MULTI-PAIR TWO-WAY AF RELAYING SYSTEMS WITH MASSIVE ARRAYS AND IMPERFECT CSI

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ABSTRACT

We consider a multi-pair two-way amplify-and-forward relaying system with a massive antenna array at the relay and estimated channel state information, assuming maximum-ratio combining/transmission processing. Closed-form approximations of the sum spectral efficiency are developed and simple analytical power scaling laws are presented, which reveal a fundamental trade-off between the transmit powers of each user/the relay and of each pilot symbol. Finally, the optimal power allocation problem is studied.

***Index Terms*—** Massive MIMO, multi-pair, power scaling law, two-way relaying.

1. INTRODUCTION

Due to the high spectral efficiency, multi-pair two-way relaying systems have attracted a great deal of research attention from both academia and industry [1–4]. The key challenge for the successful implementation of such systems is to properly compensate for the inter-pair interference. Many sophisticated techniques have been proposed in literature, including dirty-paper coding [5] and interference alignment [6]. However, these techniques significantly increase the complexity of the system.

Recently, the idea of using massive antenna arrays at the relay to mitigate inter-pair interference has been proposed [2, 4]. In [2], the spectral efficiency of the amplify-and-forward (AF) protocol was studied for both maximum ratio (MR) combining/transmission and zero-forcing (ZF) combining/transmission schemes. In parallel, [4] presented a closed-form approximation of the ergodic rate with the MR scheme, and then studied the problem of optimal user pair selection. However, both works assume the availability of perfect channel state information (CSI) at the relay, which is difficult to acquire in practice due to the finite signal-to-noise ratio (SNR) and limited channel coherence that cause non-negligible channel estimation errors.

Motivated by this, this paper studies the performance of a multi-pair two-way relaying system by considering imperfect CSI. The main contributions of this paper include: 1) We analytically characterize a lower bound on the spectral efficiency employing the MR scheme; 2) We derive simple power scaling laws, which generalize the results of [2, 4, 7]. Also, the fundamental trade-off between the transmit powers of each user/the relay and the transmit power of each pilot symbol is revealed; 3) By assuming the large scale fading is the same for all the links, we devise an optimal power allocation policy for the relay and each user, which maximizes the sum spectral efficiency of the system.

Notation: We use $(\cdot)^H$, $(\cdot)^*$, and $(\cdot)^T$ to denote the conjugate transpose, the conjugate, and the transpose, respectively. Also, $||\cdot||$ represents the Euclidian 2-norm, and $|\cdot|$ is the absolute value. In addition, $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ denotes a circularly symmetric complex Gaussian vector $\mathbf{x}$ with zero mean and covariance matrix $\mathbf{\Sigma}$. Finally, the statistical expectation operator is represented by $\mathtt{E}\{\cdot\}$.

2. SYSTEM MODEL

Consider a multi-pair two-way relaying system, where N pairs of single-antenna users, denoted as $\mathrm{T}_{A,i}$ and $\mathrm{T}_{B,i}$, for $i = 1, \ldots, N$, intend to exchange information within each pair, under the assistance of a shared relay with M antennas, denoted as T_R. We assume that the direct links between $\mathrm{T}_{A,i}$ and $\mathrm{T}_{B,i}$ do not exist due to heavy shadowing and path-loss [3]. Also, the relay operates in the half-duplex mode, i.e., it cannot transmit and receive simultaneously.

