PILOT-BASED CHANNEL ESTIMATION FOR FBMC/OQAM SYSTEMS UNDER STRONG FREQUENCY SELECTIVITY

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ABSTRACT

Scattered pilot-aided channel estimation in offset QAM-based filter bank multicarrier (FBMC/OQAM) systems has so far been only considered for channels of mild frequency selectivity. In more demanding scenarios, the classical auxiliary pilot (AP) idea has been shown to result in severe error floors. In this paper, a novel pilot-aided channel estimation method is developed which *for the first time* extends the applicability of the AP idea to highly frequency selective channels. The development relies on a Taylor series approximation of the signal model, which is able to accurately and concisely describe such scenarios. The obtained channel estimate can be viewed as a linear combination of the outputs of multiple parallel analysis filter banks each employing a derivative of the original prototype filter. The reported simulation results corroborate the analysis, demonstrating the effectiveness of the proposed method in estimating channels of strong frequency selectivity.

Index Terms— FBMC/OQAM, channel estimation, auxiliary pilots.

1. INTRODUCTION

The increasing demand for wireless services has recently prompted a renewed interest in spectrally efficient physical layer techniques [1]. Filterbank multicarrier (FBMC) modulations are an interesting alternative to cyclic prefix (CP)-based Orthogonal Frequency Division Multiplexing (OFDM), mainly because of their improved spectral localization (due to a non-rectangular pulse shaping) and also because of their possibility of suppressing the CP, which greatly improves the bandwidth and power efficiency [2]. Among all the FBMC modulations, those based on the transmission of real-valued (offset QAM) symbols (FBMC/OQAM) offer maximum spectral efficiency while maintaining the subcarrier orthogonality under frequency flat channel conditions [2, 3].

There exist two different families of training-based channel estimation methods for FBMC/OQAM: preamble-based [4], relying on a preamble of training multicarrier symbols and aiming at providing an initial channel estimate, and scattered pilot-based [6, 5], which aim at tracking the channel variations via pilot symbols scattered in the time-frequency plane. In both cases, one has to cope with the interference effect that is intrinsic to FBMC/OQAM and is well known to present a significant challenge when channels of a non-negligible frequency selectivity are to be estimated [5]. Although considerable advances in preamble-based estimation of highly frequency selective channels have been recently reported (e.g., [5]), the assumption of low frequency selectivity is crucial in almost all of the scattered pilot-based approaches, including techniques based on canceling /approximating the unknown intrinsic interference [7, 15, 16] and those relying on the so-called *help* or *auxiliary pilot (AP)* idea [6] and variants thereof [8, 9, 10, 11, 14, 17]. The latter consists in choosing the value of a symbol neighboring the pilot one so that the interference to the pilot is (almost) null. Constructing this auxiliary pilot is made possible through the assumption of the channel frequency response (CFR) being constant in both frequency and time over the neighborhood of the pilot that contributes most to the interference [6]. In the presence of less smooth channel responses, the AP scheme fails to provide good channel estimates and its performance is characterized by severe error floors at medium to high signal to noise ratios (SNR) [5]. Recent efforts in extending the AP idea to more frequency selective channels include [14], which, however, still relies on the assumption of a per-subcarrier frequency flat response, and [12], which is based on a principal component analysis of the intrinsic interference and relies on *a priori* knowledge of the channel second-order statistics to compute the generalized auxiliary pilots.

In this paper, a simple yet effective approach to generalizing the AP idea is followed, stemming from the Taylor series approximation of the received signal around the pilot frequencies and relying on a strong theoretical background for characterizing channel-induced distortion in FBMC/OQAM systems [18].

