

# BLIND CFO ESTIMATION FOR MULTIUSER OFDM UPLINK WITH LARGE NUMBER OF RECEIVE ANTENNAS

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## ABSTRACT

In this paper, we propose a new blind carrier frequency offset (CFO) estimation method for multiuser orthogonal frequency division multiplexing (OFDM) uplink transmissions. The spatial multiplexing is supported in the considered model that allows the subcarriers to be simultaneously occupied by multiple users. We propose to assign different null subcarriers to different users and design algorithm that can perform blind CFO estimation for each individual user with the aid of large number of receive antennas, which then removes the necessity of multidimensional searching. Numerical results are provided to corroborate the proposed studies.

**Index Terms**— Multiuser CFO estimation, OFDM, large number of antennas.

## 1. INTRODUCTION

The carrier frequency offset (CFO) estimation has drawn substantial research interests during past few years for orthogonal frequency division multiplexing (OFDM) systems [1]. Especially, when distributed transmitters send data streams simultaneously, multiple CFOs will be observed at receiver, making the CFO estimation and compensation more challenging. A typical scenario is the so-called orthogonal frequency division multiple access (OFDMA), where the subcarriers are exclusively occupied by the multiple users. During the past few years, a number of multiuser CFO estimation schemes for OFDMA uplink have been developed [2–4], where different CFOs could either be estimated from an iterative approach or found from some subspace methods.

With the increasing demand of enhancing spectral utilization, the multiuser uplink transmissions with spatially multiplexed (SM) OFDM have attracted considerable attention. In this case, one subcarrier could be simultaneously occupied by multiple users and thus a much higher spectral efficiency can be achieved. However, multiple users occupy

ing the same subcarrier makes the CFO estimation more difficult even compared to the OFDMA uplink transmission. There are very few results on multiple CFO estimation for multiuser SM-OFDM transmission. Besson and Stoica made the first trial in [5] while restricted their results only for flat fading channels. In [6], a semiblind method was proposed to simultaneously estimate the multiple CFOs and channels in frequency selective fading channels but the approach is only valid for zero-padding (ZP) OFDM. Another joint CFO and channel estimation for multiuser cyclic-prefix (CP) MIMO-OFDM systems was developed in [7] based on the maximum likelihood (ML) criterion. The high-complex multidimensional search is reduced from the importance sampling technique, while the remaining complexity to generate sufficient samples for importance sampling may still be high for practical implementation. Another suboptimal estimation algorithm was proposed in [8] which requires long constant amplitude zero autocorrelation (CAZAC) training sequences for accurate CFO estimation.

Another way to further improve the spectral utilization is to tremendously increase the number of antennas at the base station (BS), as was shown in [9]. Since equipping large number of antennas at BS inherently targets at serving multiple users simultaneously, the difficulty of dealing with multiple CFOs from multiuser is unavoidable. Nevertheless, to the best of our knowledge, no result about how the blind CFO estimation could be performed with large number of antennas has been reported yet.

In this paper, we present a recent finding that increasing the number of receive antennas is also helpful to resolve the multiple dimensional search required in estimating multiple CFOs. Specifically, we propose a new blind CFO estimation method for multiuser uplink transmissions with SM-OFDM modulation. One can always assign a few null subcarriers to each user, and with the large number of receive antennas, i.e., the sufficient spatial dimensions, the proposed scheme can blindly estimate each CFO for each user, which eliminates the necessity of multidimensional search. The numerical results are provided to verify the proposed scheme.

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## 2. SYSTEM MODEL

Consider a multiuser SM-OFDM uplink system that consists of  $K$  distributed users, each equipped with a single antenna, and one BS with  $M$  antennas. The total number of subcarriers is  $N$ , and, for the time being, we assume perfect time synchronization among all nodes. Denote the normalized CFO between the  $k$ th user and BS as  $\phi^{(k)}$ , which is the ratio between the real CFO and the subcarrier spacing. We consider the fractional CFO in this paper such that the region of CFO is  $\phi^{(k)} \in (-0.5, 0.5)$ .

