

MAXIMALLY IMPROPER INTERFERENCE IN UNDERLAY COGNITIVE RADIO NETWORKS

Christian Lameiro^{}, Ignacio Santamaría[†], Wolfgang Utschick[‡] and Peter J. Schreier^{*}*

^{*} Signal and System Theory Group, Universität Paderborn, Germany, <http://sst.upb.de>

[†] Advanced Signal Processing Group, Universidad de Cantabria, Spain, <http://gtas.unican.es>

[‡] Associate Institute for Signal Processing, Technische Universität München, Germany, <http://www.msv.ei.tum.de>

ABSTRACT

It is well-known that the use of improper signaling schemes can be beneficial in interference-limited networks. Here we consider an underlay cognitive radio scenario, where a multi-antenna primary user is protected by an interference temperature constraint that ensures a prescribed rate requirement. We study how the interference temperature threshold changes when the interference is constrained to be maximally improper. Since the spatial structure of the impropriety is an additional degree of freedom, we provide the maximum value of the interference threshold that ensures the rate requirement. We illustrate the potential payoffs of improper signaling with some numerical examples, which show that a secondary user can significantly improve its achievable rate with respect to the proper signaling case.

Index Terms— Improper signaling, interference temperature, underlay cognitive radio.

1. INTRODUCTION

It has been shown recently that improper Gaussian signaling, i.e., the transmission of Gaussian signals whose real and imaginary parts are correlated or have unequal power, can be beneficial in interference-limited networks. This was first observed in the degrees-of-freedom (DoF) study of the interference channel (IC) [1], where improper signaling was shown to increase the achievable DoF in the 3-user IC with constant channel coefficients. Similar results were derived for the 4-user IC in [2]. Several works have followed, showing rate improvements and proposing improper signaling schemes for different interference networks, such as the 2-user single-input single-output (SISO) IC [3, 4], the K -user SISO [5, 6] and multiple-input single-output (MISO) [7] ICs, and the Z-IC [8]. Improper signaling has also been shown to be beneficial in other scenarios, such as the broadcast channel with linear precoding [9], or the interference broadcast channel [10].

This work analyzes the impact of improper signaling in underlay cognitive radio (UCR) networks [11]. In UCR scenarios, the primary user (PU) is typically protected by an

interference temperature (IT) or interference power constraint, so that the so-called secondary users (SUs) can access the channel as long as they ensure that the interference power is below the threshold [12, 13]. Since the performance of the SUs is limited by interference (in this case, by the interference they cause to the primary receivers), they may benefit from transmitting improper Gaussian signals. In our previous work [14, 15], we analyzed the payoffs of improper signaling in a scenario comprised of an SU and a PU, both single-antenna. In such a scenario we showed that, when the PU transmits proper Gaussian signals and has a rate requirement, the instantaneous rate of the SU increases, under certain conditions, when we allow it to transmit improper signals.

In this paper we extend our analysis to the multi-antenna case, where the spatial structure of the interference strongly affects the PU performance, and, consequently, the IT threshold. Thus, we derive the maximum value of the IT threshold when the interference is constrained to be maximally improper, which is achieved when the spatial structure of the improper interference is the least detrimental to the PU. Consequently, an SU operating under such a constraint must also design its transmission scheme such that the structure of the interference's impropriety matches the best-case signature. Finally, we illustrate the potential benefits in terms of SU rate for a simple yet illustrative secondary network.

2. SYSTEM MODEL

2.1. Preliminaries

We start with some definitions and properties of improper complex random vectors that will be used throughout the paper. We refer the reader to [16] for a comprehensive treatment of the subject.