Consider a general FBMC/OQAM transmission scheme employing $2M$ uniformly spaced subcarriers, where the different filters at the transmitter and receiver are exponentially modulated versions of two real-valued prototype pulses, de-

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noted as $p[n]$ and $q[n]$, respectively.[1] Let these prototypes have a length of $2M\kappa$ samples, with κ being the overlapping factor. Denote by $y[k,n]$ the complex-valued signal at the output of the analysis filter bank (AFB), corresponding to the nth multicarrier symbol and the kth subcarrier ($k = 0, 1, \ldots, 2M-1$). Under perfect channel conditions, $y[k,n]$ can be expressed as a bidimensional convolution of the transmitted real-valued symbols $s[m,n]$ with the pulse-specific ambiguity functions $\varphi_{k,n}[m,\ell]$, namely

$$y[k,n] = \sum_{m,\ell} s\left[(k-m)_{2M}, n-\ell\right] \varphi_{k,n}[m,\ell], \quad (1)$$

where $(\cdot)_{2M}$ stands for modulo-$2M$ reduction. The ambiguity functions $\varphi_{k,n}[m,\ell]$ generally depend on the parity of the values k, n, and take on non-zero values only in the support set $\mathcal{S} = \{(m,\ell) : 0 \le m < 2M, -2\kappa < \ell < 2\kappa\}$, understood as positions in the time-frequency plane.

If the prototype pulses $p[n]$ and $q[n]$ are designed to guarantee perfect reconstruction (PR) of the transmitted signal, the ambiguity functions can be expressed as $\varphi_{k,n}[m,\ell] = \alpha_{k,n}[m,\ell] + \mathrm{j}\,\beta_{k,n}[m,\ell]$, where $\alpha_{k,n}[m,\ell] = \delta_{k,m}\delta_{n,\ell}$ and $\beta_{k,n}[m,\ell]$ is real valued, and the received signal takes the form

$$y[k,n] = s[k,n] + \\ + \mathrm{j} \sum_{(m,\ell)\in\mathcal{S}\setminus\{(k,n)\}} s\left[(k-m)_{2M}, n-\ell\right]\beta_{k,n}[m,\ell], \quad (2)$$

where the noise term is omitted for the time being for the sake of simplicity of presentation. Clearly, $\mathrm{Re}\,[y[k,n]] = s[k,n]$, as expected under PR conditions.

In the presence of a frequency selective channel, the signal model in (2) is no longer valid. However, if the channel is sufficiently flat around the kth subcarrier and locally time-invariant, one can approximate the received signal as

$$z[k,n] \simeq H(\omega_k) y[k,n] \quad (3)$$

where $H(\omega)$ is the CFR and ω_k is the kth subcarrier radial frequency. Observe that even if $s[k,n]$ has a predefined non-zero value at the transmitter, $y[k,n]$ is generally unknown due to the presence of the imaginary component in (2), which in turn depends on the neighboring *unknown* transmitted symbols. The idea behind the AP technique is to fix the value of an additional symbol in $\mathcal{S}$, usually $s[k,n\pm1]$, to guarantee that the second (purely imaginary) term of (2) is zero. Then $y[k,n] = s[k,n]$, which allows one to estimate the CFR at ω_k in a manner analogous to OFDM, namely

$$\hat{H}(\omega_k) = \frac{z[k,n]}{s[k,n]}.$$

This operation can be repeated across the spectrum by inserting similar pairs of pilots at K distinct subcarriers, say $\omega_1, \omega_2, \ldots, \omega_K$. Denoting by $\mathcal{H}$ the $K\times 1$ vector that contains the estimated CFR values $\hat{H}(\omega_k)$ and assuming that the channel has a finite impulse response $\mathbf{h} \in \mathbb{C}^{L_h\times 1}$ of length L_h, one may write $\mathcal{H} = \sqrt{2M}\mathbf{G}^H\mathbf{h}$, where $\mathbf{G}$ is a selection of the $2M$-point inverse DFT matrix corresponding to the upper L_h rows and the K columns associated with the K pilot subcarriers. Assuming that $K \ge L_h$, one should be able to retrieve $\mathbf{h}$ from the above equation, and then obtain the CFR at any subcarrier by a simple DFT. However, when the channel exhibits strong frequency selectivity, the approximation in (3) is no longer valid, and this fact results in a significant modeling error that severely distorts the channel estimate.

