MUTUAL INFORMATION BASED RADAR WAVEFORM DESIGN FOR JOINT RADAR AND CELLULAR COMMUNICATION SYSTEMS

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ABSTRACT

A joint radar/communication system is considered, where the radar adaptively designs the transmitted waveform such that the interference caused to the cellular systems is strictly controlled. In this paper, different Mutual Information based criteria for radar waveform optimization are proposed and the corresponding waveform optimization problems are formulated and solved analytically. Radar performance trade-offs for the considered Mutual Information based criteria are presented and, using simulation results, it is shown that a larger maximized Mutual Information does not guarantee an optimal detection performance. It is also emphasized the importance of exploiting the communication signals scattered off the target for the detection task when dealing with weak radar returns.

***Index Terms*—** Multicarrier Radar, Communications, Coexistence, Mutual Information, Spectrum Sharing

1. INTRODUCTION

As licensed spectrum is under consideration for release, there is potential need for radar systems to coexist with cellular radio [1]. Joint radar/communication systems are considered as a coexistence solution to the ever increasing demand for spectrum, due to services with high bandwidth requirements and the exponential increase in the number of connected devices. Such joint system allows the radar and the cellular communication systems to operate in the same bandwidth, without causing too much interference to each other. In this paper, a multicarrier waveform is considered for both the radar and the communication systems. Such waveforms have been considered for passive radar, for example in [2], but also as active sensing waveforms in [3] and [4], where it is shown that OFDM radar signals can obtain a better range and Doppler resolution than other radar signals. In a joint radar/communication scenario, the radar designs the transmitted waveforms in an agile manner. Inspired by [5, 6], we employ in [7] a Mutual Information (MI) based criterion for radar waveform optimization in a joint radar/communication setup.

In this paper, it is assumed that the scattering off the target due to the communication signals arrives at the radar receiver when a target is present, in contrast to [7]. Various MI based criteria are proposed, that can be applied for radar waveform optimization in joint radar/communication setups. These criteria are different from each other in the way the scattering due to the communication signals is considered in the radar waveform optimization: as useful energy, as interference or ignored altogether. The objective functions for each criteria are derived, the associated radar waveform optimization problems are formulated and solved analytically. Using simulation results, the achieved maximized MI for the considered criteria is shown and the obtained detection performance of the radar waveforms optimized using such criteria is compared. It is observed that a larger maximized MI does not guarantee an optimal detection performance. The importance of exploting the communication signals scattered off the target at the radar receiver is shown, especially in cases where a weak radar return is experienced. It is also shown that the same optimized radar waveform can be obtained using different MI based criteria proposed in this paper, as well as the similarity of a proposed criterion with the one in [7] for a different scenario.

This paper is organized as follows. Section 2 introduces the joint radar and communication system model and states the underlying assumptions needed in this work. In Section 3, the MI based criteria is introduced and the associated optimization problems are formulated and solved analytically. Simulation results showing the achieved maximized MI for each employed criteria are provided. In Section 4, the detection performance provided by the waveforms optimized using the proposed MI based criteria is compared. Finally, Section 5 concludes the paper.

Notation: A lower capital bold letter $\mathbf{x}$ denotes a column vector. By x_l or $x[l]$ we denote the lth element in a vector $\mathbf{x}$. The frequency domain representation of a discrete sample is $X[l]$. Symbol $*$ denotes the convolution operation, while $\prec$ and $\preceq$ denote the element-wise smaller and smaller or equal to. By $H(\cdot)$ we denote the differential entropy and by $I(\cdot)$ the MI between two or more random variables.

2. SYSTEM MODEL

Building on our model developed in [8], a joint radar and communication system is presented in Figure 1, where three communication base stations are illustrated together with a monostatic radar with a goal to detect the target. The radar receives echoes from the target due to the transmitted radar signal as well as the communication signals from the base stations, via two channels: a direct path and a path which is due to scattering off the target. It is assumed that the radar antenna beampattern is directional and steered towards the target, thus the radar signal does not arrive at the communication base stations through a direct path, but only scattered off the target.