The complementary covariance matrix of a complex random vector \mathbf{x} is defined as $\tilde{\mathbf{R}}_{xx} = \mathcal{E}\{\mathbf{x}\mathbf{x}^T\}$, where $\mathcal{E}\{\cdot\}$ denotes expectation. If $\tilde{\mathbf{R}}_{xx} = \mathbf{0}$, we call \mathbf{x} proper, otherwise improper. Without loss of generality, the complementary covariance matrix can be expressed as [16, Section 3.2.3]

$$\tilde{\mathbf{R}}_{xx} = \mathbf{R}_{xx}^{\frac{1}{2}} \mathbf{F} \mathbf{C} \mathbf{F}^T \mathbf{R}_{xx}^{\frac{T}{2}}, \quad (1)$$

where $\mathbf{R}_{xx} = \mathcal{E}\{\mathbf{x}\mathbf{x}^H\}$ is the covariance matrix, \mathbf{F} is a unitary matrix, which we will call improper signature matrix, and \mathbf{C} is a diagonal matrix containing the circularity coefficients, which measure the degree of impropriety and belong to the range $[0, 1]$. If $\mathbf{C} = \mathbf{I}$, we call \mathbf{x} maximally improper. Finally, it is usually useful to express the second-order statistics of \mathbf{x} through the augmented covariance matrix, which is defined as

$$\underline{\mathbf{R}}_{xx} = \mathcal{E}\{\underline{\mathbf{x}}\underline{\mathbf{x}}^H\} = \begin{pmatrix} \mathbf{R}_{xx} & \tilde{\mathbf{R}}_{xx} \\ \tilde{\mathbf{R}}_{xx}^* & \mathbf{R}_{xx}^* \end{pmatrix}, \quad (2)$$

where $\underline{\mathbf{x}} = [\mathbf{x}^T \ \mathbf{x}^H]^T$.

2.2. System description

Let us consider a multiple-input multiple-output (MIMO) PU link, where both transmitter and receiver are equipped with N antennas. Denoting by $\mathbf{H} \in \mathbb{C}^{N \times N}$ and $\mathbf{Q} \in \mathbb{S}_+^N$ the MIMO channel and transmit covariance matrix, respectively, and assuming that the receiver observes an interference covariance matrix given by $\mathbf{K} \in \mathbb{S}_+^N$, the achievable rate of this link can be written as

$$R(\mathbf{K}) = \log_2 \left| \mathbf{I} + (\sigma^2 \mathbf{I} + \mathbf{K})^{-1} \mathbf{H} \mathbf{Q} \mathbf{H}^H \right|, \quad (3)$$

where σ^2 is the noise power and \mathbb{S}_+^N denotes the set of $N \times N$ positive-semidefinite Hermitian matrices. Let us also assume that this user has a minimum rate requirement to be satisfied, expressed as

$$R(\mathbf{K}) \geq \bar{R}, \quad (4)$$

for a given \mathbf{Q} . An interesting question at this point is to determine the maximum tolerable interference power in order to achieve the foregoing rate constraint. That is, what is the maximum t such that $R(\mathbf{K}) \geq \bar{R}$ for all $\mathbf{K} \in \mathbb{S}_+^N$ satisfying $\text{Tr}(\mathbf{K}) \leq t$? An answer to this question was already provided in [17], where it was shown that the problem is equivalent to finding the worst-case interference covariance matrix, for which a closed-form expression was derived. The interference limit obtained this way provides the optimal protection in terms of both primary and secondary networks when there is a constraint on the total interference power. Notice that, since t is maximized, this is the least stringent constraint for the SU that ensures the PU rate with probability one (because the rate is satisfied for all \mathbf{K}). Nevertheless, for the above-described scenario there are more degrees of freedom that can be exploited to increase the interference power limit without affecting the rate of the PU. Specifically, following the lines of [14, 15] we study how a maximally improper interference would affect the interference temperature limit.

In order to consider an improper interference, we have to modify the model in (3) accordingly. For convenience, we consider the augmented complex formulation, hence, and making use of the differential entropy of an improper random variable (see [16, Eq. (2.35)]), (3) turns into

$$R(\underline{\mathbf{K}}) = \frac{1}{2} \log_2 \left| \mathbf{I} + (\sigma^2 \mathbf{I} + \underline{\mathbf{K}})^{-1} \underline{\mathbf{Q}}_{\mathbf{H}} \right|, \quad (5)$$

where $\underline{\mathbf{Q}}_{\mathbf{H}} \in \mathbb{S}_+^{2N}$ and $\underline{\mathbf{K}} \in \mathbb{S}_+^{2N}$ are the augmented covariance matrices of the desired and interference signals, respectively. Assuming that the PU transmits proper signals independently of the statistics of the interference, the augmented signal covariance matrix is given by

$$\underline{\mathbf{Q}}_{\mathbf{H}} = \begin{pmatrix} \mathbf{H} \mathbf{Q} \mathbf{H}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^* \mathbf{Q}^* \mathbf{H}^T \end{pmatrix}. \quad (6)$$

