big\}$$
$$+\sum_{k=1}^{a}\mathcal{O}(x_k^{-5}) + \sum_{j=1}^{b}\mathcal{O}(y_j^{-5}) \quad \bullet \tag{20}$$

Comparing (19) with (15), we find the correspondence

$$a = 2Mp,\ b = 2Mp,\ x_k = K,$$
$$\xi_k = 1 - k \bmod (2p) \text{ for } k = 1, 2, \cdots, a,$$
$$y_j = K \text{ and } \eta_j = 0 \text{ for } j = 1, 2, \cdots, b. \tag{21}$$

Comparing Lemmas 1 and 2, we further have

$$\beta_k = (1-\rho)K\ \forall k, \quad \epsilon_j = (1-\rho)K\ \forall j. \tag{22}$$

Furthermore, one has $E\{1/\mathcal{L}^0 \,|\, \mathcal{H}_0\} = 1$. Thus, Lemma 2 is applicable with $W = 1/\mathcal{L}$ and parameters specified in (21). Using these values in Lemma 2 and simplifying, one gets

$$\nu = 2Mp(2p-1), \quad \rho = 1 - \frac{2p+1}{3K}, \tag{23}$$

$$\sum_{k=1}^{a}\frac{B_{r+1}(\beta_k+\xi_k)}{(\rho x_k)^r} = M\sum_{l=1}^{2p}\frac{B_{r+1}((1-\rho)K+1-l)}{(\rho K)^r}, \tag{24}$$

$$\sum_{j=1}^{b} \frac{B_{r+1}(\epsilon_j + \eta_j)}{(\rho y_j)^r} = 2Mp \frac{B_{r+1}((1-\rho)K)}{(\rho K)^r}. \tag{25}$$

Therefore, we have

$$\omega_r = \frac{(-1)^{r+1} M}{r(r+1)(\rho K)^r} \Bigg\{ \Bigg(\sum_{l=1}^{2p} B_{r+1}((1-\rho)K + 1 - l) \Bigg) - 2p B_{r+1}((1-\rho)K) \Bigg\}. \tag{26}$$

It then follows from Lemma 2 that

$$\begin{aligned} P\{2\rho \, \ln(\mathcal{L}) \le z \,|\, \mathcal{H}_0\} &= P\{\chi^2_\nu \le z\} + \omega_2 \left[P\{\chi^2_{\nu+4} \le z\} \right. \\ &\left. - P\{\chi^2_\nu \le z\} \right] + \omega_3 \left[P\{\chi^2_{\nu+6} \le z\} - P\{\chi^2_\nu \le z\} \right] \\ &+ \Big\{ \omega_4 \left[P\{\chi^2_{\nu+8} \le z\} - P\{\chi^2_\nu \le z\} \right] + \frac{1}{2}\omega_2^2 \left[P\{\chi^2_{\nu+8} \le z\} \right. \\ &\left. - 2P\{\chi^2_{\nu+4} \le z\} + P\{\chi^2_\nu \le z\} \right] \Big\} + \mathcal{O}(K^{-5}) \end{aligned} \tag{27}$$

where ω_r's are given by (24)-(26), and

$$\ln(\mathcal{L}) = K \sum_{k=1}^{M} \Bigg(\Big[\sum_{\ell=1}^{p} \ln \big(\mathbf{Y}_{k;\ell\ell} \mathbf{Y}_{k;(\ell+p)(\ell+p)} \big) \Big] - \ln(|\mathbf{Y}_k|) \Bigg). \tag{28}$$

We summarize the above in the following result.

Theorem 1. The GLRT for (11) is given by $2\rho \, \ln(\mathcal{L}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau$ where ρ and $\ln(\mathcal{L})$ are given by (23) and (28), respectively. The threshold τ is picked to achieve a pre-specified $P_{fa} = 1 - P\{2\rho \, \ln(\mathcal{L}) \le \tau \,|\, \mathcal{H}_0\}$ where $P\{2\rho \, \ln(\mathcal{L}) \le \tau \,|\, \mathcal{H}_0\}$ is given by (27) and the various needed parameters are specified in (23)-(26) •

Theorem 1 allows us to calculate the test threshold analytically.