In case of a monostatic radar and N communication base stations/communication systems, the equation for the received signal at the radar can be described in continuous time as

$$y(t) = r(t) + \sum_{i=1}^{N} [r_{s_i}(t) + s_i(t)] + n(t), \tag{1}$$

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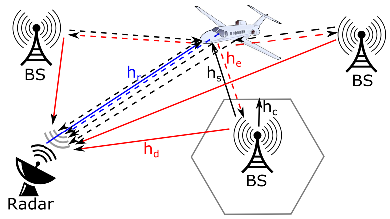

Fig. 1: System model composed of the radar and communication base stations. The regular and dashed lines represent direct paths and reflexions of the signals respectively. We use red color for interference in the direction of the arrow and blue color for the active signal transmitted by the radar and its received echo.

where $y(t)$ denotes the received signal at the radar receiver, $r(t)$ is the echo from the target due to the radar signal, $r_{s_i}(t)$ is the scattering off the target due to the communication signal corresponding to the ith base station, $s_i(t)$ is the communication signal arriving through a direct line of sight path at the radar receiver corresponding to ith base station and $n(t)$ accounts for the noise and clutter. Without loss of generality, we will focus on a single base station. However, the model and the derivations extend straightforwardly to N base stations. We assume that both the radar and the communication base stations use OFDM-type multicarrier signals with L subcarriers, that can be easily optimized [3, 9, 10]. The channels of interest are: h_r for the radar - target - radar path, h_e for the radar - target - base station path, h_s for the base station - target - radar path, h_d for the direct base station - radar path, h_c for the communication inside a base station cell. The communication signal $x_s(t)$ is assumed to be deterministic and known at the radar receiver after a previous estimation step [2]. We assume that the channels are stationary over the observation period. The channels $h_r(t)$, $h_s(t)$ and $h_e(t)$, corresponding to the target scattering, as well as the communication channels $h_d(t)$ and $h_c(t)$ are considered random and only known statistically. The radar channel impulse response is assumed to be a wide sense stationary (WSS) Gaussian process [5, 6]. Thus, Equation (1), for a single communication base station, can be written as

$$y(t) = x_r(t) * h_r(t) + x_s(t) * h_s(t) + x_s(t) * h_d(t) + n(t). \quad (2)$$

After downconversion and sampling at the Nyquist rate, the discrete time version of (2) can be written in matrix formulation as

$$\mathbf{y} = \mathbf{X}_r\mathbf{h}_r + \mathbf{X}_s\mathbf{h}_s + \mathbf{X}_s\mathbf{h}_d + \mathbf{n}, \quad (3)$$

where $\mathbf{X}_r$ and $\mathbf{X}_s$ are well approximated to $L \times L$ circulant matrices and the vectors are $L \times 1$. Linear convolution can be approximated by circular convolution for long symbol sequences, as a Toeplitz matrix can be approximated by a circulant matrix when the matrix dimensions are sufficiently large [11]. Circulant matrices are diagonalized by unitary DFT matrices. The noise and clutter $\mathbf{n}$, the radar channel taps $\mathbf{h}_r$ and the taps of the channels $\mathbf{h}_s$, $\mathbf{h}_d$, $\mathbf{h}_c$ and $\mathbf{h}_e$ are all modeled as zero mean Gaussian random vectors with known covariance matrices: $\sigma_n^2\mathbf{I}$, $\mathbf{C}_{h_r}$, $\mathbf{C}_{h_s}$, $\mathbf{C}_{h_d}$, $\mathbf{C}_{h_c}$ and $\mathbf{C}_{h_e}$ respectively.

3. MUTUAL INFORMATION BASED CRITERIA

In this section, we present three different approaches to MI maximization based radar waveform optimization for the joint radar/communication scenario in Figure 1. For each considered MI based criterion, we derive the corresponding objective function to be maximized, formulate the optimization problem and solve it analytically. We compare the achieved maximized MI values for each criterion in simulations.