Since we are now assuming an improper interference, the augmented interference covariance matrix exhibits a non-vanishing complementary covariance matrix, which, using the parameterization (1), can be expressed as

$$\tilde{\mathbf{K}} = \mathbf{K}^{\frac{1}{2}} \mathbf{F} \mathbf{C} \mathbf{F}^T \mathbf{K}^{\frac{T}{2}}, \quad (7)$$

where \mathbf{F} and \mathbf{C} are the improper signature matrix and the matrix of circularity coefficients, respectively. In the sequel, we focus on the maximally improper case.¹ That is, we will constrain the interference to be maximally improper and thus $\mathbf{C} = \mathbf{I}$. Hence, we will express hereafter the rate of the primary user as a function of both \mathbf{K} and \mathbf{F} , i.e., $R(\mathbf{K}, \mathbf{F})$, for a given \mathbf{Q} .

3. INTERFERENCE TEMPERATURE THRESHOLD

In this section, we derive the maximum tolerable interference power when the interference is maximally improper, such that the constraint in (4) is satisfied. As it can be observed from the structure of the interference complementary covariance matrix in (7), the spatial structure of the impropriety, given by \mathbf{F} , affects the achievable rate of the PU. Here we consider a limiting case that yields the maximum tolerable interference threshold, t_{max} . From the standpoint of the secondary network this implies a trade-off that must be further analyzed. That is, the interference limit is higher but the secondary network must transmit a maximally improper signal such that the interference complementary covariance matrix matches the structure of the optimal improper signature. This trade-off will be studied numerically in the next section.

The interference power limit for the maximally improper case, t_{max} , can be computed through the following optimization problem

$$\begin{aligned} \mathcal{P}: \quad & \underset{t}{\text{maximize}} \quad t, \\ & \text{subject to} \quad \max_{\mathbf{F}} R(\mathbf{K}, \mathbf{F}) \geq \bar{R}, \quad \forall \mathbf{K} \in \mathbb{K}_t, \end{aligned}$$

where the set \mathbb{K}_t is defined as

$$\mathbb{K}_t = \{\mathbf{K} \succeq \mathbf{0} : \text{Tr}(\mathbf{K}) \leq t\}. \quad (8)$$

Let us now present the following lemma, which provides some insights into the optimal solution of \mathcal{P} .

¹It has been shown [14] that, whenever improper signaling is beneficial, maximally improper signaling is optimal.

Lemma 1. For a given \mathbf{Q} , let $\phi_1 \geq \phi_2 \geq \dots \geq \phi_N$ and $\mathbf{\Gamma}$ be the eigenvalues and the matrix of eigenvectors of $\mathbf{H}\mathbf{Q}\mathbf{H}^H$, respectively. Then,

$$\min_{\mathbf{K} \in \mathbb{K}_t} \max_{\mathbf{F} \in \mathbb{U}^N} R(\mathbf{K}, \mathbf{F}) = \min_{\substack{\sum \lambda_i \leq t \\ \lambda_i \geq 0}} \sum_{i=1}^N \frac{1}{2} \log_2 \left[1 + \frac{\phi_i}{\sigma^2} \left(1 + \frac{\sigma^2 + \phi_i}{\sigma^2 + 2\lambda_i} \right) \right], \quad (9)$$

where \mathbb{U}^N is the set of $N \times N$ unitary matrices. The improper signature matrix leading to (9) is $\mathbf{F} = \mathbf{\Gamma}$.

