CONNECTIVITY FOR OVERLAID WIRELESS NETWORKS WITH OUTAGE CONSTRAINTS

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ABSTRACT

We study the connectivity of overlaid wireless networks where two users can communicate if the signal-to-interference ratio is larger than a threshold subject to an outage constraint. By using percolation theory, we first specify a 2-dimensional connectivity region defined as the set of density pairs—the density of secondary users and the density of primary users—within which the secondary network is percolated. Several interesting properties of this region are also revealed. Our work provides a new perspective for better understanding of the connectivity of large-scale overlaid networks.

1. INTRODUCTION

Connectivity is an essential requirement in wireless networks. Recently, percolation theory [1] has been adopted for connectivity study in wireless networks. The network is *percolated* if there exists a giant connected component in it, information can be disseminated to most of the users via this component.

One well-known result for the network percolation under the signal-to-interference ratio (SIR) model indicates that there exists a critical orthogonality factor $\gamma \in [0, 1]$, which stems from the imperfect orthogonality of the CDMA codes, above which the network will never percolate [2, 3]. But they implicity assume that the SIR threshold $T > 1$. With the development of detection techniques, weak signal detection becomes feasible, and reliable transmission could be more meaningful than high transmission rate in certain scenarios. For example, for a sensor network designed to collect environmental data, the reliability and durability is more important than the transmission rate.

This work was supported in part by the NSFC under Grants 61271257 and 61328102, the National Research Foundation for the Doctoral Program of Higher Education of China under Grant 20120005110007, and the China 973 Program under Grant 2012CB315801, and was also supported in part by the US National Science Foundation under Grants CNS-1016260, ECCS-1307949 and EARS-1444009. This work was done when Yang Liu was a visiting scholar at North Carolina State University (Raleigh, USA) supported by the China Scholarship Council.

The scenario becomes even more aggravated in overlaid networks where secondary users (SUs) coexist with licensed primary users (PUs) and share the same spectrum, while the PUs have the priority to access the spectrum and the SUs need to operate conservatively to limit their interference to the PUs [4]. Thus the percolation of SU networks could be much more challenging and interesting.

The connectivity has been widely explored in literature. A connectivity region for the overlaid networks under a protocol model is established in [5], but the percolation under the SIR model was not investigated. In [6], the author showed that for a small threshold, there exists a density interval for which percolation happens in a stand-alone network. But they only provided a rough range. The percolation of a multi-channel overlaid networks under SINR model was studied in [7], while the outage constraints were ignored. The local connectivity of the overlaid networks is studied in [8], but they didn't consider the percolation. To the best of our knowledge, there is no similar work for overlaid networks under the SIR model subject to outage constraints. Motivated by the results in [2, 3], we focus on the percolation of overlaid networks with $\gamma = 1$ and endeavor to investigate the percolation for small enough SIR threshold case. In this paper, we obtain a quantitative density interval for the stand-alone network, which is a new progress from [6]. Further for the overlaid networks, we propose a 2-dimensional (2D) connectivity region, which could be used to estimate the network percolation status intuitively.

The rest of the paper is organized as follows. The system model is introduced in Section II. In section III the connectivity for the stand-alone network case is studied. In section IV the connectivity for the overlaid network case is investigated. Finally the paper is concluded in Section V.

2. SYSTEM MODEL

Consider that a PU network and a SU network coexist in the same region $\mathbb{R}^2$. We assume the distribution of PUs (SUs) follows a homogeneous Poisson Point Process $\Pi_p = \{X_p(i)\}$

($\Pi_s = \{X_s(i)\}$) of density λ_p (λ_s). For simplicity, we assume all PUs (SUs) have the same transmission power P_p (P_s), and the distance between each primary (secondary) transmitter-receiver pair is r_p (r_s). For the wireless channel, we consider both the large-scale path-loss and small-scale Rayleigh fading. As such, the normalized channel power gain $g(d)$ is given as $g(d) = \gamma d^{-\alpha}$, where γ denotes the small-scale fading drawn from an exponential distribution. d is the distance between the transmitter and the corresponding receiver, and $\alpha > 2$ denotes the path-loss exponent. We use T_p (T_s) to represent the SIR threshold for the PU (SU) network, further we denote the outage constraint of the PU (SU) network as ϵ_p (ϵ_s). The thermal noise is assumed negligible in this interference-limited scenario.