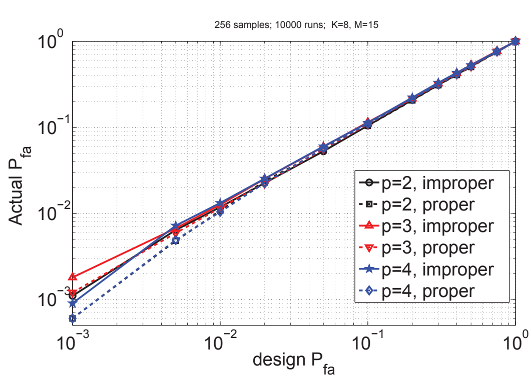

Fig. 1: Actual P_{fa} vs. design P_{fa}, $N = 256$, $K = 15$, $M = 8$

5. SIMULATION EXAMPLES

First we investigate the efficacy of Theorem 1 in computing the GLRT threshold for a given P_{fa}. We consider p antennas (p=1,2,3 or 4) with spatially uncorrelated, colored proper complex Gaussian noise $\{\mathbf{n}(t)\}$ generated by filtering p independent sequences through p separate linear filters each with impulse response $\{0.3, 1.0, 0.3\}$. To estimate the PSD of augmented $\mathbf{y}(t)$ for $N = 256$, we choose $m_t = 7$ leading to $K = 15$ and $M = 8$. In Fig. 1 we compare the actual P_{fa} and design P_{fa} based on 10,000 runs. It is seen that Theorem 1 is effective in accurately calculating the threshold value.

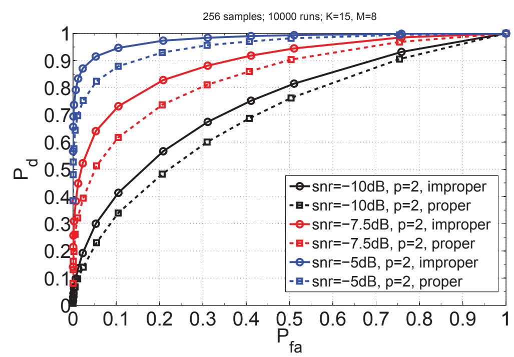

Fig. 2: ROC curve, $N = 256$, $K = 15$, $M = 8$

Next we show the receiver operating characteristic (ROC) curves. The noise $\mathbf{n}(t)$ is as in the previous example and the PU signal is given by $\mathbf{s}(t) = \sum_{l=0}^{4} \mathbf{h}(l) I(t-l)$ where $I(t)$ is a scalar BPSK sequence and vector channel $\mathbf{h}(l)$ is Rayleigh fading with 5 taps, equal power delay profile, mutually independent components. Thus signal is improper and noise is proper. The probability of detection P_d versus false-alarm rate P_{fa} results for three different SNR values and $p = 2$, based on 10,000 runs, is shown in Fig. 2; SNR is defined as ratio of the sum of signal powers at the p antennas to the sum of noise powers. In all cases we have N=256, K=15 and M=8. We also show the results of [5, 6] where we do not exploit C-PSD (labeled "proper" in Fig. 2; our proposed GLRT is labeled "improper"). It is seen that performance improves with increasing SNR, and our approach is superior to the case when impropriety of PU signal is ignored.

6. CONCLUSIONS

In this paper we investigated a method based on analysis of the multivariate PSD of augmented received noisy complex signal for spectrum sensing for multiantenna improper complex PU signals. We allow temporal correlation for both signal and noise, and also allow signal to be non-Gaussian. Our proposed approach is based on GLRT. An analytical method for calculation of the test threshold was provided and illustrated via simulations.

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