The system operates in a time-division duplex mode where channel reciprocity is assumed to hold. Therefore, the uplink and downlink channels between $\mathrm{T}_{A,i}$ and T_R can be denoted as $\mathbf{g}_{AR,i} \in \mathbb{C}^{M\times 1}$ and $\mathbf{g}^{\mathrm{T}}_{AR,i} \in \mathbb{C}^{1\times M}$, respectively. Similarly, the uplink and downlink channels between $\mathrm{T}_{B,i}$ and T_R are denoted as $\mathbf{g}_{RB,i} \in \mathbb{C}^{M\times 1}$ and $\mathbf{g}^{\mathrm{T}}_{RB,i} \in \mathbb{C}^{1\times M}$, $i = 1, \ldots, N$, respectively. Moreover, they are modeled as $\mathbf{g}_{AR,i} \sim \mathcal{CN}(\mathbf{0}, \beta_{AR,i}\mathbf{I}_M)$ and $\mathbf{g}_{RB,i} \sim \mathcal{CN}(\mathbf{0}, \beta_{RB,i}\mathbf{I}_M)$. This is known as the classical Rayleigh fading model, where $\beta_{AR,i}$ and $\beta_{RB,i}$ model the large-scale path-loss effects, which are assumed to be constant over many coherence intervals and known a priori. For notational convenience, the channel vectors are collected in the matrices $\mathbf{G}_{AR} = [\mathbf{g}_{AR,1}, \ldots, \mathbf{g}_{AR,N}] \in \mathbb{C}^{M\times N}$ and $\mathbf{G}_{RB} = [\mathbf{g}_{RB,1}, \ldots, \mathbf{g}_{RB,N}] \in \mathbb{C}^{M\times N}$.

For the considered multi-pair two-way relaying system, the entire information transmission process consists of two separate phases. In the first phase, $\mathrm{T}_{A,i}$ and $\mathrm{T}_{B,i}$ simultaneously transmit their respective signals to T_R, for $i = 1, \ldots, N$. Thus, the received sig-

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nal at the T_R is given by

$$\mathbf{y}_r = \sqrt{p_u}\sum_{i=1}^{N}\left(\mathbf{g}_{AR,i}x_{A,i} + \mathbf{g}_{RB,i}x_{B,i}\right) + \mathbf{n}_R, \quad (1)$$

where $x_{A,i}$ and $x_{B,i}$ are independent Gaussian signals distributed as $\mathcal{CN}(0,1)$ transmitted by the *i*-th user pair, p_u is the average transmit power of each user, and $\mathbf{n}_R \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is a vector of additive white Gaussian noise (AWGN) at the relay. Note that p_u has the interpretation of normalized "transmit" SNR and is therefore dimensionless.

In the second phase, T_R broadcasts $\mathbf{y}_r$ to all the users, after multiplying the received signal $\mathbf{y}_r$ with the precoding matrix $\mathbf{F}$. Thus, the transmit signal from T_R is given by $\mathbf{y}_t = \rho\mathbf{F}\mathbf{y}_r$, where $\mathbf{F} \in \mathbb{C}^{M\times M}$ is a linear processing matrix (which is a function of the channel estimates), and ρ is a normalization coefficient, which is chosen to satisfy the long-term transmit power constraint at the relay, namely, $\mathtt{E}\left\{||\mathbf{y}_t||^2\right\} = p_r$. Finally, $T_{A,i}$ and $T_{B,i}$ receive

$$z_{A,i} = \mathbf{g}_{AR,i}^T\mathbf{y}_t + n_{A,i} \quad , \quad z_{B,i} = \mathbf{g}_{RB,i}^T\mathbf{y}_t + n_{B,i}, \quad (2)$$

respectively, where $n_{A,i} \sim \mathcal{CN}(0,1)$ and $n_{B,i} \sim \mathcal{CN}(0,1)$ represent the AWGN at $T_{A,i}$ and $T_{B,i}$, respectively.

2.1. Channel Estimation

In practice, the random realizations of the channel matrices $\mathbf{G}_{AR}$ and $\mathbf{G}_{RB}$ are not known and need to be estimated at the relay. We assume that the relay forwards the channel estimates to the users through the feedback channel without error. The typical way of estimating channels is to utilize pilots [8]. To this end, $\tau_p \geq 2N$ symbols of each coherence interval τ_c (in symbols) are used for channel training. Moreover, the transmit power of each pilot symbol is p_p. As in [7, 9, 10], we assume T_R uses the minimum mean-square-error (MMSE) estimation method to estimate $\mathbf{G}_{AR}$ and $\mathbf{G}_{RB}$. As a result, $\mathbf{G}_{AR}$ and $\mathbf{G}_{RB}$ can be decomposed as

$$\mathbf{G}_{AR} = \hat{\mathbf{G}}_{AR} + \mathbf{E}_{AR} \quad , \quad \mathbf{G}_{RB} = \hat{\mathbf{G}}_{RB} + \mathbf{E}_{RB}, \quad (3)$$

respectively, where $\mathbf{E}_{AR}$ and $\mathbf{E}_{RB}$ are the estimation error matrices of $\mathbf{G}_{AR}$ and $\mathbf{G}_{RB}$. Due to the orthogonality property of MMSE estimators and the fact that $\hat{\mathbf{G}}_{AR}$, $\mathbf{E}_{AR}$, $\hat{\mathbf{G}}_{RB}$, and $\mathbf{E}_{RB}$ are Gaussian distributed, these matrices are independent of each other. By rewriting (3) in vector form, we have

$$\mathbf{g}_{AR,i} = \hat{\mathbf{g}}_{AR,i} + \mathbf{e}_{AR,i} \quad , \quad \mathbf{g}_{RB,i} = \hat{\mathbf{g}}_{RB,i} + \mathbf{e}_{RB,i}, \quad (4)$$

where $\hat{\mathbf{g}}_{AR,i}$, $\mathbf{e}_{AR,i}$, $\hat{\mathbf{g}}_{RB,i}$, and $\mathbf{e}_{RB,i}$ are the *i*-th columns of $\hat{\mathbf{G}}_{AR}$, $\mathbf{E}_{AR}$, $\hat{\mathbf{G}}_{RB}$, and $\mathbf{E}_{RB}$, respectively, which are mutually independent. Moreover, the elements of $\hat{\mathbf{g}}_{AR,i}$, $\mathbf{e}_{AR,i}$, $\hat{\mathbf{g}}_{RB,i}$, and $\mathbf{e}_{RB,i}$ are Gaussian random variables with zero mean, variance $\sigma_{AR,i}^2$, $\tilde{\sigma}_{AR,i}^2$, $\sigma_{RB,i}^2$ and $\tilde{\sigma}_{RB,i}^2$, where $\sigma_{AR,i}^2 \triangleq \frac{\tau_p p_p \beta_{AR,i}^2}{1+\tau_p p_p \beta_{AR,i}}$, $\tilde{\sigma}_{AR,i}^2 \triangleq \frac{\beta_{AR,i}}{1+\tau_p p_p \beta_{AR,i}}$, $\sigma_{RB,i}^2 \triangleq \frac{\tau_p p_p \beta_{RB,i}^2}{1+\tau_p p_p \beta_{RB,i}}$, and $\tilde{\sigma}_{RB,i}^2 \triangleq \frac{\beta_{RB,i}}{1+\tau_p p_p \beta_{RB,i}}$ [7].

2.2. Linear Processing

The relay station T_R treats the channel estimates as the true channels and utilizes them to perform linear processing. We consider the simple MR processing method to avoid computational delays and burden, hence the beamforming matrix $\mathbf{F} \in \mathbb{C}^{M\times M}$ is obtained as [4] $\mathbf{F} = \mathbf{B}^*\mathbf{A}^H$, where $\mathbf{A} = \left[\hat{\mathbf{G}}_{AR}, \hat{\mathbf{G}}_{RB}\right]$, $\mathbf{B} = \left[\hat{\mathbf{G}}_{RB}, \hat{\mathbf{G}}_{AR}\right]$. Then, we have $\rho = \sqrt{\frac{p_r}{p_u \mathtt{E}\left\{||\mathbf{FC}||^2\right\} + \mathtt{E}\left\{||\mathbf{F}||^2\right\}}}$, where $\mathbf{C} = [\mathbf{G}_{AR}, \mathbf{G}_{RB}]$.

3. SPECTRAL EFFICIENCY

In this section, we investigate the spectral efficiency (in bit/s/Hz) of the two-way AF relaying system. Without loss of generality, we only present the analytical results for $T_{A,i}$, since the results for $T_{B,i}$ can be obtained by interchanging A and B in all the expressions.

When $T_{A,i}$ receives the superimposed signal from T_R, it attempts to subtract its own transmitted message (self-interference) from the observations with the help of estimated channels and ρ. However, the self-interference term cannot be completely removed from the received signals due to imperfect CSI [11, 12], which results in residual self-interference. Therefore, after partially cancelling self-interference, namely, $\rho\sqrt{p_u}\hat{\mathbf{g}}_{AR,i}^{\mathrm{T}}\mathbf{F}\hat{\mathbf{g}}_{AR,i}x_{A,i}$, the received signal at $T_{A,i}$ can be expressed as

$$\begin{aligned}\hat{z}_{A,i} = &\underbrace{\rho\sqrt{p_u}\hat{\mathbf{g}}_{AR,i}^{\mathrm{T}}\mathbf{F}\hat{\mathbf{g}}_{RB,i}x_{B,i}}_{\text{desired signal}} \quad (5)\\ &+ \underbrace{\rho\sqrt{p_u}\left(\hat{\mathbf{g}}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{RB,i} + \mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\hat{\mathbf{g}}_{RB,i} + \mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{RB,i}\right)x_{B,i}}_{\text{estimation error}}\\ &+ \underbrace{\rho\sqrt{p_u}\left(\hat{\mathbf{g}}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{AR,i} + \mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\hat{\mathbf{g}}_{AR,i} + \mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{AR,i}\right)x_{A,i}}_{\text{residual self-interference}}\\ &+ \underbrace{\rho\sqrt{p_u}\sum_{j=1,j\neq i}^{N}\left(\mathbf{g}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{g}_{AR,j}x_{A,j} + \mathbf{g}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{g}_{RB,j}x_{B,j}\right)}_{\text{inter-user interference}}\\ &+ \underbrace{\rho\mathbf{g}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{n}_R + n_{A,i}}_{\text{compound noise}}.\end{aligned}$$

Using a standard approach based on the worst-case uncorrelated additive noise [13–15], a lower bound on the ergodic achievable spectral efficiency for $T_{A,i}$ yields

$$R_{A,i} = \frac{1}{2}\mathtt{E}\left\{\log_2\left(1 + \frac{A_i}{\mathtt{E}\left\{(B_i + C_i + D_i + E_i)\,|\hat{\mathbf{G}}_{AR}, \hat{\mathbf{G}}_{RB}\right\}}\right)\right\},$$

where the inner and outer expectations are taken over the estimation errors and channel estimates, respectively, and

$$\begin{aligned}A_i &\triangleq |\hat{\mathbf{g}}_{AR,i}^{\mathrm{T}}\mathbf{F}\hat{\mathbf{g}}_{RB,i}|^2, \quad (6)\\ B_i &\triangleq |\hat{\mathbf{g}}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{RB,i}|^2 + |\mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\hat{\mathbf{g}}_{RB,i}|^2 + |\mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{RB,i}|^2,\\ C_i &\triangleq |\hat{\mathbf{g}}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{AR,i}|^2 + |\mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\hat{\mathbf{g}}_{AR,i}|^2 + |\mathbf{e}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{e}_{AR,i}|^2,\\ D_i &\triangleq \sum_{j=1,j\neq i}^{N}\left(|\mathbf{g}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{g}_{AR,j}|^2 + |\mathbf{g}_{AR,i}^{\mathrm{T}}\mathbf{F}\mathbf{g}_{RB,j}|^2\right),\\ E_i &\triangleq ||\mathbf{g}_{AR,i}^{\mathrm{T}}\mathbf{F}||^2/p_u + 1/\left(\rho^2 p_u\right).\end{aligned}$$

Thus, the ergodic sum spectral efficiency of the multi-pair two-way AF relaying system is given by $R = \frac{\tau_c - \tau_p}{\tau_c}\sum_{i=1}^{N}(R_{A,i} + R_{B,i})$, where $R_{B,i}$ is the spectral efficiency for $T_{B,i}$, which can be derived in a similar fashion due to symmetry.

As the achievable spectral efficiency $R_{A,i}$ is difficult to obtain in closed-form for finite system dimensions, we consider the large-antenna regime and compute an approximate expression that is tight as $M \to \infty$, based on random matrix theory [16]. In what follows, we will derive a large-system approximation $\tilde{R}_{A,i}$ of $R_{A,i}$.

Theorem 1 *As the number of relay antennas grows infinitely large, then $R_{A,i} - \tilde{R}_{A,i} \overset{M\to\infty}{\longrightarrow} 0$ almost surely, where $\tilde{R}_{A,i}$ is given by*[1]

$$\tilde{R}_{A,i} = \frac{1}{2}\log_2\left(1 + \frac{M}{\tilde{B}_i + \tilde{C}_i + \tilde{D}_i + \tilde{E}_i}\right), \tag{7}$$

where $\tilde{B}_i \triangleq \frac{\tilde{\sigma}^2_{RB,i}}{\sigma^2_{RB,i}} + \frac{\tilde{\sigma}^2_{AR,i}}{\sigma^2_{AR,i}}$, $\tilde{C}_i \triangleq \frac{2\tilde{\sigma}^2_{AR,i}}{\sigma^2_{RB,i}}$, $\tilde{D}_i \triangleq \sum_{j\neq i}^{N}(D_1 + D_2 + D_3)$ *with* $D_1 \triangleq \frac{\sigma^2_{RB,j}\sigma^4_{AR,j} + \sigma^2_{AR,j}\sigma^4_{RB,j}}{\sigma^2_{AR,i}\sigma^4_{RB,i}}$, $D_2 \triangleq \frac{\sigma^2_{AR,j} + \tilde{\sigma}^2_{AR,j} + \sigma^2_{RB,j} + \tilde{\sigma}^2_{RB,j}}{\sigma^2_{RB,i}}$, *and* $D_3 \triangleq \frac{\tilde{\sigma}^2_{AR,i}\sigma^2_{AR,j}\sigma^2_{RB,j}(\sigma^2_{AR,j} + \sigma^2_{RB,j})}{\sigma^4_{AR,i}\sigma^4_{RB,i}}$, *and* $\tilde{E}_i \triangleq \frac{1}{p_u\sigma^2_{RB,i}} + \frac{1}{p_r\sigma^4_{AR,i}\sigma^4_{RB,i}}\sum_{n=1}^{N}\left(\sigma^2_{AR,n}\sigma^2_{RB,n}\left(\sigma^2_{AR,n} + \sigma^2_{RB,n}\right)\right)$.

Theorem 1 suggests that $\tilde{R}_{A,i}$ is an increasing function with respect to M, indicating that the full array gain of M can be achieved also in two-way relaying. Also, focusing on the term $\tilde{D}_i$, it can be seen that, the individual user spectral efficiency $\tilde{R}_{A,i}$ decreases with the number of user pairs N; this is anticipated since increasing the number of users generates more inter-user interference.

4. POWER SCALING LAWS

In this section, we study the potential for power saving to maintain a desired spectral efficiency level.

Theorem 2 *When $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\alpha \geq 0$, $\beta \geq 0$ (but α and β cannot be equal to zero at the same time), and $\gamma > 0$, as well as fixed E_u, E_r, and E_p, we have*

$$\tilde{R}_{A,i} - \frac{1}{2}\log_2\left(1 + \frac{1}{Q_i}\right) \overset{M\to\infty}{\longrightarrow} 0, \tag{8}$$

where $Q_i \triangleq \frac{M^{\alpha+\gamma-1}}{\tau_p E_p E_u \beta^2_{RB,i}} + \frac{M^{\beta+\gamma-1}}{\tau_p E_p E_r \beta^4_{AR,i}\beta^4_{RB,i}}\sum_{n=1}^{N}\left(\beta^2_{AR,n}\beta^2_{RB,n}\left(\beta^2_{AR,n} + \beta^2_{RB,n}\right)\right)$.

Theorem 2 provides a very encouraging result: as long as $\alpha + \gamma = 1$ and $\beta + \gamma = 1$, a non-zero $\tilde{R}_{A,i}$ can be achieved even if the transmit powers of each user, of each pilot symbol, and of the relay are simultaneously cut down inversely proportional to M^α, M^β, and M^γ, respectively. However, when $\alpha+\gamma > 1$ and/or $\beta+\gamma > 1$, $\tilde{R}_{A,i}$ converges to zero because of poor estimation accuracy or low transmit power of each user/the relay. In contrast, when $0 < \alpha + \gamma < 1$ and $0 < \beta + \gamma < 1$, $\tilde{R}_{A,i}$ grows without bound. It is worth noting that the above results rely on the assumption of ideal hardware. However, if we consider non-ideal hardware, the situation is quite different, since hardware impairments are fundamentally limiting the capacity in the many-antenna regime [17].

Corollary 1 *When $\alpha = \beta > 0$ and $\alpha + \gamma = 1$, namely, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\gamma > 0$, as well as fixed E_u, E_r, and E_p, as $M \to \infty$, $\tilde{R}_{A,i}$ converges to a non-zero limit.*

[1] Note that the results of Theorem 1 and others are provided with no proof due to space constraints.

Corollary 1 reveals a fundamental trade-off between the transmit powers of each user and of each pilot symbol; in particular, to achieve the same spectral efficiency, one can either use lower transmit power for each user and higher transmit power for each pilot symbol, or vice versa.

Corollary 2 *When $\alpha > \beta \geq 0$ and $\alpha + \gamma = 1$, namely, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\gamma > 0$, as well as fixed E_u, E_r, and E_p, $\tilde{R}_{A,i}$ converges to*

$$\tilde{R}_{A,i} \overset{M\to\infty}{\longrightarrow} \frac{1}{2}\log_2\left(1 + \tau_p E_p E_u \beta^2_{RB,i}\right). \tag{9}$$

Corollary 2 shows that $\tilde{R}_{A,i}$ is asymptotically independent of the number of user pairs N, which indicates that the sum spectral efficiency is a linear function with respect to N. In other words, a large number of user pairs will significantly boost the sum spectral efficiency in this regime of small p_u.

Corollary 3 *When $0 \leq \alpha < \beta$ and $\beta + \gamma = 1$, namely, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\gamma > 0$, as well as fixed E_u, E_r, and E_p, $\tilde{R}_{A,i}$ converges to*

$$\tilde{R}_{A,i} \overset{M\to\infty}{\longrightarrow} \frac{1}{2}\log_2\left(1 + \frac{\tau_p E_p E_r \beta^4_{AR,i}\beta^4_{RB,i}}{\sum_{n=1}^{N}\left(\beta^2_{AR,n}\beta^2_{RB,n}\left(\beta^2_{AR,n} + \beta^2_{RB,n}\right)\right)}\right).$$

Corollary 3 provides a trade-off between the transmit powers of the relay and of each pilot symbol. In addition, as can be observed, $\tilde{R}_{A,i}$ increases with E_p and E_r, while decreases with N, which suggests that when the number of user pairs increases, the relay and/or each pilot symbol should increase their power in order to maintain the same performance.

5. POWER ALLOCATION

The spectral efficiency can be further enhanced by optimally allocating the transmit powers. We assume that the design of the training phase is done in advance, i.e., the transmit power of each pilot symbol p_p is fixed. We are interested in designing a power allocation policy in the data transmission phase that maximizes the sum spectral efficiency. Let P be the total transmit power in the data transmission phase, thus we have $2Np_u + p_r \leq P$. To simplify the analysis, we assume that the large scale fading is the same for all links, i.e., $\beta_{AR,i} = \beta_{RB,i} = 1$, leading to $\sigma^2_{A,i} = \sigma^2_{B,i} = \sigma^2$, $\tilde{\sigma}^2_{A,i} = \tilde{\sigma}^2_{B,i} = \tilde{\sigma}^2$, and $\tilde{R}_{A,i} = \tilde{R}_{B,i}$.

The optimization problem can be formulated as

$$\max_{p_u,p_r} \quad \frac{\tau_c - \tau_p}{\tau_c}\sum_{i=1}^{N}\left(\tilde{R}_{A,i} + \tilde{R}_{B,i}\right) \tag{10}$$

$$\text{s.t.} \quad 2Np_u + p_r \leq P, \quad p_u \geq 0, \quad p_r \geq 0. \tag{11}$$

After some algebraic manipulations, the optimal solution can be obtained as shown in the following theorem:

Theorem 3 *The optimal power allocation policy is $p_u^{opt} = \frac{P}{4N}$, and $p_r^{opt} = \frac{P}{2}$.*

Proof: For a given p_u, $\tilde{R}_{A,i}$ is an increasing function of p_p, and for a given p_p, $\tilde{R}_{A,i}$ is an increasing function of p_u. Hence, $\tilde{R}_{A,i}$ is

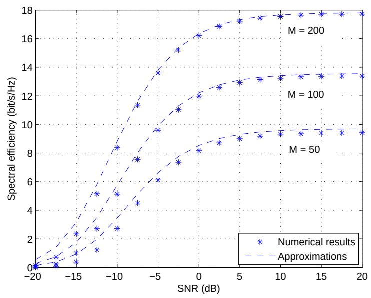

Fig. 1: Spectral efficiency versus SNR for $p_p = p_u$ and $p_r = 2Np_u$.

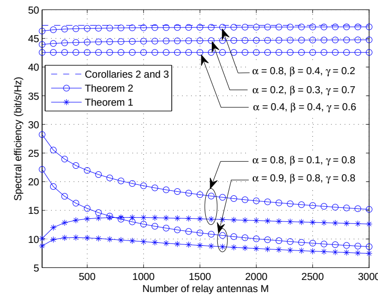

Fig. 2: Spectral efficiency versus the number of relay antennas M for $p_u = E_u/M^{\alpha}$ with $E_u = 10$ dB, $p_r = E_r/M^{\beta}$ with $E_r = 20$ dB, and $p_p = E_p/M^{\gamma}$ with $E_p = 10$ dB.

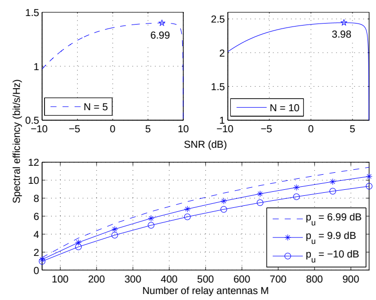

Fig. 3: Spectral efficiency with and without power allocation for $P = 20$ dB and $p_p = -20$ dB.

maximized when $2Np_u + p_r = P$ [18]. Inserting $p_r = P - 2Np_u$ and assuming that all the links are the same, the problem reduces to

$$\arg\max_{p_u} \quad \log_2\left(1 + 1/f(p_u)\right) \tag{12}$$

$$\text{s.t. } 0 \leq p_u \leq P/2N, \tag{13}$$

where $f(p_u) \triangleq a + \frac{b}{p_u} + \frac{c}{P-2Np_u}$ with $a \triangleq 4(N-1) + \frac{4N\tilde{\sigma}^2}{\sigma^2}$, $b \triangleq \frac{1}{\sigma^2}$, and $c \triangleq \frac{2N}{\sigma^2}$. Since $f''(x) = \frac{2b}{x^3} + \frac{8N^2c}{(P-2Nx)^3} \geq 0$, $f(p_u)$ is a convex function in $0 \leq p_u \leq P$. Therefore, solving $f'(x) = 0$ yields the desired results. □

Theorem 3 indicates that, for a given power budget $2Np_u + p_r = P$, half of the total power should be allocated to the relay regardless of the number of users, and the remaining half is equally allocated to the $2N$ users. Such a symmetric power allocation strategy is rather intuitive due to the symmetric system setup. In addition, it can be directly inferred that the transmit power of each user decreases monotonically as the number of users increases, which serves as a useful guideline for practical system design.

6. NUMERICAL RESULTS

For all illustrative examples, we assume that the large scale fading coefficients are $\beta_{AR} = \beta_{RB} = 1$, and choose the length of the coherence interval to be $\tau_c = 196$ (symbols), and the training length $\tau_p = 2N$. Furthermore, we define SNR $\triangleq p_u$. Also, the transmit powers E_u, E_p, E_r, p_p, and p_r are normalized to "dB" scale.

Fig. 1 shows the achievable sum spectral efficiency for different M. The "Approximations" curves are plotted according to (7), and the "Numerical results" curves are generated via Monte-Carlo simulations by averaging over 10^4 independent trials. As can be readily observed, the large-system approximations are always overestimating the performance in these simulations, but they are very accurate, especially for large antenna arrays. Moreover, increasing the number of relay antennas brings significant spectral efficiency improvement, as expected.

Fig. 2 illustrates the power scaling laws. First, focusing on the three curves on the top of the figure, we observe that the sum spectral efficiency converges to non-zero limits, as predicted by Corollaries 1–3. Moreover, the gaps between these three curves and the "Corollaries 2 and 3" curve are small, indicating the tightness of the approximations given by Corollaries 2–3.[2] Second, observing the lower set of curves, we can see that the sum spectral efficiency gradually reduces to zero in all parameter settings, i.e., $\alpha + \gamma > 1$, $\beta + \gamma < 1$, as well as $\alpha + \gamma > 1$, $\beta + \gamma > 1$, confirming the analysis in Theorem 2. In addition, when we cut down the transmit powers of each user, of the relay, and of each pilot symbol too much, i.e., $\alpha = 0.9$, $\beta = 0.8$, $\gamma = 0.8$, the sum spectral efficiency is very low.

Fig. 3 first studies the impact of N on SNR. As can be readily observed, the optimal transmit power of each user decreases with the number of user pairs N, as predicted by Theorem 3. To illustrate the benefit of power allocation policy, for $N = 5$, we plot the achievable spectral efficiency with the optimal user transmit power, namely $p_u = 6.99$ dB, against two arbitrary choose transmit power level, $p_u = 9.9$ dB and $p_u = -10$ dB. As can be observed, the optimal power allocation policy provides significant spectral efficiency enhancement, compared to the case without power allocation. In addition, this improvement is more prominent when the number of relay antennas becomes larger.

7. CONCLUSION

This paper investigated the performance of a multi-pair two-way AF relaying system with imperfect CSI. We derived a closed-form large-system approximation of the achievable spectral efficiency that is tight as $M \to \infty$, based on which, the power scaling laws were characterized. The outcome reveals some fundamental trade-offs between p_u/p_r and p_p. Finally, for a given power budget, the transmit powers of each user and of the relay were optimized to maximize the sum spectral efficiency.

[2]Note that the sum spectral efficiency of Corollary 2 and Corollary 3 are the same since we assume the large scale fading is the same for all links, and set $E_u = 10$ dB, $E_r = 20$ dB which results in $E_r = 2NE_u$.

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