3. PERCOLATION IN STAND-ALONE NETWORK

When the SU network is absent, a typical PU $X_p(i)$ is connected to its corresponding receiver $Y_p(i)$ at distance r_p if

$$\text{Prob}\left(\frac{P_p g(r_p)}{I_p} < T_p\right) \leq \epsilon_p, \tag{1}$$

where $I_p = \sum_{X_p(k)\in\prod_p \setminus\{X_p(i)\}} P_p g(\|X_p(k) - Y_p(i)\|)$ is the sum interference power from concurrent PU transmissions, and $\|\cdot\|$ is the Euclidean norm. From the above constraint, we can easily obtain a maximum PU density, denote as λ_p^{max}, below which the primary outage constraint ϵ_p can be satisfied. Thus we have

$$\lambda_p \leq \lambda_p^{max} = \frac{-\ln(1-\epsilon_p)}{2\pi k_\alpha T_p^{2/\alpha} r_p^2}, \tag{2}$$

where $k_\alpha = \frac{\pi\alpha^{-1}}{\sin(2\pi/\alpha)}$. Similarly, when the PU network is absent, we have

$$\lambda_s \leq \lambda_s^{max} = \frac{-\ln(1-\epsilon_s)}{2\pi k_\alpha T_s^{2/\alpha} r_s^2}. \tag{3}$$

According to percolation theory [1], for a stand-alone network with density λ and transmission range r, there exists a critical density $\lambda_c(r)$ ($\lambda_c(r) \approx \lambda_c(1) r^{-2}$, where $\lambda_c(1) \approx 1.44$ is the critical density for the network in which users have unit transmission range), such that the network is percolated when $\lambda > \lambda_c(r)$. We have the following theorem.

Theorem 1. When $T_s < \left(\frac{-\ln(1-\epsilon_s)}{2\pi k_\alpha \lambda_c(1)}\right)^{\frac{2}{\alpha}}$, there exists a closed density interval $\Lambda = [\lambda_c(r_s), \lambda_s^{max}]$, such that the stand-alone network is percolated if $\lambda_s \in \Lambda$.

Proof: We know $\lambda_c(r_s)$ is the critical density for the stand-alone SU network, above which the percolation happens; while λ_s^{max} is the maximum density for the stand-alone SU network, below which the outage constraint is assured. Thus when $\lambda_c(r_s) < \lambda_s^{max}$, there exists a closed density interval $\Lambda = [\lambda_c(r_s), \lambda_s^{max}]$, such that if $\lambda_s \in \Lambda$, the stand-alone SU network is percolated. In this case we have

$$\begin{aligned} \frac{\lambda_c(1)}{r_s^2} &< \frac{-\ln(1-\epsilon_s)}{2\pi k_\alpha T_s^{2/\alpha} r_s^2} \\ \Rightarrow T_s &< \left(\frac{-\ln(1-\epsilon_s)}{2\pi k_\alpha \lambda_c(1)}\right)^{\frac{2}{\alpha}}. \quad \square \end{aligned} \tag{4}$$

Remark: This result shows that for a small enough threshold, there exists a closed density interval. For example, if we choose $\alpha = 4$ and $\epsilon_s = 0.1$, according to *Theorem 1*, we know that such density interval exist if the threshold $T_s < 0.14$. When the density is chosen within this interval, the network will percolate. It is consistent with the conclusion in [6]. Since T determines the user's transmission rate, data transfers can be achieved via this giant connected component by using low rate links with strong error correction techniques.

4. PERCOLATION IN OVERLAID NETWORKS

In this case, both the outage constraints of the PU network and the SU network should be satisfied simultaneously. We have

$$\text{Prob}\left(\frac{P_p g(r_p)}{I_p + I_{sp}} < T_p\right) \leq \epsilon_p, \tag{5}$$

$$\text{Prob}\left(\frac{P_s g(r_s)}{I_s + I_{ps}} < T_s\right) \leq \epsilon_s, \tag{6}$$

where $I_{sp} = \sum_{X_s(k)\in\prod_s} P_s g(\|X_s(k) - Y_p(i)\|)$ ($I_{ps} = \sum_{X_p(k)\in\prod_p} P_p g(\|X_p(k) - Y_s(i)\|)$) is the sum interference power from concurrent SU (PU) transmissions. In order to satisfy both the constraints in (5) and (6) simultaneously, the SU density should be bounded above by:

$$\lambda_s \leq \lambda_s^{m1} = (\frac{P_p}{P_s})^{\frac{2}{\alpha}}(\lambda_p^{max} - \lambda_p). \tag{7}$$

$$\lambda_s \leq \lambda_s^{m2} = \lambda_s^{max} - \lambda_p(\frac{P_p}{P_s})^{\frac{2}{\alpha}}. \tag{8}$$