Proof. Due to the lack of space, we only provide a sketch of the proof. Let $\mathbf{H}\mathbf{Q}\mathbf{H}^H = \mathbf{\Gamma}\mathbf{\Phi}\mathbf{\Gamma}^H$ be the eigenvalue decomposition (EVD). Using majorization (we refer the interested reader to [18]), it can be shown that $\mathbf{F} = \mathbf{\Gamma}\mathbf{P}^{\frac{1}{2}}$ and $\mathbf{K} = \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^H$ hold for the optimal solution, where \mathbf{P} is a symmetric permutation matrix and $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_N)$ is the matrix of eigenvalues of \mathbf{K} . Consequently $\tilde{\mathbf{K}} = \mathbf{\Gamma}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{P}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{\Gamma}^T$, which yields

$$\min_{\mathbf{K} \in \mathbb{K}_t} \max_{\mathbf{F} \in \mathbb{U}^N} R(\mathbf{K}, \mathbf{F}) = \min_{\substack{\sum \lambda_i \leq t \\ \lambda_i \geq 0}} \max_{\pi} \frac{1}{2} \sum_{i=1}^N \log_2 \left[1 + \frac{\sigma^2 (\phi_i + \phi_{\pi(i)}) + \phi_i \phi_{\pi(i)} + \lambda_{\pi(i)} \phi_i + \lambda_i \phi_{\pi(i)}}{\sigma^2 (\sigma^2 + \lambda_i + \lambda_{\pi(i)})} \right], \quad (10)$$

where π is a symmetric permutation, i.e., $\pi(i) = j \Leftrightarrow \pi(j) = i$. Notice that π selects those pairs of signal modes that are correlated in the improper sense. To prove the lemma, we have to show that the optimal permutation satisfies $\pi(i) = i, i = 1, \dots, N$. To this end, let us consider an arbitrary permutation such that $\pi(i) = j$ (and $\pi(j) = i$), $i \neq j$. The i th and j th contributions to the summation in (10) are given by

$$r_{ij} = \frac{1}{2} \log_2 \left[1 + \frac{\sigma^2 (\phi_i + \phi_{\pi(i)}) + \phi_i \phi_{\pi(i)} + \lambda_{\pi(i)} \phi_i + \lambda_i \phi_{\pi(i)}}{\sigma^2 (\sigma^2 + \lambda_i + \lambda_{\pi(i)})} \right] + \frac{1}{2} \log_2 \left[1 + \frac{\sigma^2 (\phi_j + \phi_{\pi(j)}) + \phi_j \phi_{\pi(j)} + \lambda_{\pi(j)} \phi_j + \lambda_j \phi_{\pi(j)}}{\sigma^2 (\sigma^2 + \lambda_j + \lambda_{\pi(j)})} \right]. \quad (11)$$

Now we show that r_{ij} increases if we take permutation π' , with $\pi'(i) = i$ and $\pi'(j) = j$. Notice that applying π' is equivalent to swapping ϕ_i and λ_i of the first term in (11) with those of the second term. To this end, let us first swap ϕ_i with ϕ_j . Comparing the derivatives of the first and second terms of (11) with respect to ϕ_i and ϕ_j , respectively, it follows that r_{ij} increases if $\phi_i + \lambda_i > \phi_j + \lambda_j$. Assuming, without loss of generality, that $\phi_i \geq \phi_j$, it is easy to see through

(11) that $\lambda_i \geq \lambda_j$ must hold for the optimal solution, hence $\phi_i + \lambda_i > \phi_j + \lambda_j$ holds and, consequently, r_{ij} increases. Second, once ϕ_i and ϕ_j have been swapped, it can be shown by taking the derivatives that swapping λ_i with λ_j keeps r_{ij} unchanged. As a result, π' increases the cost function with respect to π . Applying this procedure to all other pairs, we obtain that $\pi(i) = i, i = 1, \dots, N$ is optimal, which yields (9) and concludes the proof. \square

Lemma 1 can be interpreted as follows. For the optimal improper signature, the interference at each transmission mode is a maximally improper signal, but there is no correlation among different signal modes. In other words, the PU transmission can be decomposed into a set of N signal modes each of them affected by a maximally improper interference. Now we formalize the solution of \mathcal{P} in the following theorem.