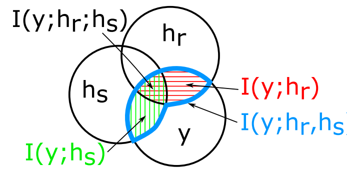

Fig. 2: Venn diagram of information theoretic measures for three random variables $\mathbf{y}$, $\mathbf{h}_r$ and $\mathbf{h}_s$. The areas with the horizontal red stripes and vertical green stripes correspond to $I(\mathbf{y};\mathbf{h}_r)$ and $I(\mathbf{y};\mathbf{h}_s)$ respectively. The area with only the horizontal red stripes corresponds to $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$, while the area with only the vertical green stripes corresponds to $I(\mathbf{y};\mathbf{h}_s|\mathbf{h}_r)$. The area marked by the thicker blue line corresponds to $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$.

The Venn diagram of information theoretic measures, for the signal at the radar receiver $\mathbf{y}$ and the target impulse response associated with the radar signal $\mathbf{h}_r$ and the communication signal $\mathbf{h}_s$, is presented in Figure 2. We assume that both impulse responses $\mathbf{h}_r$ and $\mathbf{h}_s$ partly contain common information about the target, as the radar signal and the communication signal both illuminate a common area of the target. This easily happens when dealing with a point target in the far field, for example. Such assumption implies a correlation between the corresponding channels. As illustrated in Figure 2, we can choose to maximize $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$ (the MI between $\mathbf{y}$ and, jointly, $\mathbf{h}_r$ and $\mathbf{h}_s$), $I(\mathbf{y};\mathbf{h}_r)$ (the MI between $\mathbf{y}$ and $\mathbf{h}_r$), $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$ (the MI between $\mathbf{y}$ and $\mathbf{h}_r$, conditioned on $\mathbf{h}_s$) or $I(\mathbf{y};\mathbf{h}_r;\mathbf{h}_s)$ (the multivariate MI between $\mathbf{y}$, $\mathbf{h}_r$ and $\mathbf{h}_s$). The multivariate $I(\mathbf{y};\mathbf{h}_r;\mathbf{h}_s)$ is not considered here due to the lack of space.

We are interested in optimizing the multicarrier radar waveform for the target detection task, under a total transmitted power constraint and a minimum required capacity for the communication system. For this, we use the proposed MI based criteria to obtain the objective functions to be maximized in the optimization problems and then solve these problems analytically. The presented optimization problems in this paper are convex and their solutions represent the optimum power allocation for each subcarrier in the multicarrier radar waveform.

First, we consider maximizing $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$ and, similar to [7], we can write the optimization problem for the radar waveform with the objective function

$$I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r,\mathbf{h}_s) = H(\mathbf{y}) - H(\mathbf{s}+\mathbf{n}), \quad (4)$$

where $\mathbf{y}$ is a vector corresponding to the signal at the radar receiver, $\mathbf{s}$ is a vector corresponding to the communication signal arriving at the radar on a direct path and $\mathbf{n}$ is a vector corresponding to the noise and clutter. Adding the constraints mentioned before, and after appropriate derivations as in [7], we obtain the optimization problem as

$$\begin{aligned} \max_{X_r[l]} \quad & \sum_{l=0}^{L-1} \log\left(1 + \frac{|X_r[l]|^2\sigma_{h_r}^2[l] + |X_s[l]|^2\sigma_{h_s}^2[l]}{|X_s[l]|^2\sigma_{h_d}^2[l] + \sigma_n^2[l]}\right) \\ \text{s.t.} \quad & \log\left(1 + \frac{|X_s[l]|^2\sigma_{h_c}^2[l]}{|X_r[l]|^2\sigma_{h_e}^2[l] + \sigma_n^2}\right) \geq t_l \\ & \sum_{l=0}^{L-1} |X_r[l]|^2 \leq P_T, \end{aligned} \quad (5)$$

where $|X_r[l]|^2$ and $|X_s[l]|^2$ are the power of the radar and the communication signals, respectively, for the lth subcarrier and $\sigma_{h_r}^2[l]$, $\sigma_{h_s}^2[l]$, $\sigma_{h_d}^2[l]$, $\sigma_{h_c}^2[l]$, $\sigma_{h_e}^2[l]$ and $\sigma_n^2[l]$ are the power of the corresponding channels and the noise and clutter, respectively, for the lth subcarrier. The SINR term in the objective shows that the scattering