By combining (7) and (8), we have

$$\lambda_s \leq min(\lambda_s^{m1}, \lambda_s^{m2}). \tag{9}$$

4.1. Feasible density region

Feasible density region is defined as the set of density pairs $\{(\lambda_s, \lambda_p)\}$, in which both the primary and secondary outage constraints are satisfied. In this paper, we mainly focus on the case when the secondary outage constraint dominants, i.e., $\lambda_s^{m2} \leq \lambda_s^{m1}$. Shown as the inner triangle region in Fig. 1, the feasible density region is a closed 2D area, where *line a* indicates the case when the equality (7) holds, and *line b* indicates the case when (8) is tighter than (7). The case that

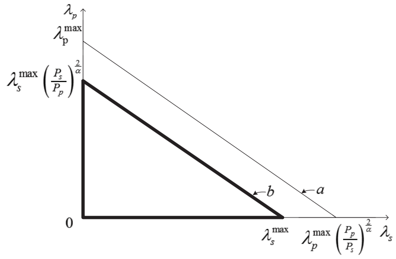

Fig. 1. The feasible density region when the secondary outage constraint dominates ($\lambda_s^{max} \leq \lambda_p^{max}\big(\frac{P_p}{P_s}\big)^{\frac{2}{\alpha}}$).

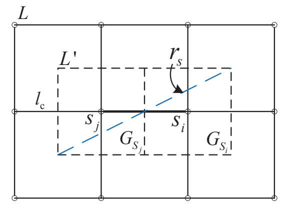

Fig. 2. The grid lattice construction.

primary outage constraint dominates will be considered in future work.

The SU's transmission range in this case can be easily obtained from (8) and (3) as

$$r_s \leq \sqrt{\frac{-\ln(1-\epsilon_s)}{2\pi k_\alpha T_s^{2/\alpha}\big(\lambda_p\big(\frac{P_p}{P_s}\big)^{\frac{2}{\alpha}}+\lambda_s\big)}}. \qquad (10)$$

4.2. Percolation based on the feasible density region: edge open probability approach

In this part, we investigate the connectivity from the percolation perspective. From [9] we know that the network is percolated if the edge open probability is larger than a threshold $P_c = 1/2$. To obtain the edge open probability, we will map the network model to a discrete percolation model. Our approach takes the following procedure. We begin by constructing a square lattice L with edge length l_c, which covers the whole network area. Let L' be the dual lattice of L, which is constructed by shifting of $l_c/2$ horizontally and vertically from L, as depicted in Fig. 2. Thus each site (vertex) s_i from L is associated with a dashed square G_{s_i}. For simplicity, we assume $l_c = r_s/\sqrt{5}$, which ensures a random SU can communicate with any SUs located in the neighboring squares.

The site s_i is said to be open when there exist at least one SU within G_{s_i}. The probability that there exist at least one SU within one dashed square is

$$P_s = 1-\exp\big(-\lambda_s r_s^2/5\big). \qquad (11)$$

A bond $l_c = s_i s_j$ is declared to be open if both s_i and s_j are open, thus the edge open probability for the overlaid SIR model can be obtained as

$$P_e = P_s^2. \qquad (12)$$

From [9], we know that if $P_e \geq P_c$, the network is percolated almost surely. Thus the SU network is percolated if the following inequality holds

$$\big[1-\exp\big(-\lambda_s r_s^2/5\big)\big]^2 \geq P_c, \qquad (13)$$

that is

$$\left[1-\exp\left(-\frac{\lambda_s}{5}\frac{-\ln(1-\epsilon_s)}{2\pi k_\alpha T_s^{\frac{2}{\alpha}}\big(\lambda_p\big(\frac{P_p}{P_s}\big)^{\frac{2}{\alpha}}+\lambda_s\big)}\right)\right]^2 \geq P_c. \quad (14)$$

After some simplification, we obtain

$$\begin{aligned}\frac{\lambda_p}{\lambda_s} &\leq \left(\frac{\ln(1-\epsilon_s)^{-1}}{10\pi k_\alpha T_s^{\frac{2}{\alpha}}\ln(1-P_c^{1/2})^{-1}}-1\right)\left(\frac{P_s}{P_p}\right)^{\frac{2}{\alpha}}\\ &\triangleq n\left(\frac{P_s}{P_p}\right)^{\frac{2}{\alpha}} \triangleq K_1,\end{aligned} \qquad (15)$$

which is represented by *line c* in Fig. 3.

