ADAPTIVE REGULARIZATION FOR BEM CHANNEL ESTIMATION IN MULTICARRIER SYSTEMS

Y. Zakharov

University of York, UK
Department of Electronics

D. Zheng

Shanghai R&D Institute, Huawei Technologies Co. Ltd.
Wireless Network Research Department

ABSTRACT

This paper addresses the BEM (Basis Expansion Model) channel estimation in receivers of multicarrier communication signals. The estimation performance can be improved by choosing optimal basis functions or an optimal number of predefined basis functions; these approaches require knowledge of channel statistics. Another approach to improve the performance is to set the number of basis functions large enough to guarantee a negligible modeling error and optimize the regularization. In this paper, we adopt the latter approach and propose an adaptive regularization scheme based on the generalized cross-validation; the scheme does not require the knowledge of channel statistics. We demonstrate by simulation for LTE uplink scenarios that the proposed scheme allows a high estimation performance in a range of channels and noise levels.

Index Terms— BEM, channel estimation, cross-validation, DPSS, LTE, OFDM

1. INTRODUCTION

The detection performance of receivers in multicarrier communication systems, such as the LTE system [1–5], depends on the accuracy of channel estimation. In this work, we consider the pilot-based channel estimation in multicarrier systems. In the channel estimation, the BEM (Basis Expansion Model) is often used [6–17]. BEMs considered in the literature include the Karhunen-Loeve functions [6, 7], discrete prolate spheroidal sequences (DPSS) [9, 15–17], complex exponentials [8–10], algebraic polynomials [18] and B-splines [11–14]. Largely popular are DPSS; if applied to the channel frequency response estimation, DPSS are optimal (Karhunen-Loeve functions) in multipath channels with a uniform power delay profile (PDP) within the channel delay spread.

The performance of BEM channel estimation can be improved by choosing optimal basis functions or an optimal number of predefined basis functions, which depends on smoothness of the channel response and the level of noise; in the latter case, the approximation (modeling) error and the statistical error are optimally balanced in the total estimation error. These two approaches require knowledge of channel statistics. Another approach to improve the estimator performance is to set the number of basis functions large enough to guarantee a negligible modeling error and optimize the regularization. In this paper, we adopt the latter approach and propose an adaptive regularization scheme. We demonstrate by simulation, for the LTE uplink scenarios with a BEM channel estimator exploiting DPSS, that the proposed scheme allows a high estimation performance for a range of channels and noise levels.

2. SIGNAL MODEL AND BEM CHANNEL ESTIMATION

The received $N \times 1$ signal vector is given by

$$\mathbf{z} = \mathbf{S}\mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathbf{h}$ is an $N \times 1$ vector describing the channel frequency response, $\mathbf{S}$ is an $N \times N$ diagonal matrix of transmitted pilot symbols, $\mathbf{n}$ is an $N \times 1$ noise vector, and N is the number of subcarriers. The noise is assumed Gaussian with independent zero mean elements of variance σ^2. The vector $\mathbf{h}$ is represented by an $M \times 1$ vector of BEM expansion coefficients $\mathbf{a}$ and the channel estimation is transformed into estimation of the vector $\mathbf{a}$ (of a reduced size compared to the vector $\mathbf{h}$). The estimate of $\mathbf{h}$ is given by $\hat{\mathbf{h}} = \mathbf{B}\hat{\mathbf{a}}$, where $\mathbf{B}$ represents the basis functions; more specifically, $\mathbf{B}$ is an $N \times M$ matrix, whose M columns are the basis functions. The expansion coefficients $\mathbf{a}$ are estimated as $\hat{\mathbf{a}} = \mathbf{A}\mathbf{z}$, where

$$\mathbf{A} = \left(\mathbf{B}^H\mathbf{R}_s\mathbf{B} + \mathbf{R}_r\right)^{-1}\mathbf{B}^H\mathbf{S}^H, \tag{2}$$

$\mathbf{R}_s = \mathbf{S}^H\mathbf{S}$, and $\mathbf{R}_r$ is a regularization matrix; e.g., if $\mathbf{R}_r = \sigma^2\mathbf{R}_a^{-1}$, where $\mathbf{R}_a = \mathbb{E}\{\mathbf{a}\mathbf{a}^H\}$ is the covariance of the expansion coefficients and $\mathbb{E}\{\cdot\}$ denotes expectation, we arrive at the LMMSE (linear minimum MSE) solution. Such a basic channel estimator is described as shown in Table 1.