Theorem 1. For a given \mathbf{Q} , let $\phi_1 \geq \phi_2 \geq \dots \geq \phi_N$ and $\mathbf{\Gamma}$ be the eigenvalues and the matrix of eigenvectors of $\mathbf{H}\mathbf{Q}\mathbf{H}^H$, respectively. Then, the optimal solution of \mathcal{P} is

$$t_{max} = \text{Tr}(\mathbf{K}_{max}), \quad (12)$$

where $\mathbf{K}_{max} = \mathbf{\Gamma}\mathbf{\Lambda}_{max}\mathbf{\Gamma}^H$ is the worst-case interference covariance matrix. $\mathbf{\Lambda}_{max}$ is a positive diagonal matrix whose entries are given by a multilevel water-filling as

$$\lambda_{max[i]} = \frac{1}{2} \left[\sqrt{\phi_i \left(\frac{1}{4} \phi_i + \mu \right)} - \left(\frac{1}{2} \phi_i + \sigma^2 \right) \right]^+, \quad (13)$$

where μ is such that the rate constraint holds with equality.

Proof. By [17, Lemma 1], the optimal solution of \mathcal{P} can be obtained through the following equivalent problem

$$\begin{aligned} \mathcal{P}_{eq} : \quad & \underset{\mathbf{K}}{\text{minimize}} && t, \\ & \text{subject to} && \max_{\mathbf{F}} R(\mathbf{K}, \mathbf{F}) \leq \bar{R}, \\ & && \mathbf{K} \in \mathbb{K}_t. \end{aligned}$$

Notice that the above problem consists of finding the worst-case interference covariance matrix, \mathbf{K} . Consequently, by Lemma 1, \mathcal{P}_{eq} can equivalently be written as

$$\begin{aligned} \tilde{\mathcal{P}}_{eq} : \quad & \underset{\{\lambda_i\}_{i=1}^N}{\text{minimize}} && \sum_{i=1}^N \lambda_i, \\ & \text{subject to} && \sum_{i=1}^N \frac{1}{2} \log_2 \left[1 + \frac{\phi_i}{\sigma^2} \left(1 + \frac{\sigma^2 + \phi_i}{\sigma^2 + 2\lambda_i} \right) \right] \leq \bar{R}, \\ & && \lambda_i \geq 0, i = 1, \dots, N. \end{aligned}$$

Since the above problem is convex and satisfies Slater's condition [19], we can find its optimal solution by solving the dual problem. Finally, the Karush-Kuhn-Tucker (KKT) conditions of $\tilde{\mathcal{P}}_{eq}$ yield (12) and (13), which concludes the proof. \square

4. NUMERICAL EVALUATION

In this section, we provide some numerical examples showing the potential benefits of an improper interference in terms of SU rate. To this end, we consider a simple secondary network, comprised solely of a point-to-point SU equipped with N antennas at both sides of the link (i.e., same model as the PU link), hence the whole network can be regarded as a 2-user IC. The transmit covariance matrix of the SU is optimized to maximize its achievable rate subject to the transmit power and interference temperature constraints. As a suboptimal but simple procedure, we first optimize the transmit covariance matrix considering a proper transmission, and then adjust the transmit complementary covariance matrix to match the improper signature $\mathbf{F} = \mathbf{\Gamma}$.

We define the transmit signal-to-noise ratio (SNR) of the PU and SU respectively as $\text{SNR}_{\text{PU}} = \frac{P_{\text{PU}}}{\sigma^2}$ and $\text{SNR}_{\text{SU}} = \frac{P_{\text{SU}}}{\sigma^2}$, being P_{PU} and P_{SU} are the PU and SU power budgets, respectively, and we take $\sigma^2 = 1$ without loss of generality. We consider that each entry of the $N \times N$ channel matrices is independently distributed as a proper complex Gaussian random variable with zero mean and unit variance, except for the SU-PU channel, whose variance is set to $\sigma_{\mathcal{I}}^2$. For all simulations, we will consider $N = 4$, $\text{SNR}_{\text{SU}} = 20$ dB and $\bar{R} = \alpha R(0)$, with $0 \leq \alpha \leq 1$.