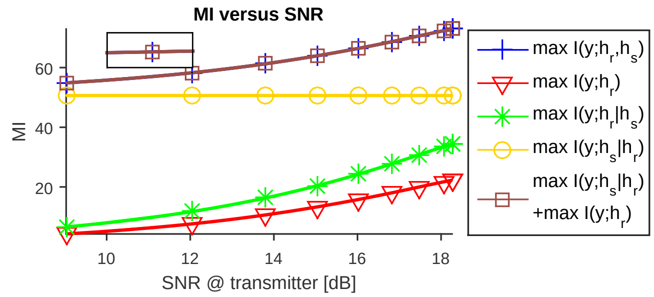

Fig. 3: Maximized $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$, $I(\mathbf{y};\mathbf{h}_r)$, $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$, $I(\mathbf{y};\mathbf{h}_s|\mathbf{h}_r)$ and the sum of $\max I(\mathbf{y};\mathbf{h}_s|\mathbf{h}_r)$ and $\max I(\mathbf{y};\mathbf{h}_r)$. We observe that $\max I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r)$

due to the communication signal is considered as useful energy. After simplifying the interference constraint and making the notation $x_l = |X_r[l]|^2$, we can rewrite the optimization problem as

$$\begin{aligned} \max_{x_l} \quad & \sum_{l=0}^{L-1} \log\left(1 + \frac{x_l}{a_l} + b_l\right) \\ \text{s.t.} \quad & \mathbf{0} \preceq \mathbf{x} \preceq \mathbf{c} \\ & \mathbf{1}^T \mathbf{x} \leq P_T, \end{aligned} \tag{6}$$

where we define $a_l = \frac{|X_s[l]|^2\sigma^2_{h_d}[l]+\sigma^2_n[l]}{\sigma^2_{h_r}[l]}$, $b_l = \frac{|X_s[l]|^2\sigma^2_{h_s}[l]}{|X_s[l]|^2\sigma^2_{h_d}[l]+\sigma^2_n[l]}$ and $c_l = \frac{1}{\sigma^2_{h_e}[l]}\left[\frac{|X_s[l]|^2\sigma^2_{h_c}[l]}{e^{t_l}-1} - \sigma^2_n[l]\right]$. This optimization problem can be solved using the technique of Lagrange multipliers and the power allocation solution is given by

$$x_l^* = \begin{cases} 0, & a_l(1+b_l) > \frac{1}{\lambda_3} \\ \frac{1}{\lambda_3} - a_l(1+b_l), & \frac{1}{\lambda_3} - c_l < a_l(1+b_l) < \frac{1}{\lambda_3} \\ c_l, & a_l(1+b_l) < \frac{1}{\lambda_3} - c_l \end{cases}, \tag{7}$$

where λ_3 is the Lagrange dual variable corresponding to the constraint on the transmitted radar power.