The propagation channel from the  $k$ th user to the  $m$ th receive antenna is modeled as a length- $L$  vector of  $\mathbf{h}_m^{(k)} = [h_m^{(k)}(1), h_m^{(k)}(2), \dots, h_m^{(k)}(L)]^T$ , whose elements are independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and power  $E[|h_m^{(k)}(l)|^2] = \frac{1}{ML}$ . Then, the total channel gain between the  $k$ th user and the BS is normalized, i.e.,  $E[\sum_{m=1}^M \sum_{l=1}^L |h_m^{(k)}(l)|^2] = 1$ . Without loss of generality, we assume both CFO and propagation channels of all users stay constant over the successive  $L_b$  OFDM block durations. In this paper, we specifically consider that the number of the receive antennas at BS is very large such that  $M \geq KL$  holds. Note that there is an increasing interest from both academy and industry to equip BS with a large scale antenna array recently [?, 9]. With tremendously enlarged spatial dimensions, such a system can provide a remarkable increase in both reliability and spectral efficiency.

Let all subcarriers be sequentially indexed with  $\{i\}$ ,  $i = 1, 2, \dots, N$ . To simplify the presentation, we consider that each user occupies  $N_d$  subcarriers for data transmission and reserves the rest  $N_\emptyset = N - N_d$  subcarriers as *null* subcarriers, while the corresponding discussion could be straightforwardly extended to more general case. Denote the index set of data subcarriers and null subcarriers for the  $k$ th user as  $\mathcal{D}^{(k)} = \{\mathcal{D}_1^{(k)}, \mathcal{D}_2^{(k)}, \dots, \mathcal{D}_{N_d}^{(k)}\}$  and  $\mathcal{V}^{(k)} = \{\mathcal{V}_1^{(k)}, \mathcal{V}_2^{(k)}, \dots, \mathcal{V}_{N_\emptyset}^{(k)}\}$ , respectively. It should be emphasized that the spatial multiplexing is considered in this paper where the subcarriers can be simultaneously occupied by multiple users.

Denote  $\mathbf{s}_g^{(k)} = [s_g^{(k)}(1), s_g^{(k)}(2), \dots, s_g^{(k)}(N_d)]^T$  as the data symbols from the  $k$ th user in the  $g$ th block. Without loss of generality, we consider the average power of data symbols is normalized, i.e.,  $E[|s_g^{(k)}(i)|^2] = 1$ . Meanwhile, define  $\mathbf{T}_d^{(k)}$  as the  $N \times N_d$  data subcarrier assignment matrix for the  $k$ th user, whose  $(\mathcal{D}_i^{(k)}, i)$ -th entry,  $i = 1, 2, \dots, N_d$ , equals one and the rest entries are zeros. Let  $\eta^g(\phi^{(k)}) = e^{j\frac{2\pi(g-1)(N+N_{cp})\phi^{(k)}}{N}}$  represent the accumulative phase shift of the  $g$ th OFDM block introduced by the CFO  $\phi^{(k)}$ , where  $N_{cp} \geq L$  is the length of CP. Denote  $\mathbf{F}$  as the  $N \times N$  normalized DFT matrix with its  $(i, j)$ -th entry

being  $\frac{1}{\sqrt{N}}e^{-j\frac{2\pi(i-1)(j-1)}{N}}$ . Moreover, denote

$$\mathbf{E}(\phi^{(k)}) = \text{diag}\left(1, e^{j\frac{2\pi\phi^{(k)}}{N}}, \dots, e^{j\frac{2\pi(N-1)\phi^{(k)}}{N}}\right) \quad (1)$$

as the  $N \times N$  diagonal matrix representing the phase rotation introduced by the CFO  $\phi^{(k)}$  inside one OFDM block.

For the  $g$ th block, the received time-domain signal at the  $m$ th antenna can then be expressed as the following length- $N$  vector:

$$\mathbf{y}_{m,g} = \sqrt{N} \sum_{k=1}^K \eta^g(\phi^{(k)}) \mathbf{E}(\phi^{(k)}) \mathbf{F}^H \mathbf{X}_g^{(k)} \mathbf{F}_L \mathbf{h}_m^{(k)} + \mathbf{w}_{m,g}$$

where  $\mathbf{X}_g^{(k)} = \text{diag}(\mathbf{T}_d^{(k)} \mathbf{s}_g^{(k)})$ ,  $\mathbf{F}_L \in \mathbb{C}^{N \times L}$  consists of the first  $L$  columns of  $\mathbf{F}$ , and  $\mathbf{w}_{m,g} \in \mathbb{C}^{N \times 1}$  denotes the corresponding additive white Gaussian noise (AWGN) vector with covariance matrix of  $E[\mathbf{w}_{m,g} \mathbf{w}_{m,g}^H] = \sigma_w^2 \mathbf{I}_N$ .