First we compare the IT thresholds for the proper and maximally improper cases. Figure 1 shows the ratio between proper and improper signaling IT as a function of α , where a significant increase can be observed, especially for low values of α and high SNR_{PU} values. To show the impact on the rates, Fig. 2 depicts the achievable rate of three different transmission schemes: the conventional proper scheme, the maximally improper scheme derived in Section 3, and an adaptative scheme; for $\alpha = 0.6$ and two different SU-PU channel gains: $\sigma_{\mathcal{I}}^2 = 1$ and $\sigma_{\mathcal{I}}^2 = 10$. The adaptative scheme follows the transmission scheme leading to the highest rate, i.e., proper or maximally improper. Notice that this scheme can easily be performed if the PU informs the SU about the two IT thresholds, so that no additional channel state information (CSI) is required at the SU. For $\sigma_{\mathcal{I}}^2 = 1$, maximally improper signaling provides significant gains for SNR_{SU} higher than 10 dB. For low SNR (low power budget), improper signaling does not yield any gain on average since proper signaling permits maximum power transmission in most channel realizations, which does not leave much room for improvement. Notice, however, that the adaptative scheme is always beneficial and does not require any additional CSI. When the interference is more significant ($\sigma_{\mathcal{I}}^2 = 10$), improper signaling provides a substantial increase in achievable rate in the whole SNR regime. Indeed, the proper scheme achieves a very low rate since the interference level at the PU is too high. In such a case, the maximally improper signaling scheme is much more capable of handling the interference, which is in agreement with the results of our previous work [14].

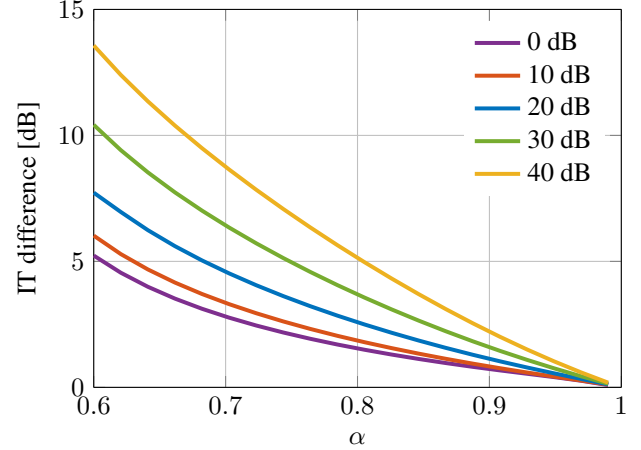


Fig. 1. Difference between the ITs of improper and proper signaling, for $N = 4$ and different values of SNR_{PU} .

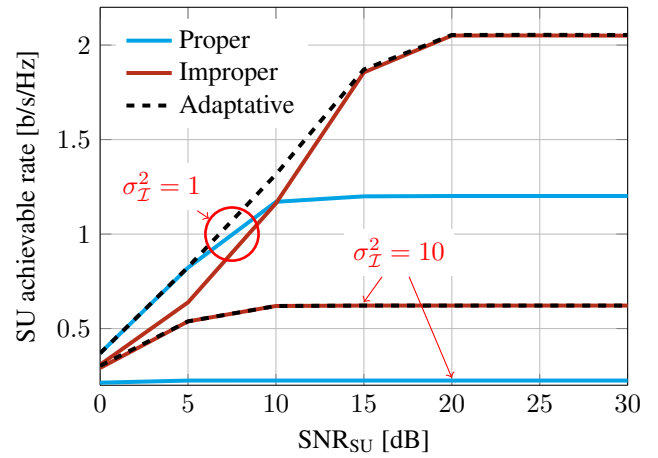


Fig. 2. SU rate for $N = 4$, $\alpha = 0.6$ and $\text{SNR}_{\text{PU}} = 20$ dB.

5. CONCLUSION

In this paper, we have studied how the IT threshold for a MIMO PU changes when we constrain the interference to be maximally improper. We have observed that the improper signature matrix, which controls the spatial structure of the impropriety, affects the interference power threshold. Therefore, we have derived the maximum tolerable interference power for the optimal improper signature, which has resulted in a maximally improper interference at each eigenmode of the PU. Additionally, the interference must be further constrained to match this specific improper signature matrix. Simulations have shown that this maximally improper scheme may provide significant improvements with respect to the proper signaling case. In light of the results, we will consider in future work the analysis of joint spatial-improper shaping constraints, to further improve the performance over conventional proper and spatially-unconstrained interference.