2. THE PROPOSED CHANNEL ESTIMATOR

In this section, the above channel estimation procedure will be generalized to the situation where the channel presents strong frequency selectivity. The approach followed is based on a Taylor series expansion of the received signal under the assumption of an asymptotically large number of subcarriers ($M\to\infty$), which nevertheless turns out to be a very accurate assumption even for moderate values of M (cf. Section 5). The prototype filters will be assumed to have been obtained through discretizing a continuous-time waveform (see [18] for further details) and to be PR-compliant. The corresponding analog waveform is assumed to be $R+1$ times continuously differentiable for some integer R and both the function itself and its first $R+1$ derivatives null out at the extremes of its support.

Under the above assumptions, one can define $q^{(r)}[n]$ as the sampled version of the rth derivative of the analog waveform. Denote then by $y^{(r)}[k,n]$ the received signal when the AFB employs $q^{(r)}[n]$ as its prototype. As previously, the corresponding received signal in the presence of a frequency selective channel $H(\omega)$ will be denoted by $z^{(r)}[k,n]$. It was proved in [18] that when $M\to\infty$ the received signal $z[k,n]$ can be expressed as

$$z[k,n] = \sum_{r=0}^{R} \frac{(-\mathrm{j})^r}{r!\,(2M)^r} H^{(r)}(\omega_k) y^{(r)}[k,n] + O\left(\frac{1}{M^{R+1}}\right)$$

where $H^{(r)}(\omega)$ is the rth derivative of the CFR. Clearly, the more frequency selective – *relatively* to the filter bank size – the channel is, the larger R needs to be to keep the modeling error small enough. It is of interest to note that (3) corresponds to the case $R=0$ and can therefore be obtained as a special case of the above formulation. More generally, considering the signals received from the mth prototype pulse derivative, one can still apply the result in [18] to write

$$\frac{z^{(m)}[k,n]}{(2M)^m} = \sum_{r=0}^{R-m} \frac{(-\mathrm{j})^r}{r!} H^{(r)}(\omega_k) \frac{y^{(r+m)}[k,n]}{(2M)^{r+m}} + O\left(\frac{1}{M^{R+1}}\right) \quad (4)$$

for $m = 0, 1, \ldots, R$. This gives a total of $R+1$ equations in $R+1$ unknowns ($H^{(r)}(\omega_k)$, $r = 0, 1, \ldots, R$), assuming

[1] This setting also covers the so-called biorthogonal frequency division multiplexing (BFDM/OQAM) [2]. Orthogonal FBMC/OQAM (also known as OFDM/OQAM) results for $p = q$.

that the values of $y^{(r)}[k,n]$ are known. As in (2), the latter will generally depend on multiple transmitted symbols $s[m,\ell]$ through a convolution operation, namely

$$y^{(r)}[k,n] = \sum_{(m,\ell)\in\mathcal{S}} s\left[(k-m)_{2M}, n-\ell\right] \varphi_{k,n}^{(r)}[m,\ell],$$

where $\varphi_{k,n}^{(r)}[m,\ell]$ is the modulation ambiguity function obtained when the AFB prototype is replaced by its rth derivative. It will be shown in the next section that, by fixing the values of $2(R+1)$ symbols around the (k,n) frequency-time position, one can guarantee that the samples $y^{(r)}[k,n]$ will take on a predefined value for every $r=0,1,\ldots,R$. Assume that these values are fixed, and let

$$\xi_{k,n}^{(r)} = \frac{y^{(r)}[k,n]}{(2M)^r} \tag{5}$$

One can then re-write (4) in matrix form

$$\mathbf{z}_k = \mathbf{\Psi}_k \boldsymbol{\mathcal{H}}_k + O(M^{-(R+1)}) \tag{6}$$

where $\mathbf{\Psi}_k$ stands for a Hankel upper triangular matrix with first row $\left[\xi_{k,n}^{(0)}, \xi_{k,n}^{(1)}, \ldots, \xi_{k,n}^{(R)}\right]$ and $\mathbf{z}_k$, $\boldsymbol{\mathcal{H}}_k$ are column $(R+1)$-vectors with $(r+1)$st entries equal to

$$\{\mathbf{z}_k\}_{r+1} = (2M)^{-r} z^{(r)}[k,n], \quad \{\boldsymbol{\mathcal{H}}_k\}_{r+1} = \frac{(-\mathrm{j})^r}{r!} H^{(r)}(\omega_k).$$