The MSE of the channel estimates is defined as

$$\epsilon = \frac{\mathrm{tr}\{\mathbb{E}[(\mathbf{h}-\hat{\mathbf{h}})(\mathbf{h}-\hat{\mathbf{h}})^H]\}}{\mathrm{tr}\{\mathbb{E}[\mathbf{h}\mathbf{h}^H]\}}, \tag{3}$$

Table 1. Basic BEM channel estimator

	Input: $\mathbf{z}, \mathbf{B}, \mathbf{S}, \mathbf{R}_r$
1	$\mathbf{R} = \mathbf{B}^H\mathbf{S}^H\mathbf{S}\mathbf{B} + \mathbf{R}_r$
2	$\boldsymbol{\xi} = \mathbf{B}^H\mathbf{S}^H\mathbf{z}$
3	Solve: $\mathbf{R}\hat{\mathbf{a}} = \boldsymbol{\xi}$
4	Channel estimate: $\hat{\mathbf{h}} = \mathbf{B}\hat{\mathbf{a}}$
	Output: $\hat{\mathbf{h}}, \hat{\mathbf{a}}$

where tr$\{\cdot\}$ is the trace operator. Let $\boldsymbol{\Upsilon} = \mathbb{E}[\mathbf{h}\mathbf{h}^H]$ be the channel covariance, then $\text{tr}\{\mathbb{E}[\mathbf{h}\mathbf{h}^H]\} = \text{tr}\{\boldsymbol{\Upsilon}\} = N\sigma_h^2$, where σ_h^2 is the channel variance. Denoting $\mathbf{P} = \mathbf{B}\mathbf{A}\mathbf{S}$, we arrive at

$$\epsilon = \frac{\text{tr}\{(\mathbf{I}_N - \mathbf{P})\boldsymbol{\Upsilon}(\mathbf{I}_N - \mathbf{P})^H + \sigma_n^2\mathbf{P}\mathbf{R}_s^{-1}\mathbf{P}^H\}}{N\sigma_h^2}, \quad (4)$$

where $\mathbf{I}_N$ is an $N \times N$ identity matrix. For a given covariance matrix $\boldsymbol{\Upsilon}$ and pilot symbols $\mathbf{S}$, the MSE is defined by the matrix $\mathbf{P} = (\mathbf{U}^{-1} + \mathbf{I}_N)^{-1}$ with $\mathbf{U} = \mathbf{B}\mathbf{R}_r^{-1}\mathbf{B}^H\mathbf{R}_s$. This implies that the same MSE will be achieved by two sets of basis functions, $\mathbf{B}_1$ and $\mathbf{B}_2$, if the corresponding regularization matrices $\mathbf{R}_{r,1}$ and $\mathbf{R}_{r,2}$ are chosen to satisfy $\mathbf{B}_1\mathbf{R}_{r,1}^{-1}\mathbf{B}_1^H = \mathbf{B}_2\mathbf{R}_{r,2}^{-1}\mathbf{B}_2^H$. This also implies that the same MSE can be obtained either by using a specific basis or by using a specific regularization.

The choice of the regularization matrix $\mathbf{R}_r$ significantly affects the estimator performance. There are different approaches for this choice. We will consider the following regularization schemes:

- LMMSE regularization ($\mathbf{R}_r = \sigma^2\mathbf{R}_a^{-1}$), where $\mathbf{R}_a$ is the covariance matrix of expansion coefficients for the true channel PDP.
- Model-based regularization ($\mathbf{R}_r = \sigma^2\mathbf{R}_{a,0}^{-1}$), where $\mathbf{R}_{a,0}$ is a covariance matrix of expansion coefficients for a model channel PDP; the model PDP can be different from the real one.
- Diagonal loading ($\mathbf{R}_r = \sigma^2\mathbf{I}_M$).

The optimal performance is achieved using the LMMSE regularization. This however requires the knowledge of the channel PDP; in particular, it requires the perfect knowledge of the number of multipath components, their delays and variances, which is difficult to accurately estimate in practice.

