A PENALTY–BSUM APPROACH FOR RATE OPTIMIZATION IN FULL–DUPLEX MIMO RELAY NETWORKS WITH RELAY PROCESSING DELAY

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ABSTRACT

This paper studies joint source transmit beamforming and relay amplification matrix design to achieve rate maximization for full-duplex (FD) MIMO amplify-and-forward (AF) relay systems with consideration of relay processing delay (RPD). The problem is difficult to solve due mainly to the self-interference constraint induced by the RPD. In this paper, we first propose a penalty-based algorithmic framework, called P-BSUM, for a class of constrained optimization problems with difficult equality constraints in addition to some convex constraints. We then apply the P-BSUM algorithm to the rate maximization problem and obtain a simple iterative algorithm. Finally, numerical results illustrate the efficiency of the proposed algorithm.

***Index Terms*—** Full-duplex relaying, MIMO, joint source-relay design, penalty method, BSUM.

1. INTRODUCTION

Since the multi-antenna technology can not only greatly improve the spectral efficiency of single antenna full-duplex relaying systems but also provide more degree of freedom for suppressing the self-interference (SI) in the spatial domain [1, 2], it is natural to combine the MIMO and FD relaying technologies to achieve higher spectral efficiency, leading to FD MIMO relaying. Recently, FD MIMO relaying has gained a lot of research interest, e.g., [1–12].

It is noted that most of the existing works [2–10] on FD MIMO relaying have assumed zero *relay processing delay* (RPD). However, the RPD is strictly positive in practice and neglecting it would cause severe causality issues in the practical implementation of relaying protocols (see [2, 13] for more discussion on the consequences of neglecting the RPD). Hence, the RPD should be taken into consideration in FD relay system design. In [11], the authors considered the RPD in *single-stream* DF MIMO AF relay systems and proposed low-omplexity joint precoding/decoding schemes to optimize the end-to-end performance. In addition, the work [12] studied the end-to-end performance optimization for *two-way* FD relay systems with processing delay, where all three nodes work in FD mode and only the relay is equipped with multiple antennas.

In this paper, as in [11], we consider a three-node FD MIMO AF relay system which consists of a multi-antenna source, a multi-antenna FD relay, and a multi-antenna destination. We extend the work [11] to the more general *multi-stream* scenario and study joint source-relay design, i.e., jointly design the source transmit beamforming $\mathbf{V}$ and relay amplification matrix $\mathbf{Q}$, to optimize the end-to-end achievable rate with the consideration of the RPD. As compared to the single-stream case in [11], the rate maximization problem in the multi-stream case is much more challenging due mainly to the difficult SI constraint $\mathbf{Q}\mathbf{H}_{RR}\mathbf{Q} = \mathbf{0}$, where $\mathbf{H}_{RR}$ is the so-called self-interference channel between the relay output and the relay input. Note that, *although the works [11, 12] have also considered the SI constraint, they assumed that the amplification matrix is of rank one to make the problem more tractable.* This often results in loss of rate performance for the multi-stream case. Hence, it is necessary to consider algorithmic design to deal with the complicated SI constraint.

To address the rate maximization problem in the multi-stream case, we first develop a penalty-based two-tier iterative optimization approach, where a penalized problem is locally solved in the inner tier using block successive upper-bound maximization/minimization (BSUM) algorithm [16] while a penalty parameter is adjusted in the outer tier so that the penalized terms gradually approach to zero. We name the proposed algorithm as penalty-BSUM (P-BSUM) algorithm. The P-BSUM algorithm is suitable for a class of constrained optimization problem with difficult equality constraints. Then, by introducing a set of auxiliary variables, we rewrite the rate maximization problem as an equivalent one that fits into the P-BSUM algorithmic framework, and accordingly propose a P-BSUM-based iterative algorithm for the rate maximization problem at hand. The benefits of the P-BSUM-based joint source-relay design over the joint design under the assumption of rank one relay amplification matrix is demonstrated using numerical examples.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a three-node full duplex MIMO relay network where a source wants to send information to a destination with the aid of a full-duplex relay. It is assumed that the source and destination are equipped with N_S and N_D antennas, while the relay is equipped with N_T transmit antennas and N_R receive antennas to enable full-duplex operation. Let $\mathbf{H}_{SR} \in \mathbb{C}^{N_R \times N_S}$ denote the channel between the source and relay, and $\mathbf{H}_{RD} \in \mathbb{C}^{N_D \times N_R}$ denote the channel between the relay and destination. In addition, let $\mathbf{H}_{RR} \in \mathbb{C}^{N_R \times N_T}$ represent the residual self-interference (SI) channel after the SI cancellation scheme is performed at the relay. We assume that all channels are subject to independent Rayleigh block-fading, i.e., they stay constant during one fading block but change independently at the beginning of the next fading block according to Rayleigh distribution.

The processing time is required at the relay to implement the FD operation. This results in relay processing delay, which we assume is given by a τ-symbol duration. Typically, the delay is much shorter than a time slot which consists of a large number of data symbols. Therefore, its effect on the achievable rate is negligible [12]. Suppose that linear processing is employed at the source and at the relay to enhance the system performance. Specifically, the source use beamforming matrix $\mathbf{V} \in \mathbb{C}^{N_S \times d}$ to send its signal while the relay uses the amplification matrix $\mathbf{Q}$ (i.e., AF relay protocol) to process its received signal. Hence, at the time instant n, the received signal $\boldsymbol{r}[n] \in \mathbb{C}^{N_R \times 1}$ at the relay is

$$\boldsymbol{r}[n] = \mathbf{H}_{SR}\mathbf{V}\boldsymbol{s}[n] + \mathbf{H}_{RR}\boldsymbol{x}_R[n] + \boldsymbol{n}_R[n] \tag{1}$$

where[1] $\boldsymbol{s}[n] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_d)$ is a vector of d transmit symbols, $\boldsymbol{n}_R[n] \sim \mathcal{CN}(0, \sigma_R^2\mathbf{I})$ denotes the complex additive white Gaussian noise (AWGN), and the term $\mathbf{H}_{RR}\boldsymbol{x}_R[n]$ represents the residual SI from the relay output to relay input. And the transmit signal $\boldsymbol{x}_R[n]$ at the relay is

$$\boldsymbol{x}_R[n] = \mathbf{Q}\boldsymbol{r}[n-\tau] \tag{2}$$

Combining (1) with (2), the relay output can be rewritten as

$$\begin{aligned}\boldsymbol{x}_R[n] &= \mathbf{Q}\mathbf{H}_{SR}\mathbf{V}\boldsymbol{s}[n-\tau] + \mathbf{Q}\mathbf{H}_{RR}\boldsymbol{x}_R[n-\tau] \\ &\quad + \mathbf{Q}\boldsymbol{n}_R[n-\tau] \\ &= \mathbf{Q}\mathbf{H}_{SR}\mathbf{V}\boldsymbol{s}[n-\tau] + \mathbf{Q}\mathbf{H}_{RR}\mathbf{Q}\boldsymbol{r}[n-2\tau] \\ &\quad + \mathbf{Q}\boldsymbol{n}_R[n-\tau]\end{aligned} \tag{3}$$

The term $\mathbf{Q}\mathbf{H}_{RR}\mathbf{Q}\boldsymbol{r}[n-2\tau]$ in (3) is a complicated function of $\mathbf{Q}$ and makes the system design very challenging. To simplify design, we impose a zero-forcing condition on $\mathbf{Q}$ to null out the residual SI from the relay output to relay input, i.e.,

$$\mathbf{Q}\mathbf{H}_{RR}\mathbf{Q} = 0 \tag{4}$$

[1] $\mathcal{CN}(\mathbf{0}, \mathbf{A})$ denotes circularly symmetric complex Gaussian distribution with zero mean and covariance matrix $\mathbf{A}$.

which is referred to as SI constraint. Plugging (4) into (3), we obtain

$$\boldsymbol{x}_R[n] = \mathbf{Q}\mathbf{H}_{SR}\mathbf{V}\boldsymbol{s}[n-\tau] + \mathbf{Q}\boldsymbol{n}_R[n-\tau]. \tag{5}$$

Consequently, the received signal at the destination is

$$\begin{aligned}\boldsymbol{y}_D[n] &= \mathbf{H}_{RD}\boldsymbol{x}_R[n] + \boldsymbol{n}_D[n] \\ &= \mathbf{H}_{RD}\left(\mathbf{Q}\mathbf{H}_{SR}\mathbf{V}\boldsymbol{s}[n-\tau] + \mathbf{Q}\boldsymbol{n}_R[n-\tau]\right) + \boldsymbol{n}_D[n]\end{aligned} \tag{6}$$

where $\boldsymbol{n}_D[n] \sim \mathcal{CN}(0, \sigma_D^2\mathbf{I})$ denotes the complex AWGN.

According to (6), the system rate can be expressed as

$$R(\mathbf{V},\mathbf{Q}) = \log\det\Bigg(\mathbf{I} + \mathbf{H}_{RD}\mathbf{Q}\mathbf{H}_{SR}\mathbf{V}\mathbf{V}^H\mathbf{H}_{SR}^H\mathbf{Q}^H\mathbf{H}_{RD}^H \times \left(\sigma_R^2\mathbf{H}_{RD}\mathbf{Q}\mathbf{Q}^H\mathbf{H}_{RD}^H + \sigma_D^2\mathbf{I}\right)^{-1}\Bigg). \tag{7}$$

The power consumption at the relay is given by

$$p_R(\mathbf{V},\mathbf{Q}) = \mathrm{Tr}\left(\mathbf{Q}\mathbf{H}_{SR}\mathbf{V}\mathbf{V}^H\mathbf{H}_{SR}^H\mathbf{Q}^H\right) + \sigma_R^2\mathrm{Tr}\left(\mathbf{Q}\mathbf{Q}^H\right) \tag{8}$$

and the power consumption at the source is $\mathrm{Tr}(\mathbf{V}\mathbf{V}^H)$.

In this paper, we are interested in joint source-relay design to optimize the system rate subject to the source/relay power constraints and the SI constraint. Mathematically, the rate maximization problem can be formulated as

$$\begin{aligned}&\max_{\mathbf{V},\mathbf{Q}} R(\mathbf{V},\mathbf{Q}) \\ &\text{s.t. } p_R(\mathbf{V},\mathbf{Q}) \le P_R, \\ &\quad \mathbf{Q}\mathbf{H}_{RR}\mathbf{Q} = 0, \\ &\quad \mathrm{Tr}(\mathbf{V}\mathbf{V}^H) \le P_S.\end{aligned} \tag{9}$$

where P_S and P_R are the allowed maximum transmission power at the souce and relay, respectively. Problem (9) is nonconvex and complicated mainly by the SI constraint. This paper aims to provide a systematic method to tackle the difficulty arising from the SI constraint.

3. PENALTY-BSUM METHOD AND ITS APPLICATION TO PROBLEM (9)

In this section, we first propose a penalty-based algorithmic framework for a class of optimization problems which have difficult equality constraints in addition to some convex inequality constraints. Then the proposed optimization framework is used to address problem (9).

3.1. Penalty-BSUM method

Consider the problem

$$\begin{aligned}(P) \qquad &\min_{\boldsymbol{x}} f(\boldsymbol{x}) \\ &\text{s.t. } \boldsymbol{h}(\boldsymbol{x}) = \mathbf{0}, \\ &\quad \boldsymbol{x} \in \mathcal{X}\end{aligned} \tag{10}$$

Table 1. Algorithm 1: P-BSUM method for problem (13)

0. initialize $\boldsymbol{x}^0 \in \mathcal{X}$, $\varrho_0 > 0$, and set $c > 1, k = 0$
1. **repeat**
2. $\boldsymbol{x}^{k+1} = \text{BSUM}(P_{\varrho_k}, \tilde{f}_{\varrho_k}, \boldsymbol{x}^k)$
3. $\varrho_{k+1} = c\varrho_k$
4. $k = k + 1$
5. **until** some termination criterion is met

where $f(\boldsymbol{x})$ is a scalar continuously differentiable function and $\boldsymbol{h}(\boldsymbol{x}) \in \mathbb{R}^{p\times 1}$ is a vector of p continuously differentiable functions; the feasible set $\mathcal{X}$ is the Cartesian product of n closed convex sets: $\mathcal{X} \triangleq \mathcal{X}_1 \times \mathcal{X}_2 \times \ldots \times \mathcal{X}_n$ with $\mathcal{X}_i \in \mathbb{R}^{m_i}$ and $\sum_{i=1}^{n} m_i = m$ and accordingly the optimization variable $\boldsymbol{x} \in \mathbb{R}^m$ can be decomposed as $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n)$ with $\boldsymbol{x}_i \in \mathcal{X}_i$ $i = 1, 2, \ldots, n$.

When the equality constraints are very difficult to handle, we can use the penalty method [17] to tackle problem (10), i.e., solving the penalized problem

$$(P_\varrho) \qquad \min_{\boldsymbol{x}} f_\varrho(\boldsymbol{x}) \triangleq f(\boldsymbol{x}) + \frac{\varrho}{2}\|\boldsymbol{h}(\boldsymbol{x})\|^2 \quad \text{s.t. } \boldsymbol{x} \in \mathcal{X}. \tag{11}$$

where ϱ is a scalar penalty parameter that prescribes a high cost for violation of the constraints. In particular, when $\varrho \to \infty$, solving the above problem yields an approximately optimum solution to problem (10) [17]. However, it is still difficult to globally solve problem (P_ϱ) when $f(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ are nonconvex functions. So an interesting question is: can we reach at least a stationary point of problem (P) by solving a sequence of problem (P_ϱ) to stationary points? This motivate us to propose the P-BSUM method.

The penalty-BSUM algorithm is summarized in Table 1, where $\text{BSUM}(P_{\varrho_k}, \tilde{f}_{\varrho_k}, \boldsymbol{x}^k)$ means that, starting from $\boldsymbol{x}^k$, the BSUM algorithm [16] is invoked to iteratively solve problem P_{ϱ_k} with a locally tight lower bound function $\tilde{f}_{\varrho_k}$ of $f(\boldsymbol{x})$. The penalty-BSUM algorithm is inspired by the penalty decomposition (PD) method which was proposed in [14, 15] for general rank minimization/sparse approximation problems, where each penalized subproblem is solved by a block coordinate descent method [17]. Different from the PD method, the penalized problem (P_ϱ) is iteratively solved using the block successive upper-bound minimization method [16] in the P-BSUM algorithm. Let $\mathcal{P}_{\mathcal{X}}\{\boldsymbol{x}\}$ denote the projection of $\boldsymbol{x}$ onto the convex set $\mathcal{X}$. The following proposition shows that any limit point of the sequence generated by the P-BSUM algorithm satisfies the first-order optimality condition of problem (P), hence a stationary point of problem (P).

Theorem 3.1. *Let $\{\boldsymbol{x}^k\}$ be the sequence generated by Algorithm 1 where the termination condition for the BSUM algorithm is $\left\|\mathcal{P}_{\mathcal{X}}\{\boldsymbol{x}^k - \nabla f_{\varrho_k}(\boldsymbol{x}^k)\} - \boldsymbol{x}^k\right\| \le \epsilon_k, \forall k$ with $\epsilon_k \to 0$ as $k \to \infty$. Suppose that $\boldsymbol{x}^*$ is a limit point of the sequence $\{\boldsymbol{x}^k\}$ and $\nabla f(\boldsymbol{x}^*)$ is bounded. In addition, assume that* Robinson condition *[15, 18] holds for problem (P) at $\boldsymbol{x}^*$, i.e., $\{\nabla \boldsymbol{h}(\boldsymbol{x}^*)\boldsymbol{d}_{\boldsymbol{x}} : \boldsymbol{d}_{\boldsymbol{x}} \in \mathcal{T}_{\mathcal{X}}(\boldsymbol{x}^*)\} = \mathbb{R}^p$ where $\mathcal{T}_{\mathcal{X}}(\boldsymbol{x}^*)$ denotes the tangent cone of $\mathcal{X}$ at $\boldsymbol{x}^*$. Then $\boldsymbol{x}^*$ is a stationary point of problem (P).*

Proof. The details of the proof are omitted due to space limitation. The key to the proof is to show that the sequence of $\boldsymbol{\mu}^k \triangleq \varrho_k \boldsymbol{h}(\boldsymbol{x}_k)$ is bounded in terms of the termination condition of the BSUM algorithm with $\epsilon_k \to 0$ and under Robinson's condition, and thus it has a convergent subsequence whose limit is a Lagrange multiplier for the equality constraints. With the boundedness of $\boldsymbol{\mu}^k$, we have $\boldsymbol{h}(\boldsymbol{x}_k) \to \boldsymbol{0}$ as $\varrho_k \to \infty$. □

3.2. The P-BSUM algorithm for (9)

Now we are ready to address problem (9) by using the P-BSUM algorithm. To efficiently make use of the BSUM algorithm, we introduce a set of auxiliary matrix variables $\{\mathbf{S}, \tilde{\mathbf{S}}, \tilde{\mathbf{V}}, \tilde{\mathbf{Q}}, \mathbf{R}\}$. Define $\mathcal{Y} \triangleq \{\mathbf{Q}, \mathbf{V}, \mathbf{S}, \tilde{\mathbf{S}}, \tilde{\mathbf{V}}, \tilde{\mathbf{Q}}, \mathbf{R}\}$. Then we can rewrite problem (9) equivalently as

$$\begin{aligned}
&\max_{\mathcal{Y}} \log\det\Bigg(\mathbf{I} + \mathbf{H}_{RD}\mathbf{S}\mathbf{S}^H\mathbf{H}_{RD}^H \times \\
&\qquad \left(\sigma_R^2\mathbf{H}_{RD}\mathbf{Q}\mathbf{Q}^H\mathbf{H}_{RD}^H + \sigma_D^2\mathbf{I}\right)^{-1}\Bigg) \\
&\text{s.t. } \mathrm{Tr}\left(\tilde{\mathbf{S}}\tilde{\mathbf{S}}^H\right) + \mathrm{Tr}\left(\tilde{\mathbf{Q}}\tilde{\mathbf{Q}}^H\right) \le P_R, \\
&\quad \mathrm{Tr}(\mathbf{V}\mathbf{V}^H) \le P_S, \\
&\quad \mathbf{Q}\mathbf{H}_{SR}\tilde{\mathbf{V}} = \tilde{\mathbf{S}}, \mathbf{R}^H\mathbf{Q} = 0, \mathbf{R}^H = \mathbf{Q}\mathbf{H}_{RR}, \\
&\quad \mathbf{S} = \tilde{\mathbf{S}}, \sigma_R\mathbf{Q} = \tilde{\mathbf{Q}}, \mathbf{V} = \tilde{\mathbf{V}}
\end{aligned} \tag{12}$$

Problem (12) falls into the class of problem (P). Thus, it can be addressed using the PBSUM algorithm. Specifically, by penalizing all the equality constraints of the above problem, we get a penalized version of problem (12) as follows

$$\begin{aligned}
&\max_{\mathcal{Y}} \log\det\Bigg(\mathbf{I} + \mathbf{H}_{RD}\mathbf{S}\mathbf{S}^H\mathbf{H}_{RD}^H \times \\
&\qquad \left(\sigma_R^2\mathbf{H}_{RD}\mathbf{Q}\mathbf{Q}^H\mathbf{H}_{RD}^H + \sigma_D^2\mathbf{I}\right)^{-1}\Bigg) \\
&\quad - \varrho\Bigg(\|\sigma_R\mathbf{Q} - \tilde{\mathbf{Q}}\|^2 + \|\mathbf{S} - \tilde{\mathbf{S}}\|^2 + \|\mathbf{V} - \tilde{\mathbf{V}}\|^2 \\
&\quad + \|\mathbf{R}^H\mathbf{Q}\|^2 + \|\mathbf{R}^H - \mathbf{Q}\mathbf{H}_{RR}\|^2 + \|\mathbf{Q}\mathbf{H}_{SR}\tilde{\mathbf{V}} - \tilde{\mathbf{S}}\|^2\Bigg) \\
&\text{s.t. } \mathrm{Tr}\left(\tilde{\mathbf{S}}\tilde{\mathbf{S}}^H\right) + \mathrm{Tr}\left(\tilde{\mathbf{Q}}\tilde{\mathbf{Q}}^H\right) \le P_R \\
&\quad \mathrm{Tr}(\mathbf{V}\mathbf{V}^H) \le P_S
\end{aligned} \tag{13}$$

where ϱ is a scalar penalty parameter.

From Table 1, it is known that, the key step of the PBSUM algorithm for problem (9) is to apply the BSUM algorithm to (13). In the following, we show how to address problem (13) using the BSUM algorithm. The basic idea behind the BSUM algorithm for a maximization (resp., minimization) problem is to successively maximize a locally tight lower (resp., upper) bound of the objective, finally reaching a stationary point of the problem [16]. Hence, the key to the BSUM algorithm applied to (13) is to find a locally tight lower bound for the objective of problem (13). For ease of exposition, we define

$$\tilde{R}(\mathbf{S},\mathbf{Q}) \triangleq \log\det\Bigg(\mathbf{I}+\mathbf{H}_{RD}\mathbf{S}\mathbf{S}^H\mathbf{H}_{RD}^H\times \left(\sigma_R^2\mathbf{H}_{RD}\mathbf{Q}\mathbf{Q}^H\mathbf{H}_{RD}^H+\sigma_D^2\mathbf{I}\right)^{-1}\Bigg), \quad (14)$$

$$\mathbb{E}(\mathbf{U},\mathbf{S},\mathbf{Q}) \triangleq \left(\mathbf{I}-\mathbf{U}^H\mathbf{H}_{RD}\mathbf{S}\right)\left(\mathbf{I}-\mathbf{U}^H\mathbf{H}_{RD}\mathbf{S}\right)^H + \sigma_R^2\mathbf{U}^H\mathbf{H}_{RD}\mathbf{Q}\mathbf{Q}^H\mathbf{H}_{RD}^H\mathbf{U}+\sigma_D^2\mathbf{U}^H\mathbf{U}. \quad (15)$$

Then, by applying the popular WMMSE algorithmic framework [19], we can obtain a locally tight lower bound of $R(\mathbf{S},\mathbf{Q})$ as follows

$$\tilde{R}(\mathbf{S},\mathbf{Q}) = \max_{\mathbf{W},\mathbf{U}} \log\det(\mathbf{W}) - \mathrm{Tr}(\mathbf{W}\mathbb{E}(\mathbf{U},\mathbf{S},\mathbf{Q})) + d$$
$$\geq \log\det(\bar{\mathbf{W}}) - \mathrm{Tr}(\bar{\mathbf{W}}\mathbb{E}(\bar{\mathbf{U}},\mathbf{S},\mathbf{Q})) + d, \forall \mathbf{Q},\mathbf{S},\bar{\mathbf{Q}},\bar{\mathbf{S}}.$$

where

$$\bar{\mathbf{U}} = \left(\sigma_R^2\mathbf{H}_{RD}\bar{\mathbf{Q}}\bar{\mathbf{Q}}^H\mathbf{H}_{RD}^H+\sigma_D^2\mathbf{I}\right)^{-1}\mathbf{H}_{RD}\bar{\mathbf{S}}, \quad (16)$$

$$\bar{\mathbf{W}} = (\mathbf{I}-\bar{\mathbf{U}}^H\mathbf{H}_{RD}\bar{\mathbf{S}})^{-1}. \quad (17)$$

Using the above result, we can obtain a locally tight lower bound for the objective of problem (13), i.e.,

$$\log\det(\bar{\mathbf{W}}) - E_\varrho(\mathcal{Y}) + d$$

where

$$E_\rho(\mathcal{Y}) \triangleq \mathrm{Tr}(\bar{\mathbf{W}}\mathbb{E}(\bar{\mathbf{U}},\mathbf{S},\mathbf{Q})) + \varrho\Bigg(\|\sigma_R\mathbf{Q}-\tilde{\mathbf{Q}}\|^2+\|\mathbf{S}-\tilde{\mathbf{S}}\|^2+\|\mathbf{V}-\tilde{\mathbf{V}}\|^2 + \|\mathbf{R}^H\mathbf{Q}\|^2+\|\mathbf{R}^H-\mathbf{Q}\mathbf{H}_{RR}\|^2+\|\mathbf{Q}\mathbf{H}_{SR}\tilde{\mathbf{V}}-\tilde{\mathbf{S}}\|^2\Bigg). \quad (18)$$

The BSUM algorithm successively maximizes this lower bound with respect to one block of variables while fixing the others, equivalently, solve the following problem in a block coordinate descent fashion

$$\min_{\mathcal{Y}} E_\varrho(\mathcal{Y})$$
$$\text{s.t. } \mathrm{Tr}\left(\tilde{\mathbf{S}}\tilde{\mathbf{S}}^H\right)+\mathrm{Tr}\left(\tilde{\mathbf{Q}}\tilde{\mathbf{Q}}^H\right)\leq P_R, \quad (19)$$
$$\mathrm{Tr}(\mathbf{V}\mathbf{V}^H)\leq P_S,$$

leading to a set of subproblems which allows closed-form solutions.The details are omitted due to space limitation.

4. NUMERICAL EXAMPLES

We present numerical results to illustrate the rate performance of the proposed joint source-relay optimization algorithms. In our numerical examples, it is assumed that there are N_{SD} antennas at both the source and destination, i.e., $N_S = N_D = N_{SD}$, and N_{TR} transmit/receive antennas at the relay, i.e., $N_T = N_R = N_{TR}$. Unless otherwise specified, we set $N_{SD} = N_{TR} = d = 5$, $\sigma_R^2 = \sigma_D^2 = 1$, and $P_S = P_R = P$. Moreover, we set $c = 2$ and $\varrho_0 = 0.01$ for the P-BSUM algorithm. We assume that the source-relay and relay-destination channels experience independent Rayleigh flat fading. Furthermore, each element of the residual SI channel $\mathbf{H}_{RR}$ is modeled as a complex Gaussian distributed random variable with zero mean and variance -20 dB. The simulation results are averaged over 100 independent channel realizations.

Figure 1 illustrates the average system rate versus the SNR which is defined as $10\log_{10}(P)$. For comparison, we also provide the rate performance of the joint design approach under the assumption of rank one amplification matrix [11]. It is observed that the P-BSUM-based joint source-relay design approach could be significantly better than the joint source-relay design approach under the rank one assumption (denoted as 'Rank-1 method' in the plot) in the high SNR region.

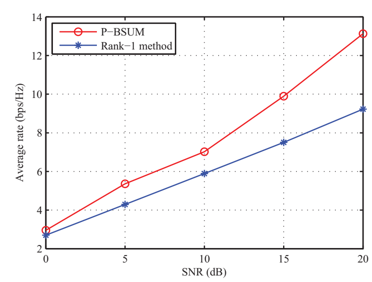

Fig. 1. The average system rate versus the SNR.

5. CONCLUSION

This paper have considered joint source-relay design for rate maximization in FD MIMO AF relay systems with consideration of relay processing delay. A simple algorithmic framework P-BSUM has been proposed to address the difficulty arising from the self-interference constraint. We remark that the P-BSUM algorithm can be used to deal with problems with difficult coupling constraints.

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7. REFERENCES

[1] D. Kim, H. Lee, and D. Hong, "A survey of in-band full-duplex transmission: from the perspective of PHY and MAC layers," *IEEE Commun. Surv. & Turotials*, early acess, 2015.

[2] T. Riihonen, S. Werner, and R. Wichman, "Mitigation of loopback selfinterference in full-duplex MIMO relays," *IEEE Trans. Signal Process.*, vol. 59, pp. 5983-5993, Dec. 2011.

[3] P. Lioliou, M. Viberg, M. Coldrey, and F. Athley, "Self-interference suppression in full-duplex MIMO relays," in *Proc. 44th Asilomar Signals, Systems and Computer-s Conference*, Pacific Grove, CA, November 2010, pp. 658-662.

[4] E. Antonio-Rodrigez, R. Lopez-Valacarce, T. Riihonen, S. Werner, and R. Wichman, "SINR optimization in wide-band full-duplex MIMO relays under limited dynamic range," in *Proc. IEEE Sensor Array and Multichannel Signal Process. Workshop (SAM)*, Jun. 2014.

[5] ——, "Subspace-constrained SINR optimization in MI-MO full-duplex relays under limited dynamic range," in *Proc. IEEE SPAWC*, June 2015.

[6] Y. Y. Kang and J. H. Cho, "Capacity of MIMO wireless channel with full-duplex amplify-and-forward relay," in *Proc. IEEE PIMRC*, pp. 117-121, Sept. 2009.

[7] J. Zhang, O. Taghizadeh and M. Haardt "Joint source and relay precoding design for one-way full-duplex MIMO relaying systems," in *Proc. 10th Int. Symp. Wireless Commun. Syst.*, pp.1-5, 2013.

[8] D. Choi and D. Park, "Effective self-interference cancellation in full duplex relay systems," *Electron. Lett.*, vol. 48, no. 2, pp. 129-130, Jan. 2012.

[9] B. Chun and H. Park, "A spatial-domain joint-nulling method of self interference in full-duplex relays," *IEEE Commun. Lett.*, vol. 16, no. 4, pp. 436-438, Apr. 2012.

[10] B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-duplex MIMO relaying: achievable rates under limited dynamic range," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 8, pp. 1541-1553, Sep. 2012.

[11] H. A. Suraweera, I. Krikidis, G. Zheng, C. Yuen, and P. J. Smith, "Low complexity end-to-end performance optimization in MIMO full-duplex relay systems," *IEEE Trans. Wireless Commun.*, vol.13, no.2, pp. 913-927, Feb. 2014.

[12] G. Zheng, "Joint beamforming optimization and power control for full-duplex MIMO two-Way relay channel," *IEEE Trans. Signal Process.*, vol. 63, no. 3, Feb. 2015.

[13] T. Riihonen, S. Werner, and R. Wichman, "Spatial loop interference suppression in full-duplexMIMO relays," in *Proc. 43rd Ann. Asilomar Conf. Signals, Syst. Comput.*, Nov. 2009.

[14] Z. Lu, Y. Zhang "Sparse approximation via penalty decomposition methods," *SIAM J. Optim.*, vol. 23, no. 4, pp. 2448-2478, 2013.

[15] Z. Lu and Y. Zhang, "Penalty decomposition methods for rank minimization," to appear in *Optimization Methods and Software*, 2015.

[16] M. Razaviyayn, M. Hong, and Z.-Q. Luo, "A unified convergence analysis of block successive minimization methods for nonsmooth optimization," *SIAM Journal on Optimization*, vol. 23, no. 2, pp. 1126-1153, 2013.

[17] D. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA: Athena Scientific, 1999.

[18] A. Ruszczynski, *Nonlinear optimization*, Princeton University Press, New Jersey, 2006.

[19] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331-4340, Sep. 2011.