Moreover, $\boldsymbol{\mathcal{H}}_k = \left(\mathbf{I}_{R+1} \otimes \mathbf{g}_k^H\right) \mathbf{\Upsilon}_R \mathbf{h}$, where $\otimes$ is the Kronecker product, $\mathbf{g}_k$ is the kth column of the matrix $\mathbf{G}$,

$$\mathbf{\Upsilon}_R = \sqrt{2M}\left[\mathbf{I}_{L_h} \; -\mathbf{\Lambda} \; \tfrac{1}{2!}\mathbf{\Lambda}^2 \cdots \tfrac{(-1)^R}{R!}\mathbf{\Lambda}^R\right]^T \tag{7}$$

and $\mathbf{\Lambda} = \mathrm{diag}(0,1,2,\ldots,L_h-1)$.

The same formulation can be applied to K distinct subcarriers ω_k so that, by denoting $\mathbf{z} = \left[\mathbf{z}_1^T, \ldots, \mathbf{z}_K^T\right]^T$, one can finally write (obviating the error term in $O(M^{-(R+1)})$)

$$\mathbf{z} = \boldsymbol{\mathcal{G}}\mathbf{h}, \quad \boldsymbol{\mathcal{G}} = \begin{bmatrix} \mathbf{\Psi}_1 \otimes \mathbf{g}_1^H \\ \vdots \\ \mathbf{\Psi}_K \otimes \mathbf{g}_K^H \end{bmatrix} \mathbf{\Upsilon}_R \tag{8}$$

Now, the system in (8) is solvable as long as the $(R+1)K \times L_h$ matrix $\boldsymbol{\mathcal{G}}$ has full column rank. With $K \geq L_h$ the matrix is certainly tall. Assuming moreover that the matrices $\mathbf{\Psi}_k$ are designed to have full rank, the following estimation is possible

$$\hat{\mathbf{h}} = \boldsymbol{\mathcal{G}}^{\#}\mathbf{z}, \tag{9}$$

where $(\cdot)^{\#}$ stands for the Moore-Penrose pseudoinverse.

3. PILOT CONSTRUCTION

A crucial step in the above channel estimation procedure is the fact that some pilots need to be inserted in order to guarantee that the signal response under ideal channel conditions takes on a predefined value, namely $y^{(r)}[k,n] = (2M)^r \xi_{k,n}^{(r)}$. To ensure this, assume that a total of K pilot clusters are inserted, each cluster consisting of a total of $2(R+1)$ pilots, and that the pilots corresponding to the kth cluster are located around the (k,n) frequency-time point. For each k, $1 \leq k \leq K$, let $\mathcal{I}_k$ be the set of $2(R+1)$ different pilot positions corresponding to the kth cluster and assume that the sets $\mathcal{I}_k$, $k=1,2,\ldots,K$, are disjoint. By generalizing (1) to the reception with the rth prototype derivative, one can write (for each r and each (k,n) point)

$$\sum_{(m,\ell)\in\mathcal{I}} s[m,\ell]\, \varphi_{k,n}^{(r)}\left[(k-m)_{2M}, n-\ell\right] = x^{(r)}[k,n], \tag{10}$$

where

$$x^{(r)}[k,n] = (2M)^r \xi_{k,n}^{(r)} - \sum_{(m,\ell)\in\mathcal{S}\setminus\mathcal{I}} s[m,\ell]\, \varphi_{k,n}^{(r)}\left[(k-m)_{2M}, n-\ell\right]$$

and $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2 \cup \cdots \cup \mathcal{I}_K$. Observe that $x^{(r)}[k,n]$ is known once $\xi_{k,n}^{(r)}$ and the values of the information symbols, namely $s[m,\ell]$ for $(m,\ell) \notin \mathcal{I}$, have been fixed.