To determine the shape of the connectivity region, we will evaluate the position of *line c*. From Fig. 3, we know that a special point $(\lambda_c(r_s), \lambda_{p,c}^{max})$ is located on *line b*, where $\lambda_{p,c}^{max} = (\frac{P_s}{P_p})^{\frac{2}{\alpha}}(\lambda_s^{max} - \lambda_c(1)r_s^{-2})$ can be obtained from (8). We define $\lambda_{p,c}^{max}$ as the maximum allowable PU density. When the PU density is above this value, no matter how large the SU density is, the SU network will never percolate.

Draw a straight line across the origin and $(\lambda_c(r_s), \lambda_{p,c}^{max})$, we obtain a new line, denote as *line d*. Further we denote the slope of *line d* as K_2, given by

$$K_2 = \Big(\frac{P_s}{P_p}\Big)^{\frac{2}{\alpha}}\left(\frac{\lambda_s^{max}-\lambda_c(1)r_s^{-2}}{\lambda_c(1)r_s^{-2}}\right). \qquad (16)$$

To evaluate the impact of *line c* on the connectivity region, we need to compare the slope of *line c* and *line d*. We have

$$\begin{aligned}\frac{K_2}{K_1} &= \frac{\big(\frac{P_s}{P_p}\big)^{\frac{2}{\alpha}}\left(\frac{\lambda_s^{max}-\lambda_c(1)r_s^{-2}}{\lambda_c(1)r_s^{-2}}\right)}{\big(\frac{P_s}{P_p}\big)^{\frac{2}{\alpha}}\left(\frac{\ln(1-\epsilon_s)^{-1}}{10\pi k_\alpha T_s^{\frac{2}{\alpha}}\ln(1-P_c^{1/2})^{-1}}-1\right)}\\ &= \frac{\lambda_s^{max}r_s^2/\lambda_c(1)-1}{\lambda_s^{max}r_s^2/\big(5\ln(1-p_c^{1/2})^{-1}\big)-1} > 1,\end{aligned} \quad (17)$$

thus we know that *line c* is located under *line d*, which means the secondary network is percolated if the density pair (λ_s, λ_p) is chosen under *line c*. Since $\lambda_s \geq \lambda_c(r_s)$, for the secondary outage constraint dominant case ($\lambda_s^{max} \leq \lambda_p^{max}\big(\frac{P_p}{P_s}\big)^{\frac{2}{\alpha}}$), the connectivity region under the overlaid SIR model is an irregular shaped area shown as the shaded region

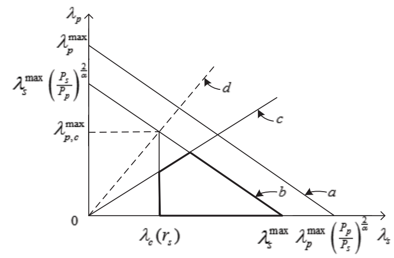

Fig. 3. Connectivity region for the overlaid networks with SIR model where the SU network outage constraint dominates.

in Fig. 3. The overlaid network with density pair (λ_s, λ_p) chosen from this connectivity region satisfies the condition for percolation of the secondary network, as well as the outage constraints for both networks.

4.3. Properties of the connectivity region

Denote the irregular shaped connectivity region as $\mathcal{C}^2$, we have the following theorem.

Theorem 2. Basic properties of the connectivity region $\mathcal{C}^2$

T2.1 $\mathcal{C}^2$ is a contiguous area, that is, for any two points $(\lambda_{s1}, \lambda_{p1})$ and $(\lambda_{s2}, \lambda_{p2})$, there is a continuous path in $\mathcal{C}^2$ connecting the two points.

T2.2 $\mathcal{C}^2$ is a convex area, that is, draw a straight line between any two points $(\lambda_{s1}, \lambda_{p1})$ and $(\lambda_{s2}, \lambda_{p2})$ in $\mathcal{C}^2$, all the points located on this line belong to $\mathcal{C}^2$.

T2.3 For the secondary outage constraint dominant case, the area of $\mathcal{C}^2$ is an increasing function of P_s, and it is maximized when $P_s = P_p\left(\lambda_p^{max}/\lambda_s^{max}\right)^{\frac{\alpha}{2}}$.

Proof: The proof of T2.1 is omitted due to limited space. Readers may refer to *Theorem 1.1* in [10] for a similar proof.