Next, we consider maximizing $I(\mathbf{y};\mathbf{h}_r)$, given by

$$I(\mathbf{y};\mathbf{h}_r) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{h}_r) = H(\mathbf{y}) - H(\mathbf{r}_s + \mathbf{s} + \mathbf{n}), \tag{8}$$

where $\mathbf{r}_s$ is a vector corresponding to the communication signal scattered off the target at the radar receiver. Following a similar procedure as before, with the same constraints, we can write the optimization problem as

$$\begin{aligned} \max_{X_r[l]} \quad & \sum_{l=0}^{L-1} \log\left(1 + \frac{|X_r[l]|^2\sigma^2_{h_r}[l]}{|X_s[l]|^2\sigma^2_{h_s}[l] + |X_s[l]|^2\sigma^2_{h_d}[l] + \sigma^2_n[l]}\right) \\ \text{s.t.} \quad & \log\left(1 + \frac{|X_s[l]|^2\sigma^2_{h_c}[l]}{|X_r[l]|^2\sigma^2_{h_e}[l] + \sigma^2_n}\right) \geq t_l \\ & \sum_{l=0}^{L-1} |X_r[l]|^2 \leq P_T. \end{aligned} \tag{9}$$

The SINR term in the objective shows that the scattering due to the communication signal is considered as interference. After simplifying the interference constraint and using the same notations, we can rewrite the optimization problem as

$$\begin{aligned} \max_{x_l} \quad & \sum_{l=0}^{L-1} \log\left(1 + \frac{x_l}{a_l(1+b_l)}\right) \\ \text{s.t.} \quad & \mathbf{0} \preceq \mathbf{x} \preceq \mathbf{c} \\ & \mathbf{1}^T \mathbf{x} \leq P_T. \end{aligned} \tag{10}$$

By using the technique of Lagrange multipliers as in (7), we solve the optimization problem and obtain the power allocation solution which is identical to the solution in (7). Thus, the radar waveform that maximizes $I(\mathbf{y};\mathbf{h}_r)$ also maximizes $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$. We have that $\max I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s) = \max I(\mathbf{y};\mathbf{h}_r) + \max I(\mathbf{y};\mathbf{h}_s|\mathbf{h}_r)$, where $\max I(\mathbf{y};\mathbf{h}_s|\mathbf{h}_r)$ is constant, regardless of the optimized radar waveform. This corresponds to the information about the target provided by the communication signals scattered off the target. As the maximization of $I(\mathbf{y};\mathbf{h}_r)$ and $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$ is done with respect to only the radar waveform, the same waveform will maximize both.

One can also choose to maximize $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$, given by

$$\begin{aligned} I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s) &= H(\mathbf{y}|\mathbf{h}_s) - H(\mathbf{y}|\mathbf{h}_r,\mathbf{h}_s) \\ &= H(\mathbf{r}+\mathbf{s}+\mathbf{n}) - H(\mathbf{s}+\mathbf{n}), \end{aligned} \tag{11}$$

where $\mathbf{r}$ is a vector corresponding to the radar echo. The SINR term in the objective shows that the scattering due to the communication signal is ignored. Proceeding as before, we can write the optimization problem as

$$\begin{aligned} \max_{X_r[l]} \quad & \sum_{l=0}^{L-1} \log\left(1 + \frac{|X_r[l]|^2\sigma^2_{h_r}[l]}{|X_s[l]|^2\sigma^2_{h_d}[l] + \sigma^2_n[l]}\right) \\ \text{s.t.} \quad & \log\left(1 + \frac{|X_s[l]|^2\sigma^2_{h_c}[l]}{|X_r[l]|^2\sigma^2_{h_e}[l] + \sigma^2_n}\right) \geq t_l \\ & \sum_{l=0}^{L-1} |X_r[l]|^2 \leq P_T. \end{aligned} \tag{12}$$

After simplifying the interference constraint and using the same notations, we can rewrite the optimization problem as

$$\begin{aligned} \max_{x_l} \quad & \sum_{l=0}^{L-1} \log\left(1 + \frac{x_l}{a_l}\right) \\ \text{s.t.} \quad & \mathbf{0} \preceq \mathbf{x} \preceq \mathbf{c} \\ & \mathbf{1}^T \mathbf{x} \leq P_T \end{aligned} \tag{13}$$

and, using again the technique of Lagrange multipliers as in (7), we obtain the power allocation solution

$$x_l^* = \begin{cases} 0, & a_l > \frac{1}{\lambda_3} \\ \frac{1}{\lambda_3} - a_l, & \frac{1}{\lambda_3} - c_l < a_l < \frac{1}{\lambda_3} \\ c_l, & a_l < \frac{1}{\lambda_3} - c_l \end{cases}. \tag{14}$$

This solution is the same with the one provided in [7], however in [7] the target scattering due to the communication signal is not considered at the radar receiver. Thus, it is presented therein as the solution that maximizes $I(\mathbf{y};\mathbf{h}_r)$. As we will see in the following, such solution offers inferior detection performance compared to the one that, for example, maximizes $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$ in the scenario in Figure 1.