## 3. PROPOSED CFO ESTIMATION

We define the following channel response matrix for the  $k$ th user:

$$\mathbf{H}^{(k)} = [\mathbf{h}_1^{(k)}, \mathbf{h}_2^{(k)}, \dots, \mathbf{h}_M^{(k)}]^T \in \mathbb{C}^{M \times L}, \quad (2)$$

whose  $m$ th row is  $\mathbf{h}_m^{(k)}$ . Let us perform CFO compensation with a trial value  $\tilde{\phi} \in (-0.5, 0.5)$  on the received signal of  $\mathbf{y}_{m,g}$ ,  $m = 1, 2, \dots, L_b$ . Then the following  $M \times N$  frequency domain signal matrix, after DFT operation, can be obtained:

$$\begin{aligned} \mathbf{Y}_g(\tilde{\phi}) &= \eta^g(-\tilde{\phi}) [\mathbf{y}_{1,g}, \mathbf{y}_{2,g}, \dots, \mathbf{y}_{M,g}]^T \mathbf{E}(-\tilde{\phi}) \mathbf{F} \\ &= \sqrt{N} \sum_{k=1}^K \eta^g(\phi^{(k)} - \tilde{\phi}) \mathbf{H}^{(k)} \mathbf{F}_L^T \mathbf{X}_g^{(k)} \mathbf{C}(\phi^{(k)} - \tilde{\phi}) + \mathbf{N}_g(\tilde{\phi}) \end{aligned}$$

where  $\mathbf{C}(\phi^{(k)} - \tilde{\phi}) = \mathbf{F}^H \mathbf{E}(\phi^{(k)} - \tilde{\phi}) \mathbf{F}$  stands for the inter-carrier interference (ICI) matrix and  $\mathbf{N}(\tilde{\phi}) \in \mathbb{C}^{M \times N}$  denotes the corresponding additive noise matrix.

Define an  $N \times N_\emptyset$  null subcarrier assignment matrix  $\mathbf{T}_v^{(q)}$  for the  $q$ th user whose  $(\mathcal{V}_i^{(q)}, i)$ -th entry,  $i = 1, 2, \dots, N_\emptyset$ ,  $q = 1, \dots, K$ , equals one and the rest entries are zeros. Define

$$\mathbf{Y}_g^{(q)}(\tilde{\phi}) = \mathbf{Y}_g(\tilde{\phi}) \mathbf{T}_v^{(q)}, \quad (3)$$

which consists of the  $N_\emptyset$  column vectors of  $\mathbf{Y}_g(\tilde{\phi})$  with the column indices corresponding the  $N_\emptyset$  null subcarriers of the  $q$ th user.

Moreover, define the covariance matrix

$$\mathbf{R}^{(q)}(\tilde{\phi}) = E \left[ \mathbf{Y}_g^{(q)}(\tilde{\phi}) (\mathbf{Y}_g^{(q)}(\tilde{\phi}))^H \right] \quad (4)$$

where the expectation is taken with respect to both the transmitted data symbols and noise items. In practice, the covariance matrix of  $\mathbf{R}^{(q)}(\tilde{\phi})$  can be approximated by

$$\hat{\mathbf{R}}^{(q)}(\tilde{\phi}) = \sum_{g=1}^{L_b} \mathbf{r}_g^{(q)}(\tilde{\phi})(\mathbf{r}_g^{(q)}(\tilde{\phi}))^H. \quad (5)$$

Similar to the case of array signal processing [10], the matrix of  $\mathbf{r}_g^{(q)}(\tilde{\phi})$  can be considered as a collection of space-domain snapshots at the null subcarriers reserved by the  $q$ th user. Based on the subspace theory, the corresponding signal subspace and noise subspace can be obtained from the eigenvalue decomposition of the covariance matrix  $\mathbf{R}^{(q)}(\tilde{\phi})$ . The following *Lemma* provides the key property to design our proposed CFO estimation.