T2.2: As shown in Fig. 3, we know that the points in $\mathcal{C}^2$ satisfy the four conditions below:

$$\begin{cases} \lambda_s \geq \lambda_c(r_s), & (18) \\ \lambda_p \geq 0, & (19) \\ \lambda_s + m\lambda_p \leq \lambda_s^{max}, & (20) \\ \lambda_p \leq n\lambda_s/m, & (21) \end{cases}$$

where $n = \left(\frac{\ln(1-\epsilon_s)^{-1}}{10\pi k_\alpha T_s^{\frac{2}{\alpha}} \ln(1-P_c^{1/2})^{-1}} - 1\right)$ and $m = \left(\frac{P_p}{P_s}\right)^{\frac{2}{\alpha}}$ are constants. Assume we have two random points $(\lambda_{s1}, \lambda_{p1})$ and $(\lambda_{s2}, \lambda_{p2})$ in $\mathcal{C}^2$, and draw a straight line between these two points. Denote the coordinates of the points located on this straight line as (λ_s', λ_p'), we have

$$\begin{cases} \lambda_s' = \theta\lambda_{s1} + (1-\theta)\lambda_{s2}, \\ \lambda_p' = \theta\lambda_{p1} + (1-\theta)\lambda_{p2}, \end{cases}$$

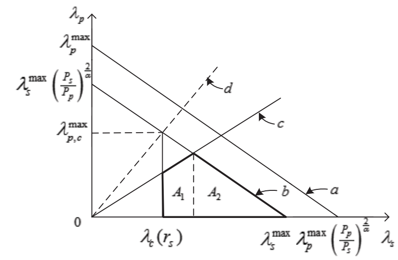

Fig. 4. Partition of $\mathcal{C}^2$. The area of $\mathcal{C}^2$ is the sum of two independent areas, denote as A_1 and A_2, respectively.

where θ $(0 \leq \theta \leq 1)$ is a constant. Thus we only need to show that (λ_s', λ_p') satisfies (18)–(21).

From $\lambda_{s1} \geq \lambda_c(r_s)$, we have $\theta\lambda_{s1} \geq \theta\lambda_c(r_s)$. Correspondingly, we have $(1-\theta)\lambda_{s2} \geq (1-\theta)\lambda_c(r_s)$. Thus we can obtain $\theta\lambda_{s1} + (1-\theta)\lambda_{s2} \geq \lambda_c(r_s)$, i.e., $\lambda_s' \geq \lambda_c(r_s)$, and (18) is satisfied. Conditions (19)–(21) can be checked similarly, which is omitted in the interest of space.

T2.3: Denote the area of $\mathcal{C}^2$ as A. As shown in Fig. 4, A is the sum of two independent areas, denote as A_1 and A_2, respectively. We have

$$A_1 = \frac{n}{2}\left(\frac{\lambda_c(1)}{r_s^2} + \frac{\lambda_s^{max}}{1+n}\right)\left(\frac{\lambda_s^{max}}{1+n} - \frac{\lambda_c(1)}{r_s^2}\right)\left(\frac{P_s}{P_p}\right)^{\frac{2}{\alpha}}, \quad (22)$$

$$A_2 = \frac{1}{2}\left(\frac{n\lambda_s^{max}}{1+n}\right)^2\left(\frac{P_s}{P_p}\right)^{\frac{2}{\alpha}}, \quad (23)$$

where n is the same constant with that in (15). Thus we know that $A = A_1 + A_2$ is an increasing function of P_s, and A is maximized when we have the maximum P_s. In this paper we assume $\lambda_s^{max} \leq \lambda_p^{max}\left(\frac{P_p}{P_s}\right)^{\frac{2}{\alpha}}$, thus the maximum area can be achieved when we choose $P_s = P_p\left(\lambda_p^{max}/\lambda_s^{max}\right)^{\frac{\alpha}{2}}$. □

Remark: $T2.1$ and $T2.2$ specify the basic structure of $\mathcal{C}^2$. $T2.3$ further reveals the impact of SU network parameters on $\mathcal{C}^2$. Thus in order to achieve the maximum area for $\mathcal{C}^2$, the SU's power should be designed carefully.

5. CONCLUSION

In this paper, we study the connectivity of overlaid wireless networks where two users can communicate if the SIR at the receiver is larger than a threshold subject to an outage constraint. We derive a connectivity region unde the SIR model, which can be used to estimate the network percolation status. Our analysis provides a new perspective for better understanding of the connectivity of large-scale overlaid networks with outage constraints.

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