More practical approach is a model-based regularization. Let $\mathbf{M}$ be a model channel covariance (in general, not necessarily equal to $\boldsymbol{\Upsilon}$) that characterizes *a priori* information on the channel. Such *a priori* information can be that the channel delays are limited to some value τ_{max} and within this limit the PDP is uniform. This information can be exploited to develop specific basis functions; e.g., for the limited delays, the optimal basis functions (providing the smallest approximation error for a fixed M) across subcarriers are DPSS. Then $\mathbf{M}$ is the covariance matrix with elements in (8). If the real channel covariance is $\mathbf{M}$, i.e. $\boldsymbol{\Upsilon} = \mathbf{M}$, DPSS provide the (optimal) Karhunen-Loeve decomposition. This however still requires the knowledge of the delay spread, which is difficult to acquire.

In practice, the diagonal loading is often used; it only requires the knowledge of the noise variance that can be available in many scenarios. This regularization, however, results in significant drop in the estimation performance (e.g., see simulation results below).

Here, we propose an algorithm for adaptive regularization that does not require the knowledge of the channel statistics.

3. ADAPTIVE REGULARIZATION OF THE CHANNEL ESTIMATOR

The model-based uniform regularization allows achieving a high estimation performance if the (true) channel delay spread is known and matches to the uniform-model delay spread. However, in practice, it is unavailable and needs to be estimated. Such estimation is a complicated problem due to the small amount of observed data.

Note that two components contribute to the estimation error: approximation (modeling) and statistical errors. The modeling error can typically be reduced by increasing the number of basis functions M. This, however, increases the statistical error. The regularization should guarantee a balance between these two components. To balance them, the generalized cross-validation (GCV) method can be used [19]. In application to the channel estimation, the GCV method can be based on minimizing the quantity

$$V = \frac{||\mathbf{z} - \mathbf{G}\mathbf{z}||^2}{(N - \text{tr}\{\mathbf{G}\})^2}, \quad (5)$$

where $\mathbf{G} = \mathbf{S}\mathbf{B}(\mathbf{B}^H\mathbf{S}^H\mathbf{S}\mathbf{B} + \mathbf{R}_r)^{-1}\mathbf{B}^H\mathbf{S}^H$.

The idea of adaptive regularization is to use a predefined set of matrices $\mathbf{R}_r$ and find within this set a matrix that minimizes V. Note that the numerator in (5) represents the energy of the residual signal and it can be easily evaluated. If the basis $\mathbf{B}$ and pilot $\mathbf{S}$ are known *a priori*, the denominator in (5) can be precomputed and stored for a set of matrices $\mathbf{G}$.

In [20], a modification to the criterion (5) was proposed to improve the estimation accuracy:

$$V = \frac{||\mathbf{z} - \mathbf{G}\mathbf{z}||^2}{(N - \alpha \cdot \text{tr}\{\mathbf{G}\})^2}, \quad (6)$$

where the parameter α is adjusted, and typically is in the range $[1, 2]$. In our simulation, we will be using the criterion in (6). The adaptive regularization algorithm is presented in Table 2. In this algorithm, a set of Q_{iter} regularization matrices $\mathbf{R}_q$, $q = 1, \ldots, Q_{\text{iter}}$, is precomputed. In our examples below, these matrices will be inverse of $Q_{\text{iter}} = 10$ covariance matrices for the uniform PDP with delay spreads

$$\frac{\tau_{max}}{\tau_{CP}} \in \{180, 156, 144, 108, 72, 36, 18, 9, 6, 3\}/144, \quad (7)$$

Table 2. Adaptive regularization algorithm

Input: $\mathbf{z}, \mathbf{S}, \mathbf{B}, Q_{\text{iter}}, \{\mathbf{R}_q\}_{q=1}^{Q_{\text{iter}}}, \sigma^2$
$\boldsymbol{\xi} = \mathbf{B}^H \mathbf{S}^H \mathbf{z}$
for $q = 1 : Q_{\text{iter}}$
$\quad \mathbf{R}_r = \sigma^2 \mathbf{R}_q$
$\quad \mathbf{F} = (\mathbf{B}^H \mathbf{S}^H \mathbf{S} \mathbf{B} + \mathbf{R}_r)^{-1}$
$\quad \mathbf{G} = \mathbf{S} \mathbf{B} \mathbf{F} \mathbf{B}^H \mathbf{S}^H$
$\quad V = \frac{\lvert\lvert \mathbf{z} - \mathbf{G}\mathbf{z} \rvert\rvert^2}{(N - \alpha \cdot \text{tr}\{\mathbf{G}\})^2}$
$\quad$ if $q = 1$
$\quad\quad V_{\min} = V, \hat{\mathbf{a}} = \mathbf{F}\boldsymbol{\xi}$
$\quad$ elseif $V < V_{\min}$
$\quad\quad V_{\min} = V, \hat{\mathbf{a}} = \mathbf{F}\boldsymbol{\xi}$
$\quad$ endif
end for

where τ_{CP} is the length of the cyclic prefix. Thus, the ratio τ_{max}/τ_{CP} is in the interval $[0.0208, 1.25]$.