6. REFERENCES

- [1] V.R. Cadambe, S.A. Jafar, and C. Wang, "Interference alignment with asymmetric complex signaling-settling the Høst-Madsen-Nosratinia conjecture," *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4552–4565, Sept. 2010.
- [2] C. Lameiro and I. Santamaría, "Degrees-of-freedom for the 4-user SISO interference channel with improper signaling," in *Proc. IEEE Int. Conf. Comm.*, Budapest, Hungary, June 2013.
- [3] Z.K.M. Ho and E. Jorswieck, "Improper Gaussian signaling on the two-user siso interference channel," *IEEE Transactions on Wireless Communications*, vol. 11, no. 9, pp. 3194–3203, Sept. 2012.
- [4] Y. Zeng, C.M. Yetis, E. Gunawan, Y.L. Guan, and R. Zhang, "Transmit optimization with improper Gaussian signaling for interference channels," *IEEE Transactions on Signal Processing*, vol. 61, no. 11, pp. 2899–2913, June 2013.
- [5] Y. Zeng, C.M. Yetis, E. Gunawan, Y.L. Guan, and R. Zhang, "Improper Gaussian signaling for the K -user SISO interference channel," in *Proceedings of the IEEE International Conference on Communications (ICC)*, June 2013, pp. 5219–5223.
- [6] H.D. Nguyen, R. Zhang, and S. Sun, "Improper signaling for symbol error rate minimization in K -user interference channel," *IEEE Transactions on Communications*, vol. 63, no. 3, pp. 857–869, Mar. 2015.
- [7] Y. Zeng, R. Zhang, E. Gunawan, and Y.L. Guan, "Optimized transmission with improper Gaussian signaling in the k -user MISO interference channel," *IEEE Transactions on Wireless Communications*, vol. 12, no. 12, pp. 6303–6313, Dec. 2013.
- [8] E. Kurniawan and S. Sun, "Improper Gaussian signaling scheme for the Z-interference channel," *IEEE Transactions on Wireless Communications*, vol. 14, no. 7, pp. 3912–3923, July 2015.
- [9] C. Hellings, M. Joham, and W. Utschick, "QoS feasibility in MIMO broadcast channels with widely linear transceivers," *IEEE Signal Processing Letters*, vol. 20, no. 11, pp. 1134–1137, Nov. 2013.
- [10] H.-Y. Shin, S.-H. Park, H. Park, and I. Lee, "A new approach of interference alignment through asymmetric complex signaling and multiuser diversity," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 880–884, Mar. 2012.
- [11] J. Mitola and G.Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, 1999.
- [12] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, Feb. 2005.
- [13] A. Goldsmith, S.A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.
- [14] C. Lameiro, I. Santamaría, and P.J. Schreier, "Benefits of improper signaling for underlay cognitive radio," *IEEE Wireless Comm. Lett.*, vol. 4, pp. 22–25, February 2015.
- [15] C. Lameiro, I. Santamaría, and P.J. Schreier, "Analysis of maximally improper signalling schemes for underlay cognitive radio," in *Proc. IEEE Int. Conf. Comm.*, London, UK, June 2015.
- [16] P.J. Schreier and L.L. Scharf, *Statistical signal processing of complex-valued data: the theory of improper and noncircular signals*, Cambridge, U.K.: Cambridge Univ. Press, 2010.
- [17] C. Lameiro, W. Utschick, and I. Santamaría, "Interference-temperature limit for cognitive radio networks with MIMO primary users," in *Proc. Asilomar Conf. Signals Syst. Computers*, Pacific Grove, CA, USA, November 2014.
- [18] A.W. Marshall, I. Olkin, and B.C. Arnold, *Inequalities: Theory of Majorization and Its Applications*, Springer Series in Statistics. Springer New York, New York, NY, 2011.
- [19] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, 2004.