*Lemma 1:* We consider  $M \geq KL$  and denote  $D$  as the dimension of the signal subspace of  $\mathbf{R}^{(q)}(\tilde{\phi})$ . When the CFO trial value equals the CFO of the  $q$ th user, i.e.,  $\tilde{\phi} = \phi^{(q)}$ , there holds

$$(K-1)L \geq D \geq \sum_{\substack{k=1 \\ k \neq q}}^K \min(L, |\mathcal{D}^{(k)} \cap \mathcal{V}^{(q)}|). \quad (6)$$

For any other  $\tilde{\phi} \neq \phi^{(q)}$ , there is

$$KL \geq D \geq \min\left(KL, L + \sum_{\substack{k=1 \\ k \neq q}}^K \min(L, |\mathcal{D}^{(k)} \cap \mathcal{V}^{(q)}|)\right). \quad (7)$$

*Proof.* See Appendix A.  $\square$

To simplify the presentation, we consider that each user exclusively occupies a few null subcarriers, i.e.,  $\mathcal{V}^{(k)} \cap \mathcal{V}^{(q)} = \emptyset$  for any  $k \neq q$  and  $\sum_{k=1}^K |\mathcal{V}^{(k)}| \leq N$ . Meanwhile, the number of null subcarriers of each user is no less than  $L$ , i.e.,  $|\mathcal{V}^{(k)}| \geq L$  for any  $k$ . As a consequence, the inequality  $|\mathcal{D}^{(k)} \cap \mathcal{V}^{(q)}| \geq L$  holds for any  $k \neq q$ . Here we should note that the following discussions can be easily applied to the more general cases when the intervals in (6) and (7) are disjoint from each other, i.e.,  $\sum_{k=1, k \neq q}^K \min(L, |\mathcal{D}^{(k)} \cap \mathcal{V}^{(q)}|) > (K-2)L$ .

Then, according to the above *Lemma*, we readily arrive at

$$\begin{cases} D = (K-1)L, & \tilde{\phi} = \phi^{(q)}, \\ D = KL, & \tilde{\phi} \neq \phi^{(q)}, \end{cases} \quad (8)$$

which implies the dimension of signal subspace of  $\mathbf{R}^{(q)}(\tilde{\phi})$  drops from  $KL$  to  $(K-1)L$  once  $\tilde{\phi} = \phi^{(q)}$ ,  $\forall q = 1, 2, \dots, K$ . In other words, the number of the smallest eigenvalues of  $\mathbf{R}^{(q)}(\tilde{\phi})$  corresponding to the noise subspace will increase from  $M - KL$  to  $M - (K-1)L$  once  $\tilde{\phi} = \phi^{(q)}$ .

Based on the above observations, we can design the CFO estimation for the  $q$ th user as follows. Denote the eigenvalues

of  $\mathbf{R}^{(q)}(\tilde{\phi})$  in ascending order as  $\gamma_1^{(q)}(\tilde{\phi}), \gamma_2^{(q)}(\tilde{\phi}), \dots, \gamma_M^{(q)}(\tilde{\phi})$ . The CFO estimate for the  $q$ th user can be obtained from the following one-dimensional search:

$$\hat{\phi}^{(q)} = \arg \min_{\tilde{\phi}} g^{(q)}(\tilde{\phi}), \quad (9)$$

where the cost function is designed as

$$g^{(q)}(\tilde{\phi}) = \sum_{m=1}^{M-(K-1)L} \gamma_m^{(q)}(\tilde{\phi}). \quad (10)$$

When  $\tilde{\phi} = \phi^{(q)}$ , the summation terms of (10) are the smallest eigenvalue corresponding to the noise subspace. While for any  $\tilde{\phi} \neq \phi^{(q)}$ , the summation of (10) will include  $L$  larger eigenvalues corresponding to the signal subspace. Hence, by finding the minimum point of the cost function in (9), we will obtain a valid CFO estimate for the  $q$ th user.

## 4. SIMULATIONS

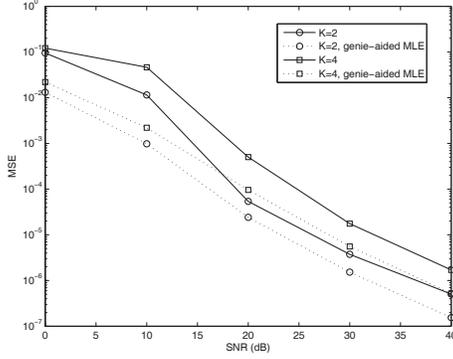
In this section, we assess the proposed multiuser frequency synchronization scheme from computer simulations. The total number of subcarriers is taken as  $N = 64$ . The normalized CFO is randomly generated from -0.4 to 0.4. 16-QAM constellation is adopted. The channel length is set as  $L = 8$ . The MSE of the normalized CFO is adopted as the figure of merit. The signal-to-noise ratio (SNR) is defined as  $1/\sigma_w^2$ . In the following we consider each user exclusively reserves eight null subcarriers, i.e.,  $N_\emptyset = 8$ . The null subcarrier index set of the  $q$ th user is expressed as  $\mathcal{V}^{(q)} = \{q, q + \frac{N}{N_\emptyset}, \dots, q + \frac{N(N_\emptyset-1)}{N_\emptyset}\}$ .