4. NUMERICAL RESULTS

For simulation, we consider LTE uplink scenarios [1] with two receive antennas at the base station. In the LTE uplink, the data are transmitted by subframes of 14 multicarrier symbols divided into two time slots, 7 symbols each. In the fourth symbol of each slot, a pilot Zadoff-Chu sequence [1] is transmitted at N subcarriers. In the other multicarrier symbols, (turbo encoded) data are transmitted on the N subcarriers using the single-carrier frequency division multiple access (SC-FDMA) with QAM symbols; e.g., see [1,21] for more detail. In our simulation below, we use $N = 48$ subcarriers and 16-QAM modulation.

The channel frequency response is estimated at the N pilot subcarriers within each of the two pilot multicarrier symbols. The estimates in the two slots are then linearly interpolated in time towards the data positions. The M basis functions, $m = 1, \ldots, M$, over subcarriers, $p = 1, \ldots, N$, are given by $(\mathbf{B})_{p,m} = \phi_m(p)$, where $\phi_m(p)$ are the basis functions (DPSS BEM in our case). For the data estimation, the following scheme is used: at every subcarrier within every multicarrier symbol, the MMSE equalizer (computed from the channel estimates) estimates the data symbols, these data estimates are mapped into the QAM constellation and turbo decoded.

DPSS (across subcarriers) are the first M eigenvectors (having largest eigenvalues) of a matrix $\mathbf{M}$ with elements

$$[\mathbf{M}]_{i,j} = \sin[W(i-j)]/[W(i-j)], \qquad (8)$$

for some $0 < W < 1/2$. For example, in application to the LTE uplink [1], if the channel delays are limited to the cyclic prefix of length $\tau_{CP} = 4.7$ μs, the PDP is uniform in the interval $[0, \ \tau_{CP}]$, and the intercarrier spacing is $f_{sc} = 15$ kHz, we have $W = \tau_{CP} f_{sc}/2 \approx 0.035$.

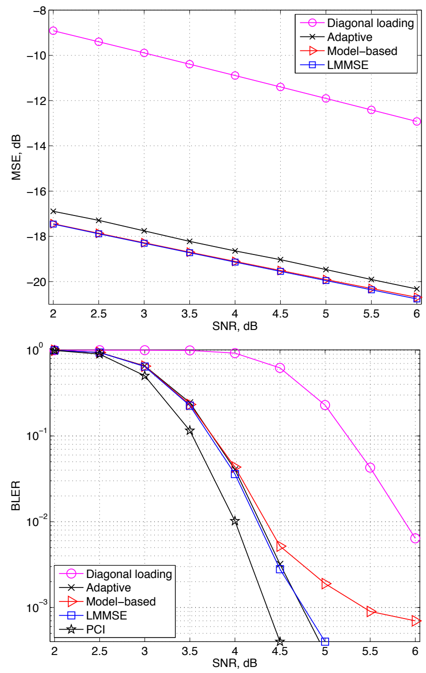

Fig. 1. Performance of the channel estimator with adaptive regularization in the channel with the EPA PDP.

We investigate the MSE channel estimation performance and block error rate (BLER) detection performance obtained in 10^4 simulation trials in scenarios with a Doppler spread of 7 Hz for the case when data are encoded with the turbo code of rate 0.4. We also assume the Jakes' model for the channel time variations. We compare the performance provided by the DPSS BEM with LMMSE, model-based, diagonal loading, and adaptive regularization. In the simulation, we use $M = 8$ basis functions and $W = 0.038$ in (8). The EPA, EVA and ETU PDPs [22] are used for computing the channel covariance and regularization in the LMMSE estimator. The uniform PDP is used for computing the model-based regularization and the adaptive regularization as explained above. For the model-based regularization, for every channel (EPA, EVA, ETU), the parameter τ_{max} is adjusted to guarantee the best performance. Thus, for this comparison, we use the optimal model-based regularization. For the adaptive regularization, τ_{max} is from the set (7) and $\alpha = 1.6$ is used.