The discrete values of the maximized MI for the considered criteria are shown in Figure 3. These values are obtained by optimizing the radar waveform for different constraints on the maximum allowed transmitted radar power P_T. It is observed in Figure 3 that $\max I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r)$. This ordering is due to the fact that the maximized MIs are functions of the optimized radar waveform and the SINR. It is observed from the SINR term of the objective functions of (5), (9) and (12) that the scattering due to the communication signal contributes to the signal part in (5), to the noise, clutter and interference part in (9) and to neither in (12). Similar ordering can be established among the SINRs, which justifies the ordering of the maximized MIs.

For the simulation results in Figure 3 and the explanations in this paper we also use $I(\mathbf{y};\mathbf{h}_s|\mathbf{h}_r)$, given by

$$\begin{aligned} I(\mathbf{y};\mathbf{h}_s|\mathbf{h}_r) &= H(\mathbf{y}|\mathbf{h}_r) - H(\mathbf{y}|\mathbf{h}_r,\mathbf{h}_s) \\ &= \sum_{l=0}^{L-1} \log\left(1 + \frac{|X_s[l]|^2\sigma^2_{h_s}[l]}{|X_s[l]|^2\sigma^2_{h_d}[l] + \sigma^2_n[l]}\right), \end{aligned} \tag{15}$$

which is independent from the radar waveform.

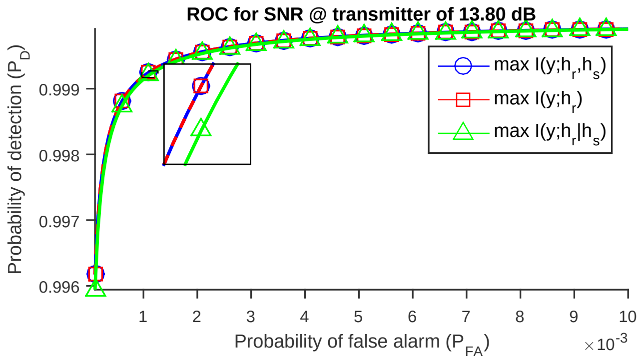

Fig. 4: ROC curves achieved with radar waveforms that maximize $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$, $I(\mathbf{y};\mathbf{h}_r)$ or $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$. The radar waveforms that maximize $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$ or $I(\mathbf{y};\mathbf{h}_r)$ provide better detection performance.

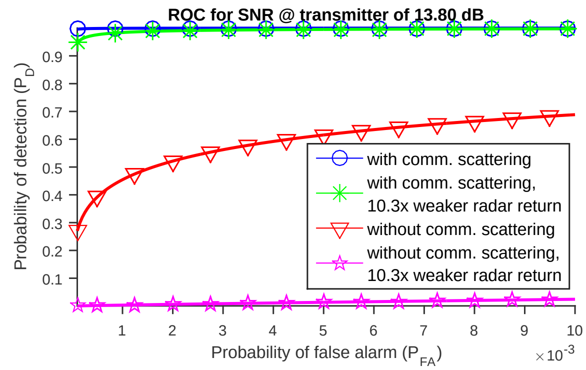

Fig. 5: ROC curves achieved with waveforms that maximize $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$ for different cases. The detection capability decreases considerably when the communication signals scattered off the target are not considered and it becomes much lower when the radar is also dealing with weak returns.