In order to obtain an equation for the pilot symbols, eq. (10) will be expressed in matrix form. Define $\boldsymbol{\phi}_{\mathcal{I}}^{(r)}[k,n]$ and $\boldsymbol{\phi}_{\mathcal{S}\setminus\mathcal{I}}^{(r)}[k,n]$ as two row vectors of dimension $|\mathcal{I}|$ and $|\mathcal{S}\setminus\mathcal{I}|$, which contain the values of $\varphi_{k,n}^{(r)}\left[(k-m)_{2M}, n-\ell\right]$ for $(m,\ell) \in \mathcal{I}$ and $(m,\ell) \in \mathcal{S}\setminus\mathcal{I}$ respectively. If $\mathbf{s}_{\mathcal{I}}$ and $\mathbf{s}_{\mathcal{S}\setminus\mathcal{I}}$ denote two associated column vectors of the same dimension containing the symbols $s[m,\ell]$ for $(m,\ell) \in \mathcal{I}$ and $(m,\ell) \in \mathcal{S}\setminus\mathcal{I}$ respectively, one can re-write (10) in compact form as $\boldsymbol{\phi}_{\mathcal{I}}^{(r)}[k,n]\,\mathbf{s}_{\mathcal{I}} = x^{(r)}[k,n]$, where $x^{(r)}[k,n] = (2M)^r \xi_{k,n}^{(r)} - \boldsymbol{\phi}_{\mathcal{S}\setminus\mathcal{I}}^{(r)}[k,n]\,\mathbf{s}_{\mathcal{S}\setminus\mathcal{I}}$. Now, define $\mathbf{\Phi}_{\mathcal{I}}[k,n]$ as an $(R+1) \times |\mathcal{I}|$ matrix formed by stacking the row vectors $\boldsymbol{\phi}_{\mathcal{I}}^{(r)}[k,n]$, $r=0,1,\ldots,R$, on top of one another and take $\bar{\mathbf{\Phi}}_{\mathcal{I}}[k,n] = \left[\mathrm{Re}^T \mathbf{\Phi}_{\mathcal{I}}[k,n], \mathrm{Im}^T \mathbf{\Phi}_{\mathcal{I}}[k,n]\right]^T$, with dimensions $2(R+1) \times |\mathcal{I}|$. Finally, introducing the matrix $\bar{\mathbf{\Phi}}_{\mathcal{I}} = \left[\bar{\mathbf{\Phi}}_{\mathcal{I}}[1,n]^T, \cdots, \bar{\mathbf{\Phi}}_{\mathcal{I}}[K,n]^T\right]^T$ of dimensions $2K(R+1) \times |\mathcal{I}|$, eq. (10) can be written as

$$\bar{\mathbf{\Phi}}_{\mathcal{I}}\mathbf{s}_{\mathcal{I}} = \bar{\boldsymbol{\xi}} - \bar{\mathbf{\Phi}}_{\mathcal{S}\setminus\mathcal{I}}\mathbf{s}_{\mathcal{S}\setminus\mathcal{I}}, \tag{11}$$

where $\bar{\boldsymbol{\xi}} = \left[\bar{\boldsymbol{\xi}}_1^T, \ldots, \bar{\boldsymbol{\xi}}_K^T\right]^T$, $\bar{\boldsymbol{\xi}}_k = \left[\mathrm{Re}^T \boldsymbol{\xi}_k, \mathrm{Im}^T \boldsymbol{\xi}_k\right]^T$, $\boldsymbol{\xi}_k = \left[\xi_{k,n}^{(0)}, \ldots, (2M)^R \xi_{k,n}^{(R)}\right]^T$. Observing the dimensions of the matrix $\bar{\mathbf{\Phi}}_{\mathcal{I}}$, it can be readily seen that the problem is solvable as long as the total number of pilots is larger than or equal to $2K(R+1)$, which corresponds to $2(R+1)$ pilots per each of the K clusters. Note that this reduces to two pilots per cluster in the case of $R=0$, which amounts to the classical AP configuration. By properly choosing the pilot position set $\mathcal{I}$, one can typically guarantee that the matrix $\bar{\mathbf{\Phi}}_{\mathcal{I}}$ will have full rank and the system in (11) will be solvable in terms of the pilot values $\mathbf{s}_{\mathcal{I}}$.