Fig. 1 shows the MSE and BLER performance in the EPA channel characterised by a small delay spread compared to the CP length: $\tau_{max}/\tau_{CP} \simeq 0.087$. The diagonal loading is inferior to the other regularization schemes, whereas the MSE performance is similar for the other three schemes. The BLER performance provided by the adaptive regularization is close to that of the LMMSE estimator, and, at BLER = 0.01, it is only 0.25 dB away from the case of the perfect channel knowledge (PCI).

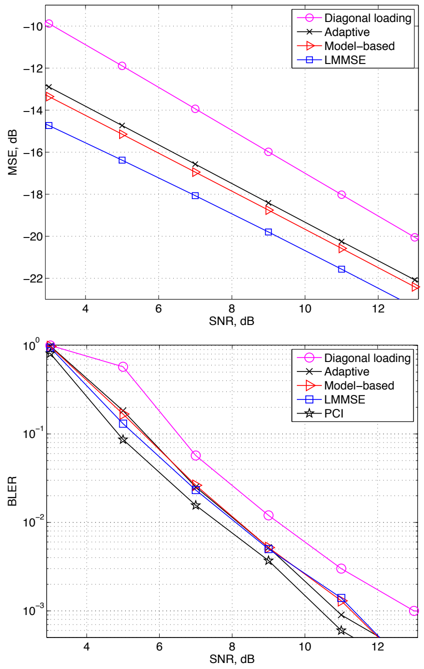

Fig. 2. Performance of the channel estimator with adaptive regularization in the channel with the EVA PDP.

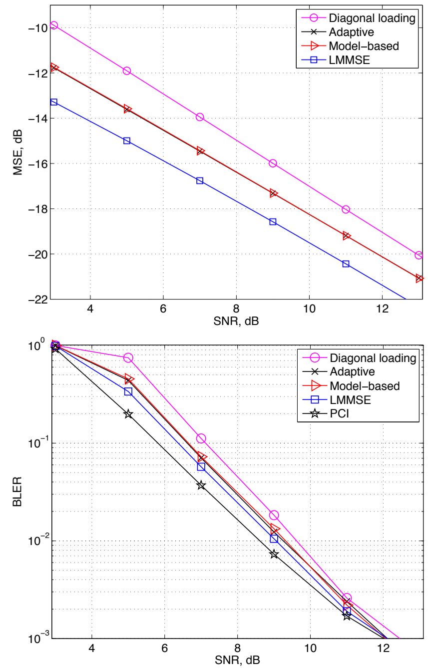

Fig. 3. Performance of the channel estimator with adaptive regularization in the channel with the ETU PDP.

Fig. 2 shows the performance in the EVA channel. The delay spread of the EVA channel is about half of the CP length: $\tau_{max}/\tau_{CP} \simeq 0.534$. It is again seen that the diagonal loading is inferior to the other regularization schemes. The MSE performance of the adaptive regularization and the model-based regularization are similar and about 1 dB away from the MSE performance of the LMMSE regularization. At BLER = 0.01, the detection performance of the adaptive regularization is close to that provided by the LMMSE estimator and only 0.5 dB away from the PCI case.

Fig. 3 shows the performance in the ETU channel, where the delay spread is slightly longer than the CP length: $\tau_{max}/\tau_{CP} \simeq 1.064$. The diagonal loading is again inferior to the other regularization schemes. The MSE performance of the adaptive regularization and the model-based regularization are similar and about 1.5 dB away from the LMMSE performance. At BLER = 0.01, the detection performance of the adaptive regularization is close to that provided by the LMMSE estimator and only about 0.7 dB away from the BLER in the case of PCI.

5. CONCLUSIONS

In this paper, we have considered the regularization in BEM-based channel estimators. We have proposed an adaptive regularization scheme that chooses a regularization matrix without any information on the channel statistics. This is based on the generalized cross-validation. The numerical results for LTE uplink scenarios show that the proposed adaptive regularization performs similarly to the model-based regularization with a perfectly known channel delay spread. For the EPA, EVA, and ETU channels, possessing different (small, medium, and high) delay spreads, the detection performance provided by the proposed channel estimator is close to that of the receiver with perfect channel knowledge.

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