4. TARGET DETECTION PERFORMANCE

In this section, we present comparisons of the three MI based criteria presented in Section 3 by considering the detection performance achieved by a waveform optimized using such criteria. It is instructive to state, which criteria is used to optimize a waveform that offers optimal detection performance. For the simulation results in this section, we use the Neyman-Pearson (NP) detector introduced in [8] in order to plot the receiver operating characteristic (ROC) curves. For each MI based criterion, the radar waveform that maximizes the MI is obtained by solving the optimization problem in CVX [12] and then the radar waveform is plugged in the detector to obtain the achieved probability of detection.

4.1. General Performance

Target detection is an important radar task. We test the detection performance of the radar waveforms optimized using the MI based criteria presented in Section 3 and the obtained simulation results are shown in Figure 4. A radar waveform that maximizes $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$ offers a slightly lower detection performance than a waveform that maximizes $I(\mathbf{y};\mathbf{h}_r)$ or $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$. Thus, when considering a joint radar/communication scenario like the one in Figure 1 it is better to optimize the waveform by maximizing $I(\mathbf{y};\mathbf{h}_r)$ or $I(\mathbf{y};\mathbf{h}_r,\mathbf{h}_s)$.

An useful result is obtained by analyzing Figures 3 and 4. We have noticed in Figure 3 that $\max I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s) > \max I(\mathbf{y};\mathbf{h}_r)$, however Figure 4 shows a better detection performance for the waveform that maximizes $I(\mathbf{y};\mathbf{h}_r)$. Thus, a larger maximized MI does not guarantee an optimal detection performance.

4.2. Performance for a Weak Radar Return

Exploiting the communication signals scattered off the target at the radar receiver has been shown, previously in [8], to improve target detection in the case of a NP detector. In this paper, we stress even more the importance of exploiting the scattering off the target due to the communication signals by considering the case where a stealth target is being illuminated. Such targets are known to provide weak radar returns when illuminated from the front, however much stronger returns when illuminated from the side, for example. We consider the case that the stealth target is illuminated from the front by the radar waveform and from the side by the communication signals. Thus, the communication signals scattered off the target would be a more significant component in target detection than the radar echo.

We have shown in Section 3 that the solution to the optimization problem that maximizes $I(\mathbf{y};\mathbf{h}_r|\mathbf{h}_s)$, considering the scenario in Figure 1, is identical to the the one that maximizes $I(\mathbf{y};\mathbf{h}_r)$ for the scenario considered in [7]. However, for the scenario considered in this paper (and the one in [8]), the communication signals scattered off the target are considered for the alternative hypothesis of the NP detector. This means that the energy corresponding to such scattering improves the detection performance regardless of how strong/weak the radar return is. As observed in Figure 5, the detection performance loss due to a 10 times weaker radar return, for example, is not very large. If the communication signals scattered off the target is not taken into account for the alternative hypothesis of the NP detector, the detected energy is reduced. Thus, a considerably lower detection performance is obtained, as seen in Figure 5. The detection performance is greatly reduced even more when dealing with a 10 time weaker radar return, as the detected energy is very low. The scenario considered in this paper takes into account the communication signals scattered off the target, which we consider more realistic than the one considered in [7]. We have also seen that the scenario considered in Figure 1 is more robust in face of a wider range of targets.

5. CONCLUSIONS

In this paper, three different MI based criteria for radar waveform optimization are proposed. For each criterion, the objective function to be maximized is derived and the associated optimization problem is formulated and solved analytically. The values of the maximized MIs for the considered criteria are provided in simulation results. It is then shown what is the detection performance for the optimized waveforms and thus, which criterion is better suited for optimal detection performance. Using simulation results, it is observed that a larger maximized MI does not guarantee an optimal detection performance. It is also shown that the same optimized radar waveform can be obtained using different MI based criteria proposed in this paper, as well as the similarity of a proposed criterion with another one proposed in a previous work. Finally, it has been shown how important it is to consider the communication signals scattered off the target at the radar receiver for target detection task, especially when dealing with weak radar returns.

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