It is easy to see from (11) that the statistical properties of the inserted pilots $\mathbf{s}_{\mathcal{I}}$ will generally be substantially different from those of the transmitted information symbols $\mathbf{s}_{\mathcal{S}\setminus\mathcal{I}}$. Depending on the ambiguity function, this may imply that the inserted pilots need to have higher power than the information symbols. This is a well known weakness of the AP scheme

and a number of approaches were recently proposed to its mitigation [14, 16]. In the strongly frequency selective scenario studied here, this question remains for future investigation.

4. PERFORMANCE CHARACTERIZATION

In practice, the system equation in (8) is not exact, due to the presence of the error term in (4), together with the noise in the observations that has so far been neglected. One can thus write

$$\mathbf{z}^{\text{noisy}} = \boldsymbol{\mathcal{G}}\mathbf{h} + \mathbf{n} + \mathbf{w}, \tag{12}$$

where $\mathbf{n}$ and $\mathbf{w}$ contain the contributions from the background noise and the model distortion error, respectively. Assume, as usual, that the information symbols $\mathbf{s}_{\mathcal{S}\setminus\mathcal{I}}$ are i.i.d. random variables with zero mean and variance 1/2. Furthermore, let $\mathbf{n}$ and $\mathbf{w}$ be written as the concatenation of K $(R+1)$-vectors, say $\mathbf{n}_k$ and $\mathbf{w}_k$, each one of them associated with the noise and distortion, respectively, at a particular pilot tone, ω_k. In what follows, a statistical characterization of these two random vectors is provided, which trivially leads to a characterization of the statistical behavior of the estimator in (9), by a simple left multiplication by $\boldsymbol{\mathcal{G}}^{\#}$.

4.1. Modeling Error

Using again the results in [18], one can further refine (6) by considering an additional term in the Taylor series expansion, namely

$$\mathbf{z}_k = \boldsymbol{\Psi}_k\mathcal{H}_k + \frac{y^{(R+1)}[k,n]}{(2M)^{R+1}}\mathbf{J}_{R+1}\mathbf{u}_k + O(M^{-(R+2)}),$$

where $\mathbf{J}_{R+1}$ is the exchange matrix of order $R+1$ and $\mathbf{u}_k = \left(\mathbf{I}_{R+1}\otimes\mathbf{g}_k^H\right)\tilde{\boldsymbol{\Upsilon}}_R\mathbf{h}$, with $\tilde{\boldsymbol{\Upsilon}}_R$ being defined as in (7) but along the index set $\{1,2,\ldots,R+1\}$.

On the other hand, using the structure of the pilots above, one can write

$$y^{(R+1)}[k,n] = \boldsymbol{\phi}_{\mathcal{I}}^{(R+1)}[k,n]\,\bar{\boldsymbol{\Phi}}_{\mathcal{I}}^{-1}\bar{\xi} + \left(\boldsymbol{\phi}_{\mathcal{S}\setminus\mathcal{I}}^{(R+1)}[k,n] - \boldsymbol{\phi}_{\mathcal{I}}^{(R+1)}[k,n]\,\bar{\boldsymbol{\Phi}}_{\mathcal{I}}^{-1}\bar{\boldsymbol{\Phi}}_{\mathcal{S}\setminus\mathcal{I}}\right)\mathbf{s}_{\mathcal{S}\setminus\mathcal{I}} \tag{13}$$

Using this, it is readily seen that the model distortion vector $\mathbf{w}$ in (12) can be written (up to an error term of order $O(M^{-(R+2)})$) as the concatenation of K non-circular random vectors $\mathbf{w}_k$, with mean and cross-covariance given by

$$\mathbb{E}[\mathbf{w}_k] = \gamma_k\mathbf{J}_{R+1}\mathbf{u}_k,$$

$$\operatorname{cov}(\mathbf{w}_k,\mathbf{w}_m) = \delta_{k,m}\mathbf{J}_{R+1}\mathbf{u}_k\mathbf{u}_m^H\mathbf{J}_{R+1},$$

where γ_k is the first term in (13) and $\delta_{k,m} = \boldsymbol{\vartheta}_k\boldsymbol{\vartheta}_m^H$ with $\boldsymbol{\vartheta}_k$ being the row vector multiplying $\mathbf{s}_{\mathcal{S}\setminus\mathcal{I}}$ in (13).