First, we assume  $M = 32$  and evaluate the CFO estimation performance of our proposed method with different number of users in Fig. 1. We also consider a ‘genie-aided MLE’ scheme for comparison, where for each user, the BS first performs MUI cancelation with perfect knowledge of all channel information and then resort the conventional ML method of [1] for CFO estimation. As expected, the estimation performance of our estimator can be improved as SNR increases, which demonstrates the validity of our estimator in spatial multiplexed multiuser uplink transmission.

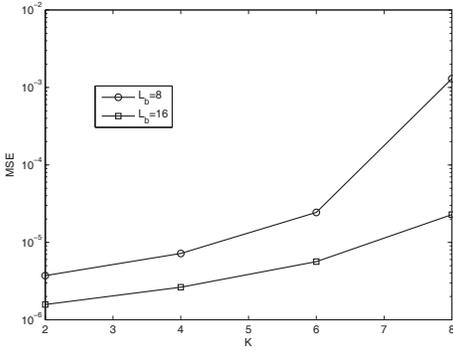
We next evaluate the estimation performance of our CFO estimator as the user number increases from  $K = 2$  to  $K = 8$  in Fig. 2. We consider  $M = 64$  and SNR = 30 dB in this example. As expected, it is seen that the estimation performance of our method gradually degrades when more users co-exist in the uplink transmissions. Nonetheless, our estimator can work well with more blocks.

## 5. CONCLUSIONS

In this paper, we developed a new blind CFO estimation method for multiuser SM-OFDM uplink transmission where



**Fig. 1.** The CFO estimation performance of our proposed method ( $M = 32$  and  $L_b = 8$ ).



**Fig. 2.** The CFO estimation performance of our method as the user number increases ( $M = 64$  and  $\text{SNR} = 30$  dB).

a large number of receive antennas are considered at the BS. The proposed scheme can perform blind CFO estimation for each user individually without any multidimensional searching procedure. The simulation results are provided which corroborate the proposed studies.

## 6. APPENDIX

Considering that the transmitted data symbols from different users are independent, we obtain

$$\begin{aligned} \mathbf{R}^{(q)}(\tilde{\phi}) &= E \left[ \mathbf{Y}_g^{(q)}(\tilde{\phi})(\mathbf{Y}_g^{(q)}(\tilde{\phi}))^H \right] \\ &= N \sum_{k=1}^K \mathbf{H}^{(k)} \mathbf{R}^{k,q}(\tilde{\phi})(\mathbf{H}^{(k)})^H + N_{\theta} \sigma_w^2 \mathbf{I}_M. \end{aligned} \quad (11)$$

where the  $L \times L$  matrix  $\mathbf{R}^{k,q}(\tilde{\phi})$  is expressed as

$$\mathbf{F}_L^T E \left[ \mathbf{X}_g^{(k)} \mathbf{C}^{(k)}(\tilde{\phi}) \mathbf{T}_v^{(q)} (\mathbf{T}_v^{(q)})^H \mathbf{C}^{(k)}(-\tilde{\phi}) (\mathbf{X}_g^{(k)})^* \right] \mathbf{F}_L.$$

It is observed that the maximum possible dimension of the signal subspace of  $\mathbf{R}^{(q)}(\tilde{\phi})$  is  $KL$ . Thus,  $M \geq KL$  is

required in this paper in order to guarantee the identifiability of the subspace method.

Denote  $\Psi_d^{(k)}$  as the data subcarrier mask matrix for the  $k$ th user with ones on the  $\mathcal{D}_i^{(k)}$ -th,  $i = 1, 2, \dots, N_d$ , diagonal entries and zeros elsewhere, and  $\Psi_v^{(k)}$  as the null subcarrier mask matrix for the  $k$ th user with ones on the  $\mathcal{V}_i^{(k)}$ -th,  $i = 1, 2, \dots, N_{\theta}$ , diagonal entries and zeros elsewhere. We can further rewrite  $\mathbf{R}^{k,q}(\tilde{\phi})$  as

$$\mathbf{R}^{k,q}(\tilde{\phi}) = \mathbf{F}_L^T \text{diag} \left( \Psi_d^{(k)} \mathbf{C}^{(k)}(\tilde{\phi}) \Psi_v^{(q)} \mathbf{C}^{(k)}(-\tilde{\phi}) \Psi_d^{(k)} \right) \mathbf{F}_L. \quad (12)$$

Note that the rank of  $\mathbf{R}^{k,q}(\tilde{\phi})$  equals that of the matrix  $\mathbf{F}_L^T \text{diag}(\Psi_d^{(k)} \mathbf{C}^{(k)}(\tilde{\phi}) \Psi_v^{(q)})$ , whose non-zero column vectors compose a full rank Vandermonde matrix. This indicates that the rank of  $\mathbf{R}^{k,q}(\tilde{\phi})$  equals the minimum value between  $L$  and the number of non-zero row vectors of matrix  $\Psi_d^{(k)} \mathbf{C}^{(k)}(\tilde{\phi}) \Psi_v^{(q)}$ .

First, we consider the case  $\tilde{\phi} = \phi^{(q)}$ . It is readily observed that  $\mathbf{R}^{q,q}(\phi^{(q)})$  becomes the zero matrix, i.e.,  $\mathbf{R}^{q,q}(\phi^{(q)}) = \mathbf{0}$ . For any  $k \neq q$ , as  $\mathbf{C}^{(k)}(\phi^{(q)})$  has no zero element when  $\phi^{(k)} \neq \phi^{(q)}$  and  $\phi^{(k)}, \phi^{(q)} \in (-0.5, 0.5)$ , we know  $\mathbf{R}^{k,q}(\phi^{(q)})$  has full rank of  $L$  in this case. Once  $\phi^{(k)} = \phi^{(q)}$ , there holds  $\mathbf{C}^{(k)}(\phi^{(q)}) = \mathbf{I}_N$  and the rank of  $\mathbf{R}^{k,q}(\phi^{(q)})$  in fact equals the minimum value between  $L$  and the number of non-zero diagonal elements of matrix  $\Psi_d^{(k)} \Psi_v^{(q)}$ , i.e.,  $\min(L, |\mathcal{D}^{(k)} \cap \mathcal{V}^{(q)}|)$ . Thus, combining both the cases of  $\phi^{(k)} \neq \phi^{(q)}$  and  $\phi^{(k)} = \phi^{(q)}$ , we obtain

$$L \geq \text{rank}(\mathbf{R}^{k,q}(\phi^{(q)})) \geq \min(L, |\mathcal{D}^{(k)} \cap \mathcal{V}^{(q)}|). \quad (13)$$

Notice that the dimension of signal subspace of  $\mathbf{R}^{(q)}(\phi^{(q)})$  can be expressed as  $D = \sum_{k \neq q} \text{rank}(\mathbf{R}^{k,q}(\phi^{(q)}))$ . According to (13), we readily arrive at (6).

Next, we consider the case  $\tilde{\phi} \neq \phi^{(q)}$ . As  $\mathbf{C}^{(q)}(\tilde{\phi})$  has no zero element when  $\tilde{\phi} \neq \phi^{(q)}$  and  $\tilde{\phi}, \phi^{(q)} \in (-0.5, 0.5)$ , we know  $\mathbf{R}^{q,q}(\tilde{\phi})$  has full rank in this case. Moreover, similar to the above discussions, the rank of  $\mathbf{R}^{k,q}(\tilde{\phi})$  equals  $L$  and  $\min(L, |\mathcal{D}^{(k)} \cap \mathcal{V}^{(q)}|)$  respectively for the case of  $\tilde{\phi} \neq \phi^{(k)}$  and  $\tilde{\phi} = \phi^{(k)}$ . Note that the dimension of signal subspace of  $\mathbf{R}^{(q)}(\tilde{\phi})$  can be expressed as  $\sum_{k=1}^K \text{rank}(\mathbf{R}^{k,q}(\tilde{\phi})) = L + \sum_{k \neq q} \text{rank}(\mathbf{R}^{k,q}(\tilde{\phi}))$ . Then, according to the above discussions, we arrive at (7). This completes the proof.

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