4.2. Noise effect

Assume now that the receiver is contaminated with circularly symmetric additive complex noise with zero mean and variance σ^2. It can readily be shown [18] that the observed noise vector at the output of the AFB, $\mathbf{n}$, is circularly symmetric noise having zero mean and covariance components $\mathbb{E}\left[\mathbf{n}_k\mathbf{n}_m^H\right] = \mathbf{C}_n^{k,m}$, with entries

$$\left\{\mathbf{C}_n^{k,m}\right\}_{r_1+1,r_2+1} = 2\sigma^2\left\{\boldsymbol{\Phi}\mathbf{F}^H\mathbf{D}_{r_1,r_2}\mathbf{F}\boldsymbol{\Phi}^*\right\}_{\zeta(k),\zeta(m)}$$

Here, $\zeta(k)$ provides the subcarrier index associated to ω_k, $\mathbf{F}$ is the DFT matrix of order $2M$, $\boldsymbol{\Phi}$ is a diagonal matrix with mth entry equal to $\exp\left(-j\pi\frac{M+1}{2M}(m-1)\right)$, $1\le m\le 2M$, and

$$\mathbf{D}_{r_1,r_2} = \operatorname{diag}\left(\mathbf{J}_{2M}\sum_{\ell=1}^{\kappa}\left[\mathbf{q}_\ell^{(r_1)}\odot\mathbf{q}_\ell^{(r_2)}\right]\right)$$

with $\mathbf{q}_\ell^{(r)} = \left[q^{(r)}[2M(\ell-1)],\ldots,q^{(r)}[2M\ell-1]\right]^T$ and $\odot$ denoting the Hadamard product.

5. NUMERICAL EVALUATION

In the simulations, an FBMC/OQAM system of $2M = 128$ subcarriers separated by 30 MHz has been considered, employing the PHYDYAS prototype filter [19] with $\kappa = 3$, at both the transmit and receive sides. The pilots were distributed in 63, 32 and 21 equispaced clusters (of 2, 4 and 6 pilots each), for $R = 0, 1, 2$ respectively, so that the total number of pilots was approximately kept constant regardless of R. The proposed channel estimation method was tested over 100 channels obeying the Extended Typical Urban (ETU) model [20]. Fig. 1 plots both the simulated (solid line) and the theoretical (dotted line) mean squared error (MSE) as a function of the SNR, for different values of R. Observe

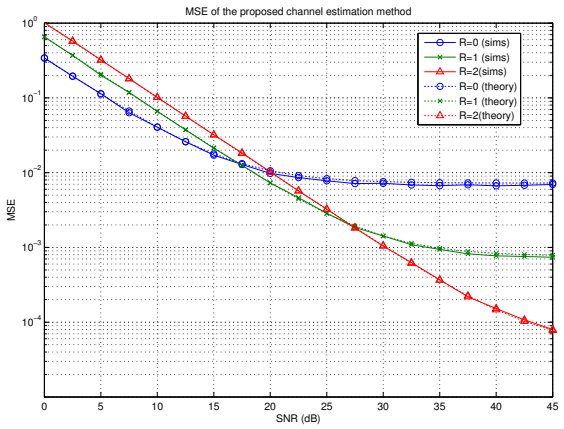

Fig. 1. Simulated and analytical MSE of the proposed channel estimation method.

that as R increases, the proposed method effectively lowers the MSE floor due to the unmodeled channel frequency selectivity. Conversely, the behavior at low SNR values, namely where the noise is strong enough to prevail over the intrinsic interference effect, is slightly penalized for increasing R, an effect that is caused by the corresponding reduction in the number of pilot clusters. Observe also that for each value of SNR there is an optimum R (and hence an optimum pilot distribution in clusters) that minimizes the MSE. At low SNR, it is preferable to distribute the pilots evenly across the spectrum, whereas at higher SNR values it is better to concentrate them into a smaller